BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - I (2008-09)

MATHEMATICS - III (MATH C241)

TEST - I (Closed-Book)

Time: 50 Minutes
Date: October 05, 2008

Max. Marks: 75
Weighage: 25 %

Note: Solve all the questions.

1. (a) Find the *general solution* of the differential equation:

$$\sec^2 y \frac{dy}{dx} + x \tan y = 4x^3.$$

   (b) Solve the following differential equation by finding integrating factor: $e^x\, dx + (e^x \cot y + 2y \csc y)\, dy = 0$.

   (c) Find the *general solution* of the differential equation: $x\, dy = (y + x^2 + 9y^2)\, dx$. (9 + 8 + 8)

2. (a) Solve the following equation by reducing the order: $y'' + k^2 y = 0$.

   (b) Solve the following differential equation : $y - x + xy \cot x + xy' = 0$.

   (c) Solve the following differential equation by reducing to a linear differential equation $x\, dy + y\, dx = xy^2\, dx$. (8 + 8 + 9)

3. (a) Find the general solution of the following differential equation: $y^{(4)} + y''' - 3y'' - 5y' - 2y = 0$.

   (b) Solve the following *initial value problem*: $y'' - 5y' + 6y = 0$ with $y(1) = e^2$ and $y'(1) = 3e^2$.

   (c) Find the general solution of the following differential equation: $x^2 y'' + 2xy' + 3y = 0$. (8+9+8)

Good Luck . . .

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DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - I (2008-09)

MATHEMATICS - III (MATH C241)

TEST - II (Open-Book)

Time: 50 Minutes
Date: November 23, 2008

Max. Marks: 60
Weighage: 20 %

Note: Solve all the questions.

1. (a) Find the eigen values λ_n and eigen function $y_n(x)$ for the following case

$$\frac{d^2y}{dx^2} + \lambda y = 0; \quad y'(0) = y(L) = 0.$$

   (b) Prove that if F(a, b, c, x) is *Hypergeometric function*, prove that

$$F(1, 2, 1, x) = \frac{1}{(1-x)^2}.$$

   (c) Determine whether the following differential equation with the given boundary conditions

$$\left(\frac{1}{x}y'\right)' + \frac{\lambda}{x}y = 0; \quad y(0) = y(e) = 0.$$

   form a *Sturm Lioville* problem. Justify your answer. (8+6+6)

2. (a) Prove the following recurrence formula for *Legendre's polynomial* $P_n(x)$

$$P_n'(x) - 2xP_{n-1}'(x) + P_{n-2}'(x) = P_{n-1}(x).$$

   (b) Find $T_5(x)$, if $T_n(x)$ is *Chebyshev polynomial.* Hence or otherwise express x^5 in terms of *Chebyshev polynomial.* (10+10)

3. (a) Find the *Forbenius series* solution for the differential equation

$$x^2y'' - (x + x^2)y' + y = 0.$$

   (b) Identify all singular points of the differential equation $(x - \pi/2)^2y'' + \cos x\, y' + \sin x\, y = 0$. Classify each as regular or irregular.

   (c) Find the *Indicial equation* and its roots for the differential equation $x^2y'' + \cos xy' + \sin y = 0$. (10+5+5)

Good Luck . . .

ID No.:-

BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - I (2008-09)

MATHEMATICS - III (MATH C241)

COMPREHENSIVE EXAM (Closed-Book)

Time: 03 Hours | Max. Marks: 120

January 03, 2009 | Weighage: 40 %

Note:- 1. All questions are compulsory and should be answered sequentially.

*2. There are three sections (**A, B and C**) in the question paper and should be answered in three separate answer sheets and, write section (**A or B or C**) on the top of the respective answer sheet in **CAPITAL BOLD LETTERS**.*

SECTION A

1. (a) Solve the differential equation $xy'' = y' + y'^3$ by *reducing the order.*

   (b) If $y_1 = x$ is an obvious solution of the equation $x^2y'' - x(x+2)y' + (x+2)y = 0$, find the general solution. (6+6)

2. (a) Find the general solution of the differential equation $y^{(4)} + 5y'' + 4y = 0$.

   (b) Find the particular solution of the differential equation $y''+2y'+5y = e^{-x}\sec 2x$ by method of *variation of parameters.* (6+6)

3. (a) Verify that the equation $y'' + y' - xy = 0$ has three term recurrence relation. Also find the general solution in terms of power series in x

   (b) Find the two independent *Forbenius series* solutions of the Bessel's equation: $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$ (8+8)

SECTION B

1. (a) Find the general solution of the differential equation $(2x^2 + 2x)y'' + (1 + 5x)y' + y = 0$ near the singular point $x = 0$.

   (b) Find the Zeros and Turning points of the *Chebychev's polynomial* $T_n(x)$.

   (c) Prove the orthogonality of the *Legendre's polynomials* using *Strum Liouville theorem.* (6+5+5)

2. (a) Prove the following relation for the *Legendre Polynomial* $P_n(x)$

$$P_{2n+1}(0) = 0 \text{ and } P_{2n}(0) = (-1)^n \frac{1 \,.\, 3 \ldots (2n-1)}{2^n n!}.$$

(b) Prove that

$$\int_{-1}^{1} \frac{dx}{1 - 2xt + t^2} = \sum_{n=0}^{\infty} \left(\int_{-1}^{1} P_n(x)^2 dx \right) t^{2n}.$$

Here $P_n(x)$ is *Legendre Polynomial.* (6+8)

3. (a) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ and hence find $J_{-3/2}(x)$.

(b) Prove that $[x^{-p} J_p(x)]' = -x^{-p} J_{p+1}(x)$

Here $J_p(x)$ denote the *Bessel's function.* (5+5)

SECTION C

1. (a) Find the Fourier series of the periodic function defined by

$$f(x) = \begin{cases} -a, & -\pi \le x < 0, \\ a & 0 \le x < \pi. \end{cases}$$

Here a is positive number.

(b) Find the Fourier series of the periodic function defined by

$$f(x) = \begin{cases} 0, & -\pi \le x < 0, \\ x & 0 \le x \le \pi. \end{cases}$$

Hence show that (6+8)

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} \cdots = \frac{\pi^2}{8}$$

2. (a) Find the fourier cosine series for the function $f(x) = x^2, \; 0 \le x \le \pi$.

(b) A string with fixed end points $x = 0$ and $x = \pi$ whose initial shape is given by $f(x) = \frac{x(\pi - x)}{\pi}$. It is released from the rest from this position. Find the displacement $y(x, t)$. (6+8)

3. (a) Solve the following differential equation by method of *Laplace Transform.*

$y'' + 2y' + 5y = e^{-t} \sin t; \quad y(0) = 0$ and $y'(0) = 1$.

(b) Establish the following formula for the *Laplace Transform*

$$L\left[\int_0^t f(t)dt\right] = \frac{F(s)}{s}$$

where $L\{f(t)\} = F(s)$. (6+6)