

BITS PILANI, DUBAI CAMPUS
INTERNATIONAL ACADEMIC CITY, DUBAI
FIRST YEAR – II SEMESTER (2013-14)

Probability and Statistics (MATH F113)
COMPREHENSIVE EXAMINATION

Date: 05.06.2014

Time: 3 hours

Max. Marks: 120

Weightage: 40 %

Answer Part A, Part B and Part C in separate Answer Books. Answer all the questions.

PART A

1. A control chart is used to monitor the average thread count produced by a machine making spandex cloth. Samples are taken periodically, and each sample is classified into one of 5 categories. These are: in control but above average, in control and average, in control but below average, out of control and high, and out of control and low. In taking a series of 20 samples, in how many ways can we obtain a series in which there are exactly 10 M
 - a) 5 samples in control but above average, 5 samples in control and average, 5 samples in control but below average, 3 samples out of control and high, and the rest out of control and low?
 - b) 18 samples in control and 2 out of control?
2. As society becomes dependent on computers, data must be communicated via public 10 M communication networks such as satellites, microwave systems and telephones. When a message is received, it must be authenticated. This is done by using a secret enciphering key. Even though the key is secret, there is always the possibility that it will fall into the wrong hands, thus allowing an unauthentic message to appear to be authentic. Assume that 95% of all messages received are authentic. Furthermore, assume that only 0.1% of all unauthentic messages are sent using the correct key and that all authentic messages are sent using the correct key. Find the probability that a message is authentic given that the correct key is used.
3. Production line workers assemble 15 automobiles per hour. During a given hour, four are produced with improperly fitted doors. Three automobiles are selected at random and inspected. Let X denote the number inspected that have improperly fitted doors. 10 M
 - a) Find the probability that at most one will be found with improperly fitted doors.
 - b) Find $E(X)$.
4. Let X be a continuous random variable with density 10 M
$$f(x) = cx^2, \quad -3 \leq x \leq 3$$
 - a) Find the value of c .
 - b) Find $P(X = 3)$
 - c) Find $P(X \geq 2)$

PART B

5. Let X denote the time in hours needed to locate and correct a problem in the software that governs the timing of traffic lights in the downtown area of a large city. Assume that X is normally distributed with mean 10 hours and variance 9. **10 M**
- a) Find the probability that the next problem will require at most 15 hours to find and correct.
b) The fastest 10% of repairs take at most how many hours to complete?
6. Let X denote the temperature ($^{\circ}\text{C}$) and let Y denote the time in minutes that it takes for the diesel engine on an automobile to get ready to start. Assume that the joint density for (X, Y) is given by $f_{XY}(x, y) = c(4x + 2y + 1)$, $0 \leq x \leq 40$ and $0 \leq y \leq 2$ **10 M**
- a) Find the value of c that makes this a joint density for two dimensional random variable.
b) Find the probability that on randomly selected day the air temperature will exceed 20°C and will take at least one minute for the car to be ready to start.
7. An engineer is studying early morning traffic patterns at a particular intersection. The observation period begins at 5.30 am. Let X denote the time of arrival of the first vehicle from north-south direction, let Y denote the first arrival time from east-west direction. Time is measured in fractions of an hour after 5.30 am. Let the joint density for (X, Y) is given by $f_{XY}(x, y) = \frac{1}{x}$, $0 < y < x < 1$. Find $E(X)$, $E(Y)$ and $E(XY)$ **10 M**
8. A study of the electromechanical protection devices used in electrical power systems showed that of 193 devices that failed when tested, 75 were due to mechanical parts failures. **10 M**
- a) Find a point estimate for p , the proportion of failure that are due to mechanical failures.
b) Find a 95% confidence interval on p .

PART C

9. A new 8-bit micro computer chip has been developed that can be reprogrammed without removal from the microcomputer. It is claimed that on an average a byte of memory can be programmed in less than 14 seconds. The following sample of 15 observations are obtained on X , the time required to reprogram a byte of memory. **12 M**

11.6 14.7 12.9 13.3 13.2

13.1 14.2 15.1 12.5 15.3

13.3 13.4 13.0 13.8 12.3

- a) Find \bar{x} , s^2 and s for this sample.
b) Set up appropriate null and alternate hypothesis needed to verify the claim.
c) Assuming normality of X test whether the null hypothesis can be rejected or not at $\alpha = 0.05$ level?

10. A company is experimenting with a new method for etching circuits that should decrease the proportion of circuits that must be etched a second time. To be cost effective the difference in proportions between the old and new methods must exceed 0.1 10 M

- a) Letting p_1 denote the proportion of circuits that must be redone using the old method, set up the null and alternative hypotheses required to verify the claim that the new method is cost effective.
- b) The following data are obtained on the number of circuits that must be etched second time under each method.

OLD	NEW
$n_1 = 25$	$n_2 = 50$
$x_1 = 4$	$x_2 = 2$

Test whether the null hypothesis H_0 can be rejected at $\alpha = 0.05$ level?

11. A research team wishes to investigate the recovery of heat normally lost to the environment in the form of exhaust gases from furnaces. This experiment is designed by fixing flow speed past heat pipes (in meters per second) and then measuring the recovery ratio. The study yielded the following data: 10 M

Flow Speed (x) (m/sec)	1	1.5	2	2.5	3	3.5	4	4.5	5
Recovery ratio(Y)	0.740	0.745	0.718	0.678	0.652	0.627	0.607	0.507	0.545

- a) Estimate the curve of regression $\mu_{(Y/x)} = \beta_0 + \beta_1 x$
- b) If the flow speed is fixed at 3.25 m/sec then predict $\mu_{(Y/x=3.25)}$

12. In studying the effect of air quality on a lake, the experimenter takes observations on the pH of the water and the air quality as measured on an air quality index. The index goes from 0 to 100 with large numbers representing high pollution. These data are obtained 8 M

pH(X)	4.5	4.1	4.8	4.0	5.0	6.0	3.5	4.9	3.2	6.1
Air quality(Y)	40	50	30	60	20	10	70	30	85	15

Estimate the correlation coefficient ρ .

Table Values (As per standard notation)

$$Z_{0.025} = 1.96, \quad Z_{0.05} = 1.645, \quad P(Z \leq 1.67) = 0.9525, \quad P(Z \leq -1.28) = 0.1, \quad t_{14,0.05} = 1.761$$

BITS PILANI, DUBAI CAMPUS
INTERNATIONAL ACADEMIC CITY, DUBAI

First Year – Semester II (2013-14)
Probability & Statistics (MATH F113)
TEST – II (Open Book)

Date: 24.04.2014
Time: 50 minutes

Max. Marks: 60
Weightage: 20%

Answer all the questions

1. Based upon past experience, 40% of all customers at an automotive station pay for their purchases with a credit card. If a random sample of 200 customers is selected, then find the probability for each of the following events using normal approximation. (15)
- A. at least 90 pay with a credit card?
 B. at most 70 pay with a credit card?
 C. between 70 and 90 customers, inclusive, pay with a credit card?

2. If X is a random variable such that $E(X) = 3$ and $E(X^2) = 13$, use Chebyshev's inequality to determine the lower bound of the probability $P(-2 < X < 8)$. (5)

3. Let the random variable X with density function is given by $f(x) = \frac{1}{2}x^2\lambda^3e^{-\lambda x}$ for $\lambda, x > 0$.

Find maximum likelihood estimation for λ . Also find maximum likelihood estimation $\hat{\lambda}$ based on these data: 7 8 4 5 9 4 9 12 8 5 (15)

4. Suppose $E(\hat{\theta}_1) = \theta$ and $E(\hat{\theta}_2) = \frac{3\theta + 4}{5}$. Check whether $\hat{\theta}_3$ given by (5)

$$\hat{\theta}_3 = \frac{6}{5}\hat{\theta}_1 - \frac{1}{3}\hat{\theta}_2 + \frac{4}{15}, \text{ is an unbiased estimator of } \theta?$$

5. If the joint probability distribution function of X and Y is as follows: (10)

Y	$X \rightarrow$	-1	1
	-1	0.125	0.5
	0	0	0.25
	1	0.125	0

- a) Find $P(X < 1, Y > 0)$
 b) Find the conditional distribution of $X=1$ given $Y=-1$.
 c) $E(X)$
 d) $E(Y)$
6. If the joint probability distribution function of X and Y is as follows: (10)
- $$f(x, y) = \begin{cases} 0.25(2x + y) & \text{for } 0 < x < 1, 0 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$
- Find the following:
- a) $P(X < 0.5, Y > 1)$
 b) $P(X=1, Y=1.5)$
 c) Marginal density of X
 d) Conditional density of Y given X
 e) Are X and Y independent? Justify.

BITS PILANI, DUBAI CAMPUS
INTERNATIONAL ACADEMIC CITY, DUBAI

First Year – Semester II (2013-14)
Probability & Statistics (MATH F113)
TEST – I (Closed Book)

Date: 06.03.2014
Time: 50 minutes

Max. Marks: 75
Weightage: 25%

Answer all the questions

1. A computer system uses password that consists of five letters followed by a single digit, then how many password consists of three A's and two B's, and end in a digit which is multiple of 3? (5)
2. (a) Two dice are thrown in an experiment. What is probability of getting the number 5 at least once, given that the sum of the outcome is 8? (10)
(b) A and B are two independent events such that $P(A) = 0.3$ and $P(A \cup B') = 0.6$, find $P(B)$? (10)
3. It has estimated that 8% of the laptops produced by a company are defective. If 20 laptops are randomly selected, then (10)
 - a) What is the probability that 5 will be defective?
 - b) What is the probability that they will all work?
 - c) What is the variance?
4. A basketball player succeeds in 80% of his free throws. The throws are independent of each other and the player throws successively until he misses. Let X = the number of free throws the player takes until he misses. (15)
 - a) Find $E(X)$.
 - b) What is the probability that he will miss in the fifth attempt?
 - c) What is the probability that the player misses his throw within the first 3 attempts?
5. Joe is a hospital manager, and he is considering the use of a new method to diagnose a rare form of particular syndrome. He knows that only 0.1% of the population suffers from that disease. He also knows that if a person has the disease, the test has 99% of chance of turning out positive. If the person doesn't have disease, the test has a 2% chance of turning positive. (10)
 - a) Find the probability that a patient is diagnosed positive?
 - b) Find the probability that the person is diagnosed positive when he does not have the disease.
6. The following table gives the probabilities that a certain computer will malfunction 0 to 6 times on any one day: (15)

No. of malfunctions (x)	0	1	2	3	4	5	6
Probability : $f(x)$	$k-0.1$	0.29	k	0.16	0.07	0.03	0.01

 - a) Find the value of k such that $f(x)$ is the probability density function.
 - b) Find the mean.
 - c) Find the moment generating function.

Quiz - 1

20.03.2014

Time: 20 Minutes

Max Marks: 24

Weightage: 8%

Name:

ID:

Faculty's Name:

There is no step marking. Each question carries 4 marks.

1. Let X is the length in minutes of a long distance telephone conversation. The density function is given by $f(x) = \frac{1}{64}x^3$ for $0 < x < 4$, then the average length of such a call is _____.
 2. Let the random variable X is Exponential with density function $f(x) = 2e^{-2x}$ for $x > 0$. The variance is given by _____.
 3. A particular nuclear plant releases a detectable amount of radioactive gases twice a month on average. Probability that there will be at least one such emission during a month _____.
 4. If the mean and variance of a Gamma distribution is 12 and 48 respectively, then the parameters α and $\beta =$ _____ also moment generating function (m.g.f.) is given by _____.
 5. Let the density for X is given by $f(x) = ce^{-3x}$ for $x > 0$, then the value of c _____ such that $f(x)$ is the probability density function.
 6. Let the random variable X is Hyper geometric with $N = 10$, $r = 5$ and $n = 3$, then variance _____.
-

BITS, Pilani - Dubai Campus
Dubai International Academic City, Dubai
First year – II Semester 2013 – 2014
Probability and Statistics (MATH F113)

B

Quiz - 1

20.03.2014

Time: 20 Minutes

Max Marks: 24

Weightage: 8%

Name:

ID:

Faculty's Name:

There is no step marking. Each question carries 4 marks.

1. If the mean and variance of a Gamma distribution is 6 and 12 respectively, then the parameters α and $\beta =$ _____ also moment generating function (m.g.f.) is given by _____
 2. Let the density for X is given by $f(x) = ce^{-x}$ for $x > 0$, then the value of c _____ such that $f(x)$ is the probability density function.
 3. A particular nuclear plant releases a detectable amount of radioactive gases thrice a month on average. Probability that there will be at least one such emission during a month _____
 4. Let the random variable X is Hyper geometric with $N = 10$, $r = 7$ and $n = 3$, then variance _____
 5. Let X is the length in minutes of a long distance telephone conversation. The density function is given by $f(x) = \frac{1}{4}x^3$ for $0 < x < 2$, then the average length of such a call is _____
 6. Let the random variable X is Exponential with density function $f(x) = 5e^{-5x}$ for $x > 0$. The variance is given by _____
-