BITS-PILANI, DUBAI CAMPUS
KNOWLEDGE VILLAGE, DUBAI

Comprehensive Examination, January 2004

Math UC241 - MATHEMATICS III

13.01. 2004 **Max:40 marks**

Time: 3 hours

Section – A

Answer any Five of the following. Each question carries Two marks.

1. Find the orthogonal trajectories of the family of curves $r = c(1+\cos\theta)$
2. Solve the differential equation $x^2y'' = 2xy' + (y')^2$ using the method of reduction of order.
3. Find the indicial equation and its roots of the differential equation $4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$
4. Find the normal form of the Bessel's differential equation $x^2y'' + xy' + (x^2 - p^2)y = 0$
5. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
6. Find the eigen values and eigen functions for the equation $y'' + \lambda y = 0$ given $y(0) = 0$ and $y(2\pi) = 0$

Section – B

Answer any Five of the following. Each question carries Six marks.

7. (a) Solve the differential equation $xdy = (y + x^2 + 9y^2)dx$ using a suitable differentiation formula

(b) Show that $y_1 = x^{-\frac{1}{2}} \sin x$ is a solution of the differential equation $x^2y''+xy'+\left(x^2-\frac{1}{4}\right)y=0$ and find the general solution

8. (a) Find the general solution of $x^2y''+xy'-2y=0$
(b) Using the method of undetermined coefficients solve the differential equation $y''+4y = 4\cos 2x + 6\cos x + 8x^2 - 4x$

9. Verify that the origin is a regular singular point of the differential equation $2xy''+(3-x)y'-y=0$ and show that $y_1(x)=\sum_{n=0}^{\infty}\frac{x^n}{1.3.5...(2n+1)}$ and $y_2(x)=x^{-\frac{1}{2}}e^{\frac{x}{2}}$ are its Frobenius solutions.

10. Show that $y = c_1F(a,b,c,x)+c_2x^{1-c}F(a-c+1,b-c+1,2-c,x)$ is the general soltuion of the hypergeometric equation $x(1-x)y''+[c-(a+b+1)x]y'-aby=0$ near x=0. Discuss the convergence characteristics of your solution atleast for one Frobenius solution.

11. (a) Bring out the relationship between $J_0(x)$ and $J_1(x)$ .Show that between any two positive zeroes of $J_0(x)$ there is a zero of $J_1(x)$
(b) Show that the cosine for $x^2$is given by $x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2}, -\pi \le x \le \pi$

12. (a) Solve the non-homogeneous system of equations (fully)

$$\frac{dx}{dt}=x+y-5t+2$$

$$\frac{dy}{dt}=4x-2y-8t-8$$

(b) Solve the differential equation $y''+2y'+2y=2, y(0)=0, y'(0)=1$ using the method of Laplace Transforms.

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Course Code : UC241

Course Title : Maths III

Date : 19.10.2003

Duration : 50 minutes

Time : 10 – 10.50

Maximum : 20 marks

Test 1

Answer any Four of the following. Each carries Five marks

1. **Solve the following differential equation by finding an integrating factor.**
$$(y\log y - 2xy)dx + (x+y)dy = 0$$

2. **Find the particular solution of the equation**
$(x^2 + 2y')y'' + 2xy' = 0, y = 1$ **and** $y' = 0$ **when** $x = 0$

3. **Show that** $y = c_1e^x + c_2e^{2x}$ **is the general solution of** $y'' - 3y' + 2y = 0$ **on any interval , and find the particular solution for which** $y(0) = -1$ **and** $y'(0) = 1$

4. **Examine whether the differential equation** $xy'' + (x^2 - 1)y' + x^3y = 0$ **can be transformed into a differential equation with constant coefficients. If possible solve it.**

5. **Solve the differential equation** $y'' + y' = 10x^4 + 2$

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Date : 30.11.2003 | **Duration : 50 minutes**

Time : 9.30 – 10.20 | **Maximum : 20 marks**

Test 2

Answer any Four of the following. Each carries Five marks

1. Show that every nontrivial solution of $y''+xy=0$ has infinitely many positive zeroes and at most one negative zero and also obtain the series solution of the given equation

2. Solve the differential equation $y^{(3)}-2y'+y=2x^3-3x^2+4x+5$

3. Show that the equation $4x^2y''-8x^2y'+(4x^2+1)y=0$ has only one Frobenius solution and find the general solution of the given equation

4. Get the general solution of the differential equation $(1-x^2)y''-xy'+p^2y=0$ neat $x=1$

5. Using the method of variation of parameters find the general solution of the differential equation $(x^2-1)y''-2xy'+2y=(x^2-1)^2$

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Time : 9.30 – 10.20 | **Maximum : 20 marks**

Test 2 – Makeup

Answer any Four of the following. Each carries Five marks

1. **Show that** $x^2 y'' + xy' + (x^2 - 1)y = 0$ **has only one Frobenius solution and find the same**

2. **Solve the differential equation** $y^{(3)} + y'' = 9x^2 - 2x + 1$

3. **Obtain the series solution of the differential equation** $(1 + x^2)y'' + 2xy' - 2y = 0$ **interms of elementary functions**

4. **Show that any differential equation of the form** $y'' + P(x)y' + Q(x)y = 0$ **can be reduced to normal form**

5. **Using the method of variation of parameters find the general solution of the differential equation** $x^2 y'' - 2xy' + 2y = xe^{-x}$

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QUIZ IN MATHS III

5^{TH} NOVEMBER 2003

TIME : 10-10.50 **MAX : 10 MARKS**

ANSWER ALL QUESTIONS

(1) Name the four methods employed in operator method for finding a particular solution of a linear differential equation with constant coefficients (1)

(2) Mention the use of "Method of variation of Parameter" pointing out the situation in which it can be use (1)

(3) Match the following (For every item in the ~~right~~ left hand side there is one and only option in the right hand side. Match the corresponding items) (1)

1. Wronskian	(a) Exact equations
2. Superposition	(b) Power series
3. Integrating factors	(c) Operator method
4. Analytic functions	(d) Combining solutions
	(e) Independence of solutions

(4) Write the general solution a 9^{th} degree homogeneous linear differential equation with constant coefficients for which $2,3,1,2,3,1+i,1-i,1+i,1-i$ are roots of the auxiliary equation. (1)

(5) Show that under usual notations $P(D)e^{kx}g(x) = e^{kx}P(D+k)g(x)$ (1)

(6) Without using any formula, show that the derivative of any solution of $y''+py'+qy=0$ is also a solution, where p and q are constants (1)

(7) Show that under usual notations,

$$\frac{1}{D-r}f(x) = e^{rx}\int e^{-rx}f(x)dx$$ (1)

(8) Check the validity of the following statements (1)

(i) If an equation is not exact then it can be made exact by a suitable modification

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QUIZ IN MATHS III

22nd December 2003

TIME : 10-10.50 **MAX : 10 MARKS**

ANSWER ALL QUESTIONS

(1) Find the value of $L(\sin^2 3x) + L(\cos^2 3x)$

(2) Find $L^{-1}\left(\frac{p}{p^2 - a^2}\right)$

(3) Find the convolution of e^t, e^{3t}

(4) Find $L\left[x^2 \sin x\right]$

(5) Obtain the auxiliary equation of $\frac{dx}{dt} = x + y, \frac{dy}{dt} = 4x - 2y$

(6) Write one set of solution of the system $\frac{dx}{dt} = 3x - 4y, \frac{dy}{dt} = x - y$

(7) Write down the formula used for finding the particular solution of a non-homogenous system of equations

(8) Obtain the solution of y'+4y=1 using Laplace transform given y(0)=0

(9) Obtain the constant term in the Fourier series expansion of $f(x) = 2, -\pi \le x \le \pi$

(10) Find the Fourier series of $f(x) = \begin{cases} 0, -\pi \le x < 0 \\ \pi, 0 \le x \le \pi \end{cases}$