BITS, PILANI - DUBAI
Dubai International Academic City, DUBAI
II-Year, Semester- I (2009-10)
MATHEMATICS- III (MATH C241)
COMPREHENSIVE EXAMINATION

Time: 3 Hrs | Max.Marks:120
Date: 23-12-2009. | Weightage:40%

Note: This Paper contains THREE SECTIONS (**A, B and C**).
Answer each section in **Separate** Answer Book. All questions are *compulsory* and should be answered *sequentially*.

Section – A

1 a) Reducing the order of the differential equation solve the equation $xy'' + y' = 4x$.

b) Solve the *differential equation* $(x \log x)y' + y = 3x^3$ (8 + 6)

2 a) Find the solution of the initial value problem
$y'' - 6y' + 5y = 0$, $y(0) = 3$ and $y'(0) = 11$.

b) Using the method of variation of parameters, find a particular solution of
$y'' + y = \sec x \tan x$. (6 + 8)

3 a) Using method of undetermined coefficients find the general solution of
$y'' + 3y' - 10y = 6e^{4x}$.

b) Using operator method, find a particular solution of $y''' - 2y'' + 10y = x^4 + 2x + 5$.
(6 + 6)

Section – B

1 a) Find two independent *power series* solutions of the *Chebyshev's equation*
$(1 - x^2)y'' - xy' + p^2 y = 0$ near $x=0$.

b) Find the general solution of the differential equation, $(2x^2 + 2x)y'' + (1 + 5x)y' + y = 0$, near the singular point $x=0$, in the Hyper geometric form. (7 + 8)

2 a) If $P_n(x)$ is a *Legendre polynomial* show that $\int_{-1}^{1} P_n^2(x)dx = \frac{2}{2n+1}$.

b) If $J_p(x)$ is *Bessel function* of the first kind of *order p*, show that

$$\frac{d}{dx}(x^p J_p(x)) = x^p J_{p-1}(x) \text{ and } \frac{d}{dx}(x^{-p} J_p(x)) = -x^{-p} J_{p+1}(x).$$ (7 + 8)

3 a) Find the general solution of the system of first order equations

$$\frac{dx}{dt} = -3x + 4y$$

$$\frac{dy}{dt} = -2x + 3y$$

b) Find the eigenvalues λ_n and eigenfunctions $y_n(x)$ for the boundary value problem $y'' + \lambda y = 0$, $y(a) = 0$, $y(b) = 0$ *when* $a < b$. (5 + 5)

Section - C

1 a) Find the *Laplace Transform* of $\{x^{1/2}\}$,using $L(x^{-1/2})$. (5)

b) Using convolution theorem for *Laplace Transform* solve $y'' + 5y' + 6y = 5e^{3t}$, $y(0) = 0; y'(0) = 0$. (10)

2 a) Find the Fourier Series of the periodic function defined by

$$f(x) = \begin{cases} 0; & -\pi \le x < 0 \\ x^2; & 0 \le x < \pi \end{cases}$$

and deduce the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. (10)

b) Find the *Cosine Series* for the function $f(x) = \pi - x;\ 0 \le x \le \pi$. (5)

3 Solve the vibrating string problem $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, if the initial shape of the string is given by the function $f(x) = 2cx/\pi;\quad 0 \le x \le \pi/2,$

$2c(\pi - x)/\pi,\quad \pi/2 \le x \le \pi$ (10)

BITS PILANI, DUBAI CAMPUS
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II-Year, Semester- I (2009-10)
MATHEMATICS- III (MATH C241)
Test –II (OPEN BOOK)

Time: 50 Minutes
Date: 22-11-2009.

Max.Marks:60
Weightage:20%

Note: Answer the questions in **serial order**.
Only the **prescribed text book** and **hand written** notes are allowed.

1 a) Verify that the origin is a *regular singular* point of $4xy'' + 2y' + y = 0$ and find *only one Frobenius Series* solution of it.

b) If $F(a,b,c,x)$ is *Hypergeometric function,* then prove that
$$\log\frac{(1+x)}{(1-x)} = 2xF(\frac{1}{2},1,\frac{3}{2},x^2).$$

c) Find the general solution of the differential equation $(x^2-1)y'' + 2xy' - 6y = 0$, about $x=1$, by changing it into *Hypergeometric* equation. (10+5+5)

2 a) If $P_4(x)$, $P_2(x)$ *and* $P_0(x)$ are *Legendre Polynomials,* show that
$x^4 = \frac{8}{35}P_4(x) + \frac{4}{7}P_2(x) + \frac{1}{5}P_0(x)$. By using the orthogonal properties of *Legendre Polynomials* find $\int_{-1}^{1} x^4 P_2(x)dx$.

b) If $T_n(x)$ is the *Chebyshev polynomial,* show that

$T_{m+n}(x) + T_{m-n}(x) = 2T_m(x)T_n(x)$ and $2\,(T_n(x))^2 = 1 + T_{2n}(x)$ (10+10)

3 a) Express the *Bessel function* $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$.

b) Find the *eigenvalues* and *eigenfunctions* of the *Boundary Value Problem*

$y'' + \lambda y = 0,\ y'(0) = 0$ and $y'(L) = 0, L > 0$. (10+10)

BITS PILANI, DUBAI CAMPUS
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II-Year, Semester- I (2009-10)
MATHEMATICS- III (MATH C241)
Test –I (CLOSED BOOK)

Time: 50 Minutes | Max.Marks:75
Date:4-10-2009. | Weightage:25%

1 a) Solve the differential equation $x^2 \frac{dy}{dx} = 3(x^2 + y^2)\tan^{-1}(\frac{y}{x}) + xy$.

b) Solve the differential equation $(xy - 1)dx + (x^2 - xy)dy = 0$ by finding an integrating factor.

c) Find a solution of the differential equation $(1 + x^2)dy + 2xydx = (\cot x)dx$. (9+8+8)

2 a) Find the general solution of the differential equation $xy'' = y' + (y')^3$, by reducing the order of the equation.

b) If $y_1 = x$, is one solution of $x^2y'' + 2xy' - 2y = 0$, then find its general solution.

c) Find the general solution of the differential equation $4y'' - 8y' + 7y = 0$. (9+8+8)

3 a) Find the general solution of $x^2y'' + 2xy' - 16y = 0$

b) Solve the initial value problem $y'' + 4y' + 5y = 0, y(0) = 1, y'(0) = 0$

c) Find the general solution of $y^{(4)} + 2y''' + 2y'' + 2y' + y = 0$ (8+8+9)

Set – A

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MATHEMATICS- III (MATH C241)
Quiz –II (CLOSED BOOK)

Time: 20 Minutes | Max.Marks:21
Date: 8-12-2009. | Weightage:7%

Note: Write [illegible]

NAME: **ID. No:**

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1. Find the *Laplace Transform* of $f(x) = \cos^2 x + x$. *(3)*

2. Find the *Inverse Laplace Transform* of $F(p) = \dfrac{1}{p(p-1)}$ (3)

3. Find the *Inverse Laplace Transform* of $F(p) = \dfrac{16}{(p+4)^3}$. (3)

4. Find the *Laplace Transform* of $f(x) = x \sin 4x$. (3)

5. Find the convolution of $f(t) = t$ and $g(t) = e^{at}$. (5)

6. Find ***A(t)*** in solving the *differential equation* $y'' - y' - 6y = -t$, $y(0) = y'(0) = 0$. (4)

Set – B

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MATHEMATICS- III (MATH C241)
Quiz – II (CLOSED BOOK)

Time: 20 Minutes | Max.Marks:21
Date: 8-12-2009. | Weightage:7%

Note: [illegible]

NAME: **ID. No:**

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1. Find the *Laplace Transform* of $f(x) = \sin^2 x + e^{3x}$. *(3)*

2. Find the *Inverse Laplace Transform* of $F(p) = \dfrac{1}{(p-2)(p-1)}$ (3)

3. Find the *Inverse Laplace Transform* of $F(p) = \dfrac{5}{(p+2)^3}$. (3)

4. Find the *Laplace Transform* of $f(x) = x\cos 3x$. (3)

5. Find the convolution of $f(t) = -t$ and $g(t) = e^{-at}$. (5)

6. Find ***A(t)*** in solving the *differential equation* $y'' + y' - 6y = t,\ y(0) = y'(0) = 0$. (4)

Set – A

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MATHEMATICS- III (MATH C241)
Quiz –I (CLOSED BOOK)

Time: 20 Minutes Max.Marks:24
Date: 13-10-2009. Weightage:8%

Each Question carries FOUR marks.

1. Find Wronskian $W(e^{3x}\cos 6x, e^{3x}\sin 6x)$.

2. If $y'' + 4y = Tan2x$, find v_1 of $v_1 y_1 + v_2 y_2$ where y_1 and y_2 are general solutions of homogeneous part.

3. Find the singular points of $x^2(x^2-1)y'' + 2x(x-1)y' - 16y = 0$ and classify them.

4. Find the recurrence relation for the coefficients in the series solution of $4xy' + 5y = 0$

5. Find a particular solution of the differential equation $(D+2)^3 y = \dfrac{Sin(-6x)}{e^{2x}}$

6. Find a particular solution of the differential equation $y^{(5)} + y = x^5$.

Set – B

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MATHEMATICS- III (MATH C241)
Quiz – I (CLOSED BOOK)

Time: 20 Minutes | Max.Marks:24
Date: 13-10-2009. | Weightage:8%

Each Question carries FOUR marks.

1. Find the Wronskian $W(e^{2x}\cos 4x, e^{2x}\sin 4x))$.

2. For $y''+9y = Sec3xTan3x$ find v_1 of $y = v_1y_1+v_2y_2$.

3. Find the singular points of $(3x+1)xy''+3(x+1)y'-4=0$ and classify them.

4. Find the indicial equation and its roots for the following differential equation $x^2y''-4xy'=0$.

5. Find a particular solution of the differential equation. $(D-1)^3 y = \frac{\cos(5x)}{e^{-x}}$

6. Find a particular solution of the differential equation $y'''-\frac{y'}{4}=x$.