BITS, PILANI – DUBAI CAMPUS
Knowledge Village, Dubai

II Year- Semester-I (2005-2006) COMPREHENCIVE EXAMINATION
Mathematics-III (MATHUC 241)
(Closed Book)

Full Marks : 80
Weightage : 40 %

Date: 27-12-2005
Duration : 3 Hours

Note: 1. All questions are compulsory and should be answered in the given sequence.
2. There are two sections (A & B) in the question paper and should be answered in two separate sheets and should write A or B on the top of the answer sheet.

SECTION A

Q1. (a) Solve the differential equation $(y-x^3)dx+(x+y^3)dy=0$ (4)

(b) Solve completely the differential equation $xy''+y'=4x$ by using the reduction of order method. (4)

(c) Find the general solution of the differential equation $y'''-3y''+4y'-2y=0$ (2)

Q2. (a) Show that $y=c_1x+c_2x^2$ is the general solution of $x^2y''-2xy'+2y=0$ on any interval not containing 0, and find the particular solution for which $y(1)=3$ and $y'(1)=5$. (4)

(b) By the method of variation of parameter solve the differential equation
$y''+y=\operatorname{cosec} x$ (3)

(c) Find the particular solution of the differential equation $y''-y=e^{-x}$ by successive integration. (3)

Q3. (a) Find the general solution of the equation $(1+x^2)y''+2xy'-2y=0$ in terms of power series in x. (5)

(b). Locate and classify the singular point and calculate the two independent Frobinius series solution of the equation $4xy''+2y'+y=0$ (6)

Q4. (a) Find the general solution of the differential equation
$x(1-x)y''+\left(\frac{3}{2}-2x\right)y'+2y=0$ near the singular point x=0 (4)

(b). prove the recursion formula

$(n+1)P_{n+1}(x)=(2n+1)xP_n(x)-nP_{n-1}(x)$ hence evaluate $\int_{-1}^{1} xP_n(x)P_{n-1}(x)$ where $P_n(x)$ is the Legenders Polynomial.

(5)

SECTION B

Q5.(a) Prove the following recurrence formula

$J'_p(x)+\frac{p}{x}J_p(x)=J_{p-1}(x)$, where $J_p(x)$ is the Bessel Function of first kind.

(4)

(b). Find the general solution of the system of homogenous equations,

$$\frac{dx}{dt}=3x-4y; \qquad \frac{dy}{dt}=x-y$$ (6)

without converting it into second order linear equation.

Q6.(a) Use impulse functions to solve the following differential equation,

$$y''+y'-6y=t \qquad y(0)=y'(0)=0$$ (5)

(b). Solve the following differential equation by the method of Laplace Transform,

$$xy''+(1-2x)y'-2y=0 \text{ if } y(0)=1$$ (5)

Q7(a) Find the Fourier series for the function defined by

$$f(x)=\begin{cases}0 & -\pi\le x<0\\ \sin x & 0\le x\le\pi\end{cases}$$

Sketch the graph of the sum on the interval $-2\pi\le x\le 2\pi$ (5)

(b) Find the Fourier Series for the function defined by

$$f(x)=|x| \qquad -2\le x\le 2$$ (5)

Q8 (a) Find the solution of one dimensional heat equation

$$a^2\frac{\partial^2 w}{\partial x^2}=\frac{\partial w}{\partial t}$$

subject to the condition $w(0,t)=w(\pi,t)=0$ *and* $w(x,0)=f(x)$ (5)

(b). State the Sturm-Liouville Problem. Find the eigen-value & eigen functions of

$$y''+\lambda y=0 \qquad y(-L)=0,\ y(L)=0 \text{ when } L>0$$ (5)

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II - Year - SEMESTER - I (2005-06)

MATHEMATICS - III (MATH UC 241)

TEST - II (Open-Book)

Time: 50 Minutes

Date: December 5, 2005

Max.Marks: 20

Weighage: 20 %

Note: 1. Text-book and class notes are allowed. 2.Solve all the four questions.

1. (a) Prove that that the if $T_n(x)$ is Chebyshev polynomial then prove that $x^4 = [T_4(x) + 4T_2(x) + 3T_0(x)]/8$.

   (b) Solve the system of differential equation by differentiating the first equation with respect to t and eliminating y

$$\begin{aligned} \frac{dx}{dt} &= 5x + 4y \\ \frac{dy}{dt} &= -x + y. \end{aligned}$$

(2 + 3)

2. (a) Find $L[e^{iat}]$ hence find $L[\sin at]$.

   (b) Graph the following function and finds its Laplace Transform:

$$f(t) = \begin{cases} \cos t, & 0 \le t \le \pi, \\ 0 & x > \pi. \end{cases}$$

(2 + 3)

3. (a) Express $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$ using appropriate recurrence formula and hence compute $J_3(6.2)$. Given that $J_0(6.2) = 0.20175$ and $J_1(6.2) = -0.23292$.

   (b) Show that $y = xJ_1(x)$ is a solution of the differential equation

$$xy'' - y' - x^2 J'_0(x) = 0.$$

(2 + 3)

4. (a) Use recurrence formula and properties of Legendres' Polynomials to show that

$$\int_{-1}^{1} x^2 P_n^2(x)dx = \frac{1}{8(2n-1)} + \frac{3}{4(2n+1)} + \frac{1}{8(2n+3)}.$$

   (b) Determine whether the following differential equation with the given boundary conditions

$$\left(\frac{1}{x}y'\right)' + (x + \lambda)y = 0; \qquad y(0) = 0, \ y'(1) = 1$$

form a Sturm Lioville problem.

(3 + 2)

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II-Year –SEMESTER-I (2005-06)
MATHEMATICS-III (MATHUC 241)
Quiz-II(Closed-Book)-MAKEUP

Time: 30 Minutes November 28, 2005 Max Marks: 10

IDNo: Section No: Name:

Note: (1) Write ID No. Name, Section No and Answer in the space provided
(2) Overwriting will be treated as wrong Answer

Q.No	1	2	3	4	5	6	7	8	9	10
Ans:										

1. The formula $\Gamma(P+1) = P\,\Gamma(P)$ is true for

(a) $P=0$ (b) $P<0$ (c) $P>0$ (d) none

2. The value of the $\frac{d}{dx}[x J_1(x)]$ is

(a) $x J_0(x)$ (b) $-x J_0(x)$ (c) $J_0(x)$

(d) none of these

3. The value of $x J_{1/2}(x)$ is given by

(a) $\sqrt{\frac{2}{\pi x}}\sin x$ (b) $\sqrt{\frac{2x}{\pi}}\sin x$ (c) $\sqrt{\frac{2}{\pi x}}\cos x$

(d) none of these

4. The value of $P_{2n+1}(0)$ is given by

(a) $\dfrac{(-1)^n\, 1.3.\cdots(2n-1)}{2^n \underline{|n}}$ (b) 0 (c) -1

(d) none of these

5. The value of $P_3(x)$ is given by

(a) $\frac{1}{2}(3x^2-1)$ (b) $\frac{1}{4}(6x^3-3x^2)$ (c) $\frac{1}{2}(5x^3-3x)$

(d) none of these

6. The one of the zero of the chebyshev polynomial is

(a) $\cos\frac{k\pi}{n}$ (b) $\sin\frac{2k\pi}{n}$ (c) $\sin\frac{(2k+1)\pi}{2n}$

(d) none of these

7. All the eigenvalues of a sturm-Liouvelle problem are

(a) Integer (b) complex (c) real

(d) none of these

8. The Hypergeometric function $xF(\frac{1}{2},1,\frac{3}{2},-x^2)$ is repersented by

(a) $\log(1+x)$ (b) $(1+x)^{-1}$ (c) $\sin^{-1}x$ (d) none

9. The eigen value λ_n for the equ^n $y''+\lambda y=0$ when $y(0)=0$ and $y(1)=0$ is given by

(a) $n^2\pi^2$ (b) $\frac{n^2}{\pi^2}$ (c) $n\pi$ (d) none of these

10. The value of $\|P_0(x)\|^2$ is given by

(a) -2 (b) 2 (c) $\frac{1}{2}$ (d) none of these

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II - Year - SEMESTER - I (2005-06)

MATHEMATICS - III (MATH UC 241)

QUIZ II - (Closed-Book)

Time: 30 Minutes　　November 16, 2005　　Max. Marks: 10

ID No:　　Section No:　　Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	a	b	a	c	b	b	c	a	c	a

1. The Hypergeometric function $F(1, 1, 1, x)$, $|x| < 1$ represents the function
   (a) $(1-x)^{-1}$　(b) $(1+x)^{-1}$　(c) $(1+x)^{-1/2}$　(d) None of these

2. The zeros of the Chebyshev polynomial are
   (a) $\cos(\frac{2k\pi}{2n})$　(b) $\cos(\frac{(2k+1)\pi}{2n})$　(c) $\cos(\frac{4k\pi}{2n})$　(d) None of these

3. The value of the $J_{-3/2}(x)$ is
   (a) $\sqrt{\frac{2}{\pi x}}\left(-\frac{\cos x}{x} - \sin x\right)$　(b) $\sqrt{\frac{2}{\pi x}}\left(-\frac{\sin x}{x} - \cos x\right)$　(c) $\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x} - \sin x\right)$
   (d) None of these

4. The generating function for the Legendre's polynomial is
   (a) $(1-2xt-t^2)^{-1/2}$　(b) $(1+2xt-t^2)^{-1/2}$　(c) $(1-2xt+t^2)^{-1/2}$
   (d) None of these

5. The eigen values for the equation $y'' + \lambda y = 0$ with boundary condition $y(0) = 0$ and $y(2\pi) = 0$ are

(a) $4n^2$ (b)$n^2/4$ (c) $n^2/2$ (d) None of these

6. The value of the $P_n(-x)$ is

(a) $(-1)^n P_n(-x)$ (b)$(-1)^n P_n(x)$ (c) $-P_n(x)$ (d) None of these

7. The value of the $\frac{d}{dx}[J_0(x)] =$

(a) $xJ_1(x)$ (b) $-xJ_1(x)$ (c) $-J_1(x)$ (d) None of these

8. The Bessel's function $J_p(x)$ is defined as

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{(x/2)^{p+2n}}{n!(n+p)!}$ (b) $\sum_{p=0}^{\infty}(-1)^n \frac{(x/2)^{p+2n}}{n!(n+p)!}$ (c) $\sum_{p=0}^{\infty}(-1)^n \frac{(x/2)^{n+2p}}{n!(n+p)!}$

(d) None of these

9. The *Rodrigues' formula* for generating Legendre polynomials is given by

(a) $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)$ (b) $P_n(x) = \frac{1}{2^n n!}\frac{d}{dx}(x^2 - 1)^n$

(c) $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$ (d) None of these

10. The eigenfunctions $y_n(x)$ for the d.e. $y'' + \lambda y = 0$ with boundary condition $y(a) = 0,\ y(b) = 0\ (a < b)$, is

(a) $\sin\frac{n\pi(x-a)}{b-a}$ (b) $\sin\frac{n\pi x}{b-a}$ (c) $\sin\frac{n\pi x}{2(b-a)}$ (d) None of these

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II - Year - SEMESTER - I (2005-06)

MATHEMATICS - III (MATH UC 241)

QUIZ II - (Closed-Book)

Time: 30 Minutes November 16, 2005 Max. Marks: 1C

ID No: Section No: Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	a	c	b	a	b	b	c	a	a	c

1. The value of the $J_{-3/2}(x)$ is

   (a) $\sqrt{\frac{2}{\pi x}}\left(-\frac{\cos x}{x} - \sin x\right)$ (b) $\sqrt{\frac{2}{\pi x}}\left(-\frac{\sin x}{x} - \cos x\right)$ (c) $\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x} - \sin x\right)$

   (d) None of these

2. The generating function for the Legendre's polynomial is

   (a) $(1-2xt-t^2)^{-1/2}$ (b) $(1+2xt-t^2)^{-1/2}$ (c) $(1-2xt+t^2)^{-1/2}$

   (d) None of these

3. The eigen values for the equation $y''+\lambda y=0$ with boundary condition $y(0)=0$ an. $y(2\pi)=0$ are

   (a) $4n^2$ (b) $n^2/4$ (c) $n^2/2$ (d) None of these

4. The Hypergeometric function $F(1,1,1,x)$, $|x| < 1$ represents the function

(a) $(1-x)^{-1}$ (b) $(1+x)^{-1}$ (c) $(1+x)^{-1/2}$ (d) None of these

5. The zeros of the Chebyshev polynomial are

(a) $\cos(\frac{2k\pi}{2n})$ (b) $\cos(\frac{(2k+1)\pi}{2n})$ (c) $\cos(\frac{4k\pi}{2n})$ (d) None of these

6. The value of the $P_n(-x)$ is

(a) $(-1)^n P_n(-x)$ (b)$(-1)^n P_n(x)$ (c) $-P_n(x)$ (d) None of these

7. The value of the $\frac{d}{dx}[J_0(x)] =$

(a) $xJ_1(x)$ (b) $-xJ_1(x)$ (c) $-J_1(x)$ (d) None of thes

8. The eigenfunctions $y_n(x)$ for the d.e. $y'' + \lambda y = 0$ with boundary condition $y(a) = 0$, $y(b) = 0$ $(a < b)$, is

(a) $\sin \frac{n\pi(x-a)}{b-a}$. (b) $\sin \frac{n\pi x}{b-a}$ (c) $\sin \frac{n\pi x}{2(b-a)}$ (d) None of these

9. The Bessel's function $J_p(x)$ is defined as

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{(x/2)^{p+2n}}{n!(n+p)!}$ (b) $\sum_{p=0}^{\infty}(-1)^n \frac{(x/2)^{p+2n}}{n!(n+p)!}$ (c) $\sum_{p=0}^{\infty}(-1)^n \frac{(x/2)^{n+2p}}{n!(n+p)!}$

(d) None of these

10. The *Rodrigues' formula* for generating Legendre polynomials is given by

(a) $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)$ (b) $P_n(x) = \frac{1}{2^n n!}\frac{d}{dx}(x^2-1)^n$

(c) $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$ (d) None of these

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II – Year – SEMESTER – I (2005-06)

MATHEMATICS – III (MATH UC 241)

TEST – I (MAKE-UP)

Time: 50 Minutes

Date: . . . . Max Marks: 20

Weightage: 20%

Note: Solve all the four questions.

P-94 1. (a) Verify that one solution of $xy'' - (2x+1)y' + (x+1)y = 0$ is given by $y_1 = e^x$, and find the general solution.

P-98 (b) Find the general solution of the differential equation

$$x^2 y'' + 2xy' - 6y = 0$$

(2+3)

P-135 2. (a) Find the particular integral of the differential equ^n

$$y'' - 2y' - 3y = 6e^{5x}$$

P-106 (b) Apply the method of variation of parameter to find the particular solution of $y'' + y = \sec x$

(2+3)

3 (a) Find the two independent Frobenius series solution of the d.e. $4xy'' + 2y' + y = 0$

(b) Verify by examining the series expansions of the functions on the left side

$$\sin x = x\left[\lim_{a \to \infty} F\left(a, a, \frac{3}{2}, \frac{-x^2}{4a^2}\right)\right]$$

(3+3)

4. When $P = 0$, Bessel's equ^n becomes $x^2y'' + xy' + x^2y = 0$. Show that its indicial equation has only one root and deduce that $y = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(\lfloor n)^2} x^{2n}$ is the corresponding Frobenius series solution.

(4)

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Knowledge Village, DUBAI

II - Year - SEMESTER - I (2005-06)

MATHEMATICS - III (MATH UC 241)

TEST - 1 (Closed-Book)

Time: 50 Minutes
Date: October 16, 2005

Max. Marks: 20
Weighage: 20 %

Note: Solve all the four questions.

1. (a) Given that $y_1 = x^{-1/2}$ is one solution of the differential equation

$$x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0,$$

Find the general solution.

(b) Find the general solution of the differential equation

$$x^2y'' - 3xy' + 4y = 0.$$

(2 + 3)

2. (a) Find the particular integral of the differential equation

$$y'' - 2y' = 12x - 10.$$

(b) Show the method of variation of parameter applied to equation $y'' + y = f(x)$ leads to particular solution

$$y_p(x) = \int_0^x f(t)\sin(x-t)dt.$$

(2 + 3)

3. (a) Consider the differential equation

$$x(1-x)y'' + [p - (p+2)x]y' - py = 0,$$

Find the general solution near $x = 0$ in terms of the hypergeometric function.

(b) *Chebyshev's Equation* is

$$(1-x^2)y'' - xy' + p^2y = 0,$$

(3 + 3)

where p is constant. Find the general solution in terms of the power series in x valid for $|x| < 1$.

4. Find the *Forbenius series* solution for the differential equation

$$x^2y'' - 3xy' + 4(x+1)y = 0$$

(4)

Good Luck . . .

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BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

II - Year - SEMESTER - I MATHEMATICS - III (MATH UC 241)

QUIZ I - (Closed-Book)

Time: 30 Minutes   Dated: September 05, 2005   Max. Marks: 10

ID No: Section No: Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. The integrating factor for the d.e. $(y\log y - 2xy)dx + (x+y)dy = 0$ is ....$\frac{1}{y}$....
2. The general solution of the d.e. $xdy + ydx + x^2y^5dy = 0$ is .$x(y^5 + cy) = 4$
3. The general solution of the d.e. $y'' = 1 + (y')^2$ is...$y = -\log[\cos(x+c_1)] + c_2$
4. The equation $y' + y = \frac{1}{1+e^{2x}}$ has general solution ..$y = e^{-x}\tan^{-1}e^{x} + ce^{-x}$
5. If $y_1 = x^2$ is one solution of the d.e. $x^2y'' + xy' - 4y = 0$, then the general solution is.....$y = c_1x^2 + c_2x^{-2}$
6. The general solution of the d.e. $4y'' - 12y' + 9y = 0$ is ...$y = c_1e^{\frac{3x}{2}} + c_2xe^{\frac{3x}{2}}$
7. The d.e. $y'' + 3xy' + x^2y = 0$ can be transformed into d.e. with constant coefficient. State True or False ......False

8. The d.e. $y'' - 2y' + y = e^x$ has a particular integral ....$y = \frac{1}{2}x^2 e^x$

9. The Wronskian $W(y_1, y_2)$ is equal to ....-1 or 1...., where y_1 and y_2 are solution of the d.e. $y'' + y = 0$.

10. The general solution of the d.e. $y''' + y = 0$ is....$y = c_1 e^{-x} + e^{x/2}(c_2 \cos\frac{1}{2}\sqrt{3}x + c_3$

Good Luck . . .

B

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

II - Year - SEMESTER - I MATHEMATICS - III (MATH UC 241)

QUIZ I - (Closed-Book)

Time: 30 Minutes Dated: September 05, 2005 Max. Marks: 10

ID No: Section No: Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. The equation $y' + y = \frac{1}{1+e^{2x}}$ has general solution ..$y = e^{-x} \tan^{-1} e^{x} + c\,e^{-x}$

2. If $y_1 = x^2$ is one solution of the d.e. $x^2y'' + xy' - 4y = 0$, then the general solution is....$y = c_1 x^2 + c_2 x^{-2}$

3. The integrating factor for the d.e. $(y \log y - 2xy)dx + (x + y)dy = 0$ is ......$\frac{1}{y}$..

4. The general solution of the d.e. $xdy + ydx + x^2y^5dy = 0$ is ..$x(y^5 + cy) = 4$

5. The general solution of the d.e. $y'' = 1 + (y')^2$ is...$y = -\log[\cos(x+c_1)] + c_2$

6. The d.e. $y'' + 3xy' + x^2y = 0$ can be transformed into d.e. with constant coefficient. State True or False ....False

7. The d.e. $y'' - 2y' + y = e^x$ has a particular integral .....$y = \frac{1}{2}x^2 e^x$

8. The Wronskian $W(y_1, y_2)$ is equal to ....-1 or 1...., where y_1 and y_2 are solution of the d.e. $y'' + y = 0$.

9. The general solution of the d.e. $y''' + y = 0$ is.$y = c_1 e^{-x} + e^{x/2}(c_2 \cos\frac{1}{2}\sqrt{3}x + c_3 \sin\frac{1}{2}\sqrt{}$

10. The general solution of the d.e. $4y'' - 12y' + 9y = 0$ is ..$y = c_1 e^{\frac{3x}{2}} + c_2 e^{\frac{3x}{2}}$

Good Luck . . .