BITS PILANI, DUBAI CAMPUS
DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI
II YEAR – I SEMESTER 2012-13
DISCRETE STRUCTURES FOR COMPUTER SCIENCE (CS F222)
COMPREHENSIVE EXAMINATION (Closed Book)

Time: 3 Hours | Max. Marks: 40

Date: December 31, 2012 | Weightage: 40%

Answer all questions

1. Let A, B, C be three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$. **[2]**

2. Let R be the relation defined by $R = \{(x, y) / x \in N, y \in N, x + 2y = 10\}$. Find the domain, range of R and also find R^{-1}. **[2]**

3. In the set of all integers the relation R is defined as $a\ R\ b$ means $a \equiv b \pmod{m}$. Is R an equivalence relation? Justify your answer. **[3]**

4. Use mathematical induction to prove that $3n^5 + 5n^3 + 7n$ is divisible by 15 for each positive integer n. **[2]**

5. Use strong mathematical induction to prove that for each positive integer n, the n^{th} Fibonacci number F_n is less than $(7/4)^n$. **[2]**

6. Find the coefficient of x^{16} in $\left(1 + x^4 + x^8\right)^{10}$. **[2]**

7. Calculate $B(x) = \sum_{r=0}^{\infty} b_r x^r = \dfrac{1}{x^2 - 5x + 6}$. **[2]**

8. Solve the inhomogeneous recurrence relation $a_n + 3a_{n-1} - 10a_{n-2} = n^2 + n + 5$ using the method of undetermined coefficients. **[3]**

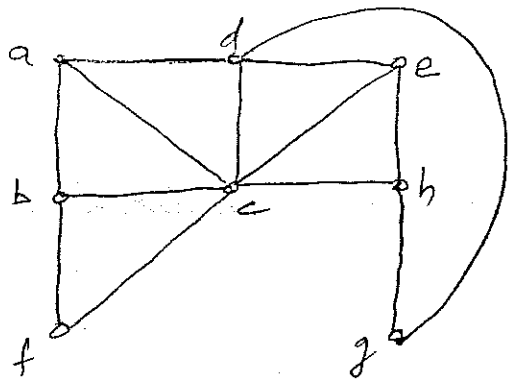
9. Draw a poset diagram for $(\{2, 3, 4, 9, 12, 18\}; /)$ and determine all maximal and minimal elements and greatest and least elements if they exist. **[2]**

10. Let $(A, \leq)$ be a lattice. Then prove $\forall a, b, c \in A, (a \wedge b) \wedge c = a \wedge (b \wedge c)$ and $a \wedge (a \vee b) = a$. **[3]**

11. What is the largest possible number of vertices in a graph with 35 edges and all vertices of degree at least 3? **[2]**

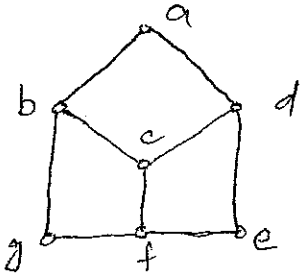
12. Prove that a graph G is a tree if and only if G has no cycles and $|E| = |V| - 1$. **[2]**

13. For the graph below find the spanning tree using breadth-first search. **[3]**



14. If G is a connected plane graph then prove that $|V|-|E|+|R|=2$. **[2]**

15. Is there a Hamiltonian cycle in the following graph? Justify your answer. **[2]**



16. Prove that every group of prime order is cyclic. **[2]**

17. Is $(R^*,\bullet)$ is isomorphic to $(R,+)$? Justify your answer. **[2]**

18. Let $(R,+,\bullet)$ be a ring. Prove that $a\bullet 0=0\bullet a=0$ for every $a\in R$. **[2]**

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II YEAR – I SEMESTER 2012-13

DISCRETE STRUCTURES FOR COMPUTER SCIENCE (CS F222)

TEST – II (Open Book)

Time: 50 Minutes | Max. Marks: 20

Date: November 18, 2012 | Weightage: 20%

Answer all questions

1. If $A_1, A_2, \cdots A_n$ are any *n* sets, show by mathematical induction that $\overline{\bigcup_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} \overline{A_i}$ **[3]**

2. A jigsaw puzzle consists of a number of pieces. Two or more pieces with matched boundaries can be put together to form a big piece. Block refers to either a single piece or a number of pieces with matched boundaries that are put together to form a big piece. Thus blocks with matched boundaries can be put together to form another block. Finally when all pieces are put together as one single block, the jigsaw puzzle is solved. Use the principle of strong mathematical induction to prove that for a jigsaw puzzle with *n* pieces, it will always take $n-1$ moves to solve the puzzle. **[3]**

3. (i) Suppose that we know the degrees of the vertices of a nondirected graph *G*. Is is possible to determine the order and size of *G*? Explain.

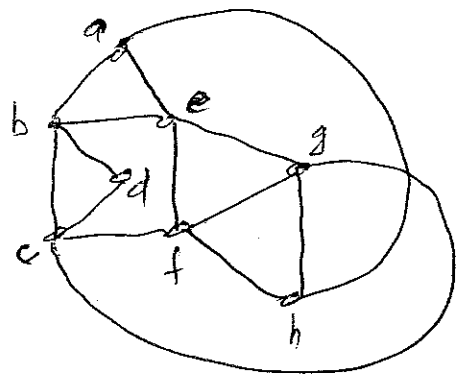
   (ii) Let *G* be a (p,q) graph all of whose points have degree k or $k+1$. If *G* has $t>0$ points of degree k, show that $t = p(k+1)-2q$. **[4]**

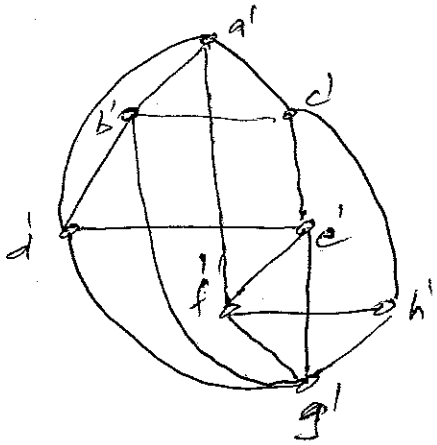
4. Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group. **[3]**

5. Check whether the sequence (6, 6, 5, 4, 3, 3, 1) is graphic? Justify your answer. **[2]**

6. The length of the longest simple path between two distinct vertices in a graph *G* is called the diameter of *G*. The length of the shortest cycle in *G* is the girth of *G* while the length of the longest cycle is the circumference of *G*. Find the diameter, girth and circumference of the graph $K_{3,3}$. **[2]**

7. Is the following pairs of nondirected graphs are isomorphic? Justify your answer. **[3]**





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DISCRETE STRUCTURES FOR COMPUTER SCIENCE (CS F222)

TEST – I (Closed Book)

Time: 50 Minutes | Max. Marks: 25

Date: September 30, 2012 | Weightage: 25%

Answer all questions

1. Using $A-B=A\cap\overline{B}$ prove that $A-(A-B)=A\cap B$. **[4]**

2. Let *A*, *B* and *C* be subsets of *U*. Prove or disprove: $(A\cup B)\cap(B\cup\overline{C})\subset A\cap\overline{B}$. **[3]**

3. Let *L* be the set of lines in the Euclidean plane and let *R* be the relation in *L* defined by "*x* is parallel to *y*". Is *R* an equivalence relation? Justify. **[3]**

4. Show that the function *f* from the reals into reals defined by $f(x)=x^3+1$ is a one-to-one, onto function and find f^{-1}. **[3]**

5. In R^* (non zero reals) we define $a*b=\frac{ab}{2}$. Prove that $(R^*,*)$ is a group. **[3]**

6. Let *G* be a group. Let $a,b\in G$. Show that $(ab)^{-1}=b^{-1}a^{-1}$. **[3]**

7. In an abelian group prove that $(ab)^2=a^2b^2$. **[3]**

8. Let *H* be a subgroup of *G*. Show that the identity element of *H* is the same as that of *G* and for each $a\in H$ the inverse of *a* in *H* is the same as the inverse of *a* in *G*. **[3]**

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DISCRETE STRUCTURES FOR COMPUTER SCIENCE (CS F222)

QUIZ – II (Closed Book)

Time: 20 Minutes | Max. Marks: 7

Date: December 12, 2012 | Weightage: 7%

ID.No: Name:

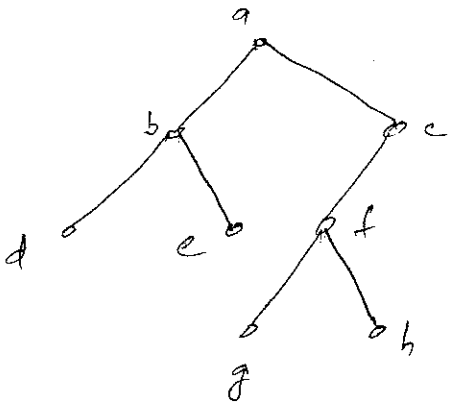
Answer all questions

1. Let $T_1 = (V_1, E_1), T_2 = (V_2, E_2)$ be two trees where $|E_1| = 17$ and $|V_2| = 2|V_1|$ determine $|V_1|, |V_2|, |E_2|$. [2]

2. A book has 3 chapters C1, C2, C3. C1 has two sections S1.1 and S1.2. C3 has three sections S3.1, S3.2 and S3.3. Section S3.2 has two subsections S3.2.1 and S3.2.2. Draw a rooted tree to represent the table of contents of the book. [2]

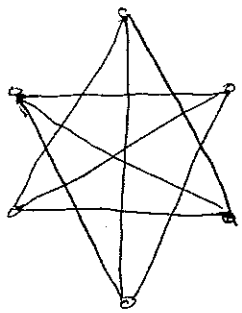
3. A graph G is critical planar if G is nonplanar but any subgraph obtained by removing a vertex is planar. Is $K_{3,3}$ critical planar? Justify your answer. [1]

4. Determine whether or not the following tree is height balanced. If not, explain why not. [1]



5. Is the following graph planar? Justify.

[1]



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DISCRETE STRUCTURES FOR COMPUTER SCIENCE (CS F222)

QUIZ – I (Closed Book)

Time: 20 Minutes | Max. Marks: 8

Date: October 24, 2012 | Weightage: 8%

ID.No: Name:

Answer all questions

1. Is $(nZ,+)$ a cyclic group? Justify your answer. **[1]**

2. Find the order of -1 and 3 in $(R^*,\cdot)$. **[1]**

3. Find all the distinct left cosets of $\{0,4,8\}$ in $(Z_{12},\oplus)$. **[2]**

4. Can a group of order 12 contain a subgroup of order 8? Justify your answer. **[1]**

5. Is $(Z,+)$ isomorphic to $(2Z,+)$? Justify your answer. **[2]**

6. Define a ring. **[1]**