

**BITS PILANI, DUBAI CAMPUS**  
**DUBAI INTERNATIONAL ACADEMIC CITY**  
**FIRST YEAR – II SEMESTER (2012-13)**  
**MATHEMATICS-II (MATH F112/MATH C192)**

**COMPREHENSIVE EXAMINATION (CLOSED BOOK)**

**Date: 30.05.2013**

**Max. Marks: 120**

**Time: 3 hours**

**Weightage: 40 %**

**Answer Part A, Part B and Part C in separate Answer Books.**

**Answer all the questions.**

**PART A**

1. Solve the system of linear equations by Gauss-Jordan method:

$$-5x_1 - 2x_2 + 2x_3 = 14, \quad 3x_1 + x_2 - x_3 = -8, \quad 2x_1 + 2x_2 - x_3 = -3 \quad [10]$$

2. Determine whether  $S = \{t^3 + t^2 - 2t + 1, t^2 + 1, t^3 - 2t, 2t^3 + 3t^2 - 4t + 3\}$  forms a basis for  $P_3$ . If not, find the basis and dimension of the subspace they span. [10]

3. Let  $L: R^3 \rightarrow P_2$  be defined by  $L(a,b,c) = (a+b)x^2 + (b+c)x + (a+c)$ . Find bases for  $\text{Ker } L$  and  $\text{Range } L$ . Also verify the dimension theorem. [10]

4. Find the eigenvalues and eigenvectors of the following matrix. [10]

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

**PART B**

5. Show that  $f'(z)$  does not exist, for the following function. [10]

$$f(z) = \begin{cases} \frac{\bar{z}^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

6. Show that the function  $f(z) = \frac{1}{z^4}$ , ( $z \neq 0$ ) is differentiable in the indicated domain and also find  $f'(z)$ . [10]

7. If C is the boundary of the triangle with vertices at the points 0, 3i and -4, oriented in the counter clockwise direction then find upper bound of  $\left| \int_C (e^z - \bar{z}) dz \right|$ . [10]
8. Find all the roots of the equation  $z^4 + 64 = 0$  [10]

### PART C

9. Evaluate the integral  $\int_C \frac{1 - \cos z}{\sin z} dz$  where  $C : |z| = 5$ . [10]
10. Find the value of the integral  $\int_C \frac{\cos z}{(z-1)^3(z-5)^2} dz$  where C is taken counter clockwise around the circle  $|z-4| = 2$  [10]
11. Find the Laurent's series expansions for  $f(z) = \frac{z}{(z-2)(z+3)}$  in the regions  
 a)  $2 < |z| < 3$  (b)  $|z-2| < 1$ . [10]
12. Find the value of the improper integral  $\int_0^\infty \frac{x \sin x}{(x^2 + 1)^2} dx$  [10]

All the Best

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**COMPREHENSIVE EXAMINATION (CLOSED BOOK)**

ANSWER KEY

30.5.13

**PART A**

$$1. \quad [A \mid B] \sim \left[ \begin{array}{ccc|c} -5 & -2 & 2 & 14 \\ 3 & 1 & -1 & -8 \\ 2 & 2 & -1 & -3 \end{array} \right]$$

$$\sim \left[ \begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array} \right] \quad \boxed{[7M]}$$

Since  $r(A \mid B) = r(A) = \text{no. of unknowns}$ , the system is consistent with unique solution

The solution is  $x = -2, y = 3, z = 5 \quad \boxed{[3M]}$

2.

Let  $A = \left[ \begin{array}{cccc} 1 & 1 & -2 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -2 & 0 \\ 2 & 3 & -4 & 3 \end{array} \right]$

$$\sim \left[ \begin{array}{cccc} 1 & 1 & -2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \boxed{[6M]}$$

Since  $r(A) = 2 < \dim(P_3)$ ,  $S$  does not form the basis for  $P_3$ . Let  $S_1 = \{t^3 + t^2 - 2t + 1, t^2 + 1\}$ , then  $S_1$  is linearly independent and forms the basis for  $W = \text{Span } S$ .  $\dim(\text{Span } S) = 2 \quad \boxed{[4M]}$

$$3. \quad L(a,b,c) = (a+b)x^2 + (b+c)x + (a+c)$$

$$\text{Ker } L = \{(a,b,c) \mid L(a,b,c) = 0\}$$

$$L(a,b,c) = 0 \Rightarrow a+b=0$$

$$b+c=0$$

$$a+c=0$$

$$\text{Let } A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Since  $r(A) = 3 = \text{no. of unknowns}$ , the system is consistent with unique solution

$$\text{i.e. } a=0, b=0, c=0$$

$$\therefore \text{Ker } L = (0, 0, 0)$$

$$\dim(\text{Ker } L) = 0 \quad \text{--- (4M)}$$

$$\text{Range } L: \{P(x) \in P_2 \mid L(a,b,c) = P(x)\}$$

$$(a+b)x^2 + (b+c)x + (a+c) = Ax^2 + Bx + C$$

Let  $S = \{x^2, x, 1\}$ . Since  $S$  is linearly independent,  $S$  forms the basis for Range  $L$ .

$$\dim(\text{Range } L) = 3.$$

Or Let  $S = \{x^2+1, x^2+x, x+1\}$ . Show that  $S$  is linearly independent. Then  $S$  forms the basis for Range  $L$ .

$$\dim(\text{Range } L) = 3. \quad \text{--- (4M)}$$

Dimension Theorem:

$$\dim(\text{Ker } L) + \dim(\text{Range } L) = \dim(\text{domain})$$

$$= \dim(\mathbb{R}^3)$$

$$\text{--- (2M)}$$

4.

The characteristic eq is

$$\lambda^3 + \lambda^2 - 12\lambda = 0 \quad \text{--- (2M)}$$

The eigen values are  $\lambda = 0, -4, 3 \quad \text{--- (2M)}$

when  $\lambda = 0$ , the eigen vector is  $\begin{bmatrix} 1 \\ 6 \\ -13 \end{bmatrix} \quad \text{--- (2N)}$

when  $\lambda = 3$ , the eigen vector is  $\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \quad \text{--- (2N)}$

when  $\lambda = -4$ , the eigen vector is  $\begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} \quad \text{--- (2M)}$

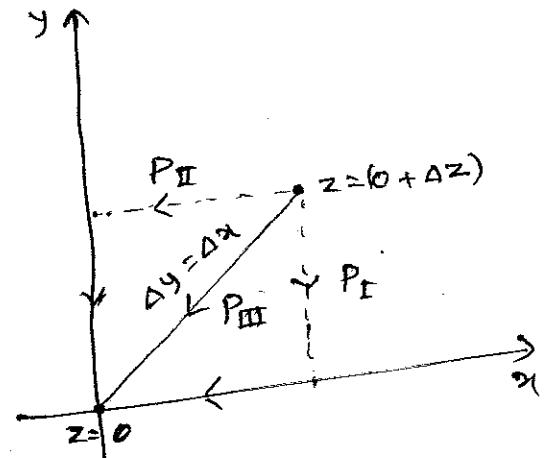
(3)

$$5. \quad w = f(z) = \begin{cases} \frac{(z)^2}{z} & ; z \neq 0 \\ 0 & ; z = 0 \end{cases}$$

$$\Delta w = f(z + \Delta z) - f(z)$$

when  $z = 0$        $\Delta w = f(\Delta z) - f(0) = f(\Delta z) = \frac{(\Delta z)^2}{\Delta z}$

$$\frac{\Delta w}{\Delta z} = \frac{(\Delta x - i\Delta y)^2}{(\Delta x + i\Delta y)^2}$$



Along Path P\_I

$$\frac{\Delta w}{\Delta z} = 1$$

Along Path P\_II       $\frac{\Delta w}{\Delta z} = -1$

Along Path P\_III       $\frac{\Delta w}{\Delta z} = -1$

$$f'(0) = \lim_{\Delta z \rightarrow 0} \frac{f(\Delta z) - f(0)}{\Delta z} = \lim_{\Delta z \rightarrow 0} \frac{\Delta w}{\Delta z} = \begin{cases} 1 \text{ along } P_I, P_E \\ -1 \text{ along } P_{II} \end{cases}$$

Hence  $f'(0)$  does not exist

$$6. f(z) = \frac{1}{z^4} ; z \neq 0$$

$$= \frac{1}{r^4 e^{4i\theta}} = \frac{1}{r^4} (\cos 4\theta - i \sin 4\theta)$$

$$u_r = -\frac{4}{r^5} \cos(4\theta) ; v_r = \frac{4}{r^5} \sin(4\theta)$$

$$u_\theta = -\frac{4}{r^4} \sin(4\theta) ; v_\theta = -\frac{4}{r^4} \cos(4\theta)$$

Cauchy-Riemann Conditions (1e)  $u_r = \frac{1}{r} v_\theta$

and  $v_r = -\frac{1}{r} u_\theta$  also valid

everywhere in the 2-plane  
except  $z=0$

Hence  $f'(z)$  exist everywhere except  $z=0$

$$f'(z) = e^{i\theta} (u_r + i v_r)$$

$$= e^{i\theta} \left[ -\frac{4}{r^5} \cdot e^{-i(4\theta)} \right] = -\frac{4}{r^5 e^{i(5\theta)}} = -\frac{4}{(r^5 e^{i\theta})^5}$$

$$\underline{\underline{f'(z) = -\frac{4}{z^5}}} \quad (z \neq 0)$$

7. Length of the curve

$$L = OA + AB + BO$$

$$L = 3 + 5 + 4$$

$$L = 12 \quad \text{--- [3 M]}$$

$$f(z) = e^z - \bar{z}$$

$$|f(z)| = |e^z - \bar{z}| \leq |e^z| + |\bar{z}|$$

$$|e^z| = |e^{x+iy}| = e^x |e^{iy}| = e^x \cdot 1 = e^x$$

$$|\bar{z}| = |z| = \sqrt{x^2+y^2}$$

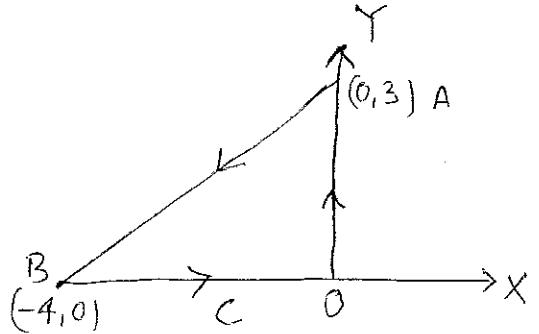
For  $x: -4 \text{ to } 0$  ;  $|e^z| \leq 1$  and  $|\bar{z}| \leq 4$

$$\Rightarrow |f(z)| \leq 4 + 1 \leq 5 (= M) \quad \text{--- [5 M]}$$

$$\left| \int_C f(z) dz \right| = \left| \int_C (e^z - \bar{z}) dz \right| \leq M \times L \leq 5 \times 12 = 60$$

--- [2 M]

[6]



$$8. \quad z^4 + 64 = 0$$

$$\Rightarrow z^4 = -64 = 64 e^{i(\pi+2k\pi)}$$

$$z = 4 e^{i(\frac{\pi}{4} + \frac{k\pi}{2})}$$

where  $k = 0, 1, 2, 3$

At  $k=0$

—— [2 M]

$$z_0 = 4 e^{i\pi/4}$$

$$z_0 = 2\sqrt{2}(1+i) \quad \text{—— [2 M]}$$

At  $k=1$

$$z_1 = 4 e^{i(3\pi/4)}$$

$$z_1 = 2\sqrt{2}(-1+i) \quad \text{—— [2 M]}$$

At  $k=2$

$$z_2 = 4 e^{i5\pi/4}$$

$$z_2 = -2\sqrt{2}(1-i) \quad \text{—— [2 M]}$$

At  $k=3$

$$z_3 = 4 e^{i-7\pi/4}$$

$$z_3 = 2\sqrt{2}(1-i) \quad \text{—— [2 M]}$$

[45]

Ex Calculate the integral

①  $\int_C \frac{1-\cos z}{\sin z} dz$  where  $C: |z|=5$

soln Singularities inside  $|z|=5$  are

$$z_0 = -\pi, z_1 = 0, z_2 = \pi$$

Residues at these singularities

$$\text{Res}_{z=\pi} \frac{1-\cos z}{\sin z} = -2$$

$$\text{Res}_{z=0} \frac{1-\cos z}{\sin z} = 0$$

(6 M)

$$\text{Res}_{z=\pi} \frac{1-\cos z}{\sin z} = -2$$

$$\begin{aligned} \int_{|z|=5} \frac{1-\cos z}{\sin z} dz &= 2\pi i (-2 + 0 - 2) \\ &= -8\pi i \end{aligned}$$

[8]

Q9  
10)

Find

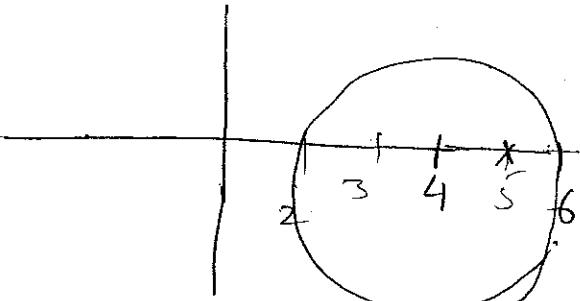
$$I = \int_C \frac{\cos z}{(z-1)^3(z-5)^2} dz$$

where

$$C: |z-4|=2$$

Ans

$$\int_C \frac{\cos z}{(z-1)^3(z-5)^2} dz$$



$$\phi(z) = \frac{\cos z}{(z-1)^3}$$

$$\phi'(z) = -\frac{\sin z (z-1)^3 - 3\cos z (z-1)^2}{(z-1)^6}$$

$$I =$$

$$I = \int_C = \frac{2\pi i}{6!} \left[ \frac{-\sin z (z-1)^3 - 3(z-1)^2 \cos z}{(z-1)^6} \right]_{z=5}$$

$$= \frac{2\pi i}{6!} \left[ \frac{\sin 5 \cdot 4^3 - 3 \cdot 4^2 \cos 5}{4^6} \right]$$

Ans.

[9]

11.

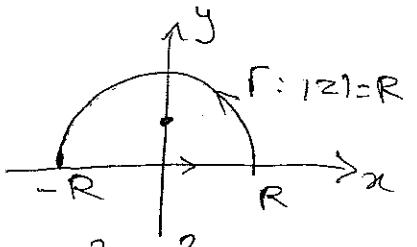
$$\textcircled{a} \quad f(z) = \frac{z}{(z-2)(z+3)} \\ = \frac{2}{5(z-2)} + \frac{3}{5(z+3)} \quad \text{—— (2M)}$$

$$= \frac{2}{5z\left(1-\frac{2}{z}\right)} + \frac{3}{5(3)\left(1+\frac{2}{3}\right)} \\ = \frac{2}{5z} \left(1-\frac{2}{z}\right)^{-1} + \frac{1}{5} \left(1+\frac{z}{3}\right)^{-1} \\ = \frac{2}{5z} \left\{ 1 + \left(\frac{2}{z}\right) + \left(\frac{2}{z}\right)^2 + \dots \right\} \\ + \frac{1}{5} \left\{ 1 - \frac{z}{3} + \left(\frac{z}{3}\right)^2 - \dots \right\} \quad \text{—— (4M)}$$

$$\textcircled{b} \quad f(z) = \frac{2}{5(z-2)} + \frac{3}{5(z+3)} \\ = \frac{2}{5(z-2)} + \frac{3}{5(z-2+5)} \\ = \frac{2}{5(z-2)} + \frac{3}{25} \left(1 + \left(\frac{z-2}{5}\right)\right)^{-1} \\ = \frac{2}{5(z-2)} + \frac{3}{25} \left[1 + \left(\frac{z-2}{5}\right)\right]^{-1} \\ = \frac{2}{5(z-2)} + \frac{3}{25} \left\{ 1 - \left(\frac{z-2}{5}\right) + \left(\frac{z-2}{5}\right)^2 - \dots \right\} \quad \text{—— (4M)}$$

12. Let  $f(x) = \frac{x}{(x^2+1)^2}$

$$f(z) = \frac{z}{(z^2+1)^2}$$



The poles are given by  $(z^2+1)^2=0$

$$\Rightarrow z = \pm i \text{ order 2} \quad (2M)$$

Only the pole  $z=i$  lies in the upper half of the plane.

Let  $C: (-R, R) \cup \Gamma: |z|=R$  as shown in the diagram. (1M)

$\therefore$  By Cauchy Residue theorem

$$\int_C e^{iz} f(z) dz = \int_{-R}^R f(z) e^{iz} dz + \int_{\Gamma} f(z) e^{iz} dz$$

$$= 2\pi R, \quad \boxed{\Gamma} \quad (1M)$$

$$\text{where } R_1 = \operatorname{Res}_{z=i} f(z) e^{iz}$$

$$= \frac{e^{-1}}{4} \quad (2M)$$

$$\therefore \int_{-R}^R e^{ix} f(x) dx + \int_{\Gamma} e^{iz} f(z) dz = 2\pi i \cdot \frac{e^{-1}}{4}$$

$$= \frac{\pi i}{e} \quad (1M)$$

Taking the limit  $R \rightarrow \infty$ ,

$$\int_{-\infty}^{\infty} e^{ix} f(x) dx + \lim_{R \rightarrow \infty} \int_{\Gamma} e^{iz} f(z) dz = \frac{\pi i}{e}$$

Show that by Jordan's lemma,

$$\lim_{R \rightarrow \infty} \int_{\Gamma}^{iz} e^z f(z) dz = 0 \quad \text{--- (M)}$$

$$\therefore \int_{-\infty}^{\infty} e^{ix} f(x) dx = \frac{\pi i}{e}$$

Taking the imaginary parts on both sides,

$$\int_{-\infty}^{\infty} f(x) \sin x dx = \frac{\pi i}{e} \quad \text{--- (M)}$$

**Mathematics II (MATH F112/MATH C 192)**

**Test - 2 (Open Book)**

**Date:** 25.04.2013

**Time:** 50 Minutes

**Max. Marks:** 60

**Weightage:** 20%

**Answer ALL the Questions**

1. Find the eigenvalues and the corresponding eigenvectors for the matrix (12)

$$\begin{pmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{pmatrix}$$

2. Find the kernel and range of L for the linear transformation  $L: R^3 \rightarrow R^2$  given by

$$L \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}. \text{ Also verify the Dimension theorem (Rank- Nullity theorem). (10)}$$

3. Let  $L: P_2 \rightarrow R^2$  be defined by  $L(p(x)) = \left( p'(1), \int_0^1 p(x) dx \right)$  with respect to the ordered basis  $S = \{x^2 + x, x^2 + 1, x\}$  and  $T = \{(1, 1), (1, 2)\}$  for  $P_2$  and  $R^2$  respectively. Find the matrix of the linear transformation L with respect to the bases S and T. (13)

4. Find all the roots of the equation  $z^4 + 1 - i\sqrt{3} = 0$  in Cartesian form. (10)

5. Find the upper and lower bound of  $\left| \frac{z+5}{z^2+4z+3} \right|$  if z lies on the circle  $|z| = 2$ . (8)

6. Show that all the values of  $\left( \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)^i$  lie on the real axis of the complex plane. (7)

**Good luck!**

Mathematics II (MATH F112/MATH C 192)

Test - 2 (Open Book)

25.04.13

Answer Key

1.

Characteristic eqn :  $\lambda^3 - 18\lambda^2 + 19\lambda - 162 = 0$  (3 M)

Eigen values  $\lambda = 3, 6, 9$ . (3 M)

Eigen vectors

for  $\lambda = 3$  is

$$\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

for  $\lambda = 6$  is

$$\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

(or any scalar)

(6 M)

for  $\lambda = 9$  is

$$\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

2.

$$\text{Ker } L = \left\{ \mathbf{x} \in \mathbb{R}^3 \mid L \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solving the homogeneous system

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = K \begin{bmatrix} -9/5 \\ 7/5 \\ 1 \end{bmatrix}$$

$$\therefore \text{Basis for Ker } L = \begin{bmatrix} -9/5 \\ 7/5 \\ 1 \end{bmatrix} \text{ or any scalar}$$

(4 M)

$$\dim(\text{Ker } L) = 1$$

$$\begin{aligned}
 k=0, C_0 &= 2^{1/4} \left[ \frac{\sqrt{3}}{2} + i \frac{1}{2} \right] \\
 k=1, C_1 &= 2^{1/4} \left[ -\frac{1}{2} + i \frac{\sqrt{3}}{2} \right] \\
 k=2, C_2 &= 2^{1/4} \left[ -\frac{\sqrt{3}}{2} - i \frac{1}{2} \right] \\
 k=3, C_3 &= 2^{1/4} \left[ +\frac{1}{2} - i \frac{\sqrt{3}}{2} \right]
 \end{aligned}
 \quad \boxed{(8M)}$$

5.

$$| |z| - 5 | \leq |z + 5| \leq |z| + 5$$

$$3 \leq |z + 5| \leq 7 \quad \text{--- (2M)}$$

$$|z^2 + 4z + 3| = |z+1||z+3|$$

$$| |z|-1 | \cdot | |z|-3 | \leq |z^2 + 4z + 3| \leq (|z|+1)(|z|+3)$$

$$\frac{1}{15} \leq \frac{1}{|z^2 + 4z + 3|} \leq 1 \quad \text{--- (4M)}$$

$$\therefore \frac{1}{5} \leq \left| \frac{z+5}{z^2 + 4z + 3} \right| \leq 7 \quad \text{--- (2M)}$$

$$\begin{aligned}
 \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} &= e^{i(-\frac{\pi}{4} + 2n\pi)}, \quad n = 0, \pm 1, \pm 2, \dots \\
 \left( \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)^i &= \left[ e^{i(-\frac{\pi}{4} + 2n\pi)} \right]^i \\
 &= e^{i(\frac{\pi}{4} - 2n\pi)}, \quad n = 0, \pm 1, \pm 2, \dots
 \end{aligned}
 \quad \boxed{(4M)}$$

Since the complex no. does not contain the imaginary part, for different values of  $n$ , it lies on the real axis. --- (3M)

**BITS Pilani, Dubai Campus**  
**Dubai International Academic City, Dubai**

**First year – Second Semester 2012 – 2013**  
**Mathematics II (MATH F112/MATH C 192)**

**Test - 1 (Closed Book)**

**Date: 07.03.2013**

**Time: 50 Minutes**

**Max. Marks: 75**

**Weightage: 25%**

**Answer ALL the Questions**

1. Solve the following system of linear equations by using Gauss Elimination method. (12)

$$x + 2y + 3z = 9$$

$$2x - y + z = 8$$

$$3x - z = 3$$

2. Solve the following linear homogeneous system of equations using Gauss Jordan method: (13)

$$x_1 + 2x_2 - x_3 + 3x_4 = 0$$

$$2x_1 + 2x_2 - x_3 + 2x_4 = 0$$

$$x_1 + 3x_3 + 3x_4 = 0$$

3. Find the inverse of the matrix  $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$  using Gauss Jordan Method, if it exists. (13)

4. Let  $V$  be the set of all polynomials of the form  $at^2 + bt + c$ , where  $a, b$  and  $c$  are real numbers with  $b = a + 1$ , defined under the usual addition and scalar multiplication. Check whether  $V$  is a vector space. If not, justify all the properties which fail. (12)

5. Consider the set  $W$  of all vectors in  $R^4$  of the form  $(a, b, c, d)$ , where  $b = 3a - 5d$  and  $c = d + 4a$ . Under usual addition and scalar multiplication, is  $W$  a subspace of  $R^4$ ? Justify your answer. (10)

6. (a) Let  $S = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} -3 & 1 \\ 0 & 1 \end{pmatrix} \right\}$ . Determine whether  $v = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  belongs to  $\text{Span } S$ ? Justify your answer. (8)

- (b) Let  $S = \{x^2 + 4x - 3, 2x^2 + x + 5, 7x - 11\}$ . Check whether  $S$  spans  $P_2$ . Justify your answer. (7)

**Good luck!**

First year – Second Semester 2012 – 2013

Mathematics II (MATH F112/MATH C 192)

7.3.13

Test - 1 (Closed Book) Answer Key

1. Gauss Elimination Method :

$$(A|b) : \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 2 & -1 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{array} \right]$$

$$\sim \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$\therefore r(A|b) = r(A) = \text{no. of unknowns}$ , the system is consistent with unique solution.

Using backward solution,  $x=2, y=-1, z=3$

2. Gauss Jordan Method :

$$(A) : \left[ \begin{array}{cccc} 1 & 2 & -1 & 3 \\ 2 & 2 & -1 & 2 \\ 1 & 0 & 3 & 3 \end{array} \right]$$

$$\sim \left[ \begin{array}{cccc} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 8/3 \\ 0 & 0 & 1 & 4/3 \end{array} \right]$$

$\therefore r(A) = 3 < \text{no. of unknowns}$ , the system is consistent with infinite solution.

Let  $x_4 = k$ , then  $x_1 = k$

$$x_2 = -\frac{8}{3}k$$

$$x_3 = -\frac{4}{3}k$$

3.

$$\left[ \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array} \right]$$

$$\sim \left[ \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 3 & 2 & -1 & 0 \\ 0 & 2 & -5 & 1 & 0 & -1 \end{array} \right]$$

$$\sim \left[ \begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array} \right]$$

$$\sim \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & -13 & 5 & 3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array} \right]$$

$$\therefore A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ -13 & 5 & 3 \\ 5 & -2 & -1 \end{bmatrix}$$

4. V is not a vector space under usual  $\oplus$  &  $\odot$  and the following properties are not satisfied.

- i) closure property wr to  $\oplus$
- ii) closure property wr to  $\odot$
- iii) Existence of additive identity
- iv) Existence of additive inverse.

5.

$$W = \{(a, b, c, d) \mid b=3a-5d \text{ & } c=d+4a\}$$

W is a subspace of  $\mathbb{R}^4$ . — (2 M) (Answer)

Prove the axioms

— (8 M)

6. a)  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = c_1 \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} + c_2 \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} + c_3 \begin{pmatrix} -3 & 1 \\ 0 & 1 \end{pmatrix}$

$$(A|b) : \left[ \begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 2 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{array} \right]$$

— (6 M)

$$\sim \left[ \begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & 5 & -7 & 2 \\ 0 & 0 & 8 & 2 \\ 0 & 0 & 0 & 70 \end{array} \right]$$

∴  $r(A|B) \neq r(A)$ , the system is inconsistent.

$$\therefore \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \notin \text{Span } S. — (2 M)$$

b)

$$\text{Let } ax^2+bx+c \in P_2$$

(5 M) —

$$ax^2+bx+c = c_1(x^2+4x-3) + c_2(2x^2+x+5) + c_3(7x-11)$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & 0 & a \\ 4 & 1 & 7 & b \\ -3 & 5 & -11 & c \end{array} \right] \sim \left[ \begin{array}{ccc|c} 1 & 2 & 0 & a \\ 0 & 7 & -7 & 4a-b \\ 0 & 0 & 0 & 21a-11b+7c \end{array} \right]$$

∴  $r(A|b) \neq r(A)$ ,  $P_2$  is not spanned by S.

— (2 M)

BITS PILANI, DUBAI CAMPUS  
DUBAI INTERNATIONAL ACADEMIC CITY  
SECOND SEMESTER 2012-2013  
TEST -1 (CLOSED BOOK)

COURSE NO. BIOF 111      10-3-2013(Weightage25%)      MAXIMUM MARKS: 50  
COURSE NAME; GENERAL BIOLOGY      DURATION: 50 Mins

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- Q1. (a) Differentiate between the following (6 Marks)  
(i) Chromatin and Chromosome (1 major difference)  
(ii) Lysosomes & Peroxisomes (2 major differences)  
(iii) DNA & RNA (2 major differences)
- (b) What factors are important in deciding the permeability of the plasma membrane? Explain. (3 Marks)
- (c) Structure and function of proteins are interrelated, justify the statement with 2 examples. (5 Marks)
- (d) How an organism is able to differentiate between what belongs to its body and what is foreign? Explain with an example. (3 Marks)
- Q2. (a) As an Engineering student justify how the study of biology is helpful in raising the living standard and contributing in future developments? (Answer in Points only) (4 marks)
- (b) Mention the mode of transport of the following molecules; Oxygen, Glucose, Sodium ions, across cell membrane in a tabular column and differentiate between the methods of transport. (6 Marks)
- (c) What is individual adaptation, explain with 3 examples? (5Marks)
- Q3 (a) Cellulose is an important polysaccharide used in constructing call walls of plant cells. In which it is useful for humans? Explain. (2Marks)
- (b) What will happen if Microtubules are removed from an animal cell? Explain (points). (4 Marks)
- (c) List out the components of a bacterial cell and mention their function. (8 Marks)
- (d) In which way the process of phagocytosis is significant for an organism /cell? Explain with 2 examples. (4 Marks)

xxxxxxxxxxxxxxxxxxxxxxxxxxxxx GOOD LUCK xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

BITS PILANI, DUBAI CAMPUS  
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 FIRST YEAR – SECOND SEMESTER 2012- 2013

**B**

Quiz 2

Course Code: MATH F112/MATHC192  
 Course Title: MATHEMATICS II  
 Duration: 20 minutes

Date: 16.5.2013  
 Max Marks: 21  
 Weightage: 7%

Name: ..... ID No: ..... Name of the Faculty: .....  
 Answer the following questions:

1. Check whether  $f(z) = \sqrt{r} e^{i\theta/2}$ , ( $r \neq 0$ ) is analytic. If so, find  $f'(z)$  (6)

$$u = \sqrt{r} \cos(\theta/2) \quad v = \sqrt{r} \sin(\theta/2)$$

$$u_r = \frac{1}{2} r^{-1/2} \cos(\theta/2) \quad v_r = \frac{1}{2} r^{-1/2} \sin(\theta/2)$$

$$u_\theta = -\frac{1}{2} \sqrt{r} \sin(\theta/2) \quad v_\theta = \frac{1}{2} \sqrt{r} \cos(\theta/2)$$

Since the partial derivatives are continuous and satisfies the CR eqs  $u_r = v_\theta$  &  $u_\theta = -v_r$ ,  
 $f(z)$  is analytic

$$\begin{aligned} f'(z) &= e^{-i\theta/2} (u_r + i v_r) \\ &= \frac{1}{2} r^{-1/2} e^{-i\theta/2} \\ &= \frac{1}{2\sqrt{r}} \end{aligned}$$

PTO

2. If  $u(x, y) = e^x \cos y$  is harmonic, find its harmonic conjugate  $v(x, y)$ . (5)

$$u_x = e^x \cos y$$

$$u_y = -e^x \sin y.$$

$u$  &  $v$  satisfies the CR eqs.

$$v_x = -u_y$$

$$= e^x \sin y$$

Sing partially wrt to  $x$ .

$$v(x, y) = e^x \sin y + \phi_1(y) \quad \textcircled{1}$$

$$v_y = u_x$$

$$= e^x \cos y$$

Sing partially wrt to  $y$

$$v(x, y) = e^x \cos y + \phi_2(x) \quad \textcircled{2}$$

From  $\textcircled{1}$  &  $\textcircled{2}$

$$v(x, y) = e^x \cos y + C.$$

PTO

3. Show that  $\text{Log}(-1-i)^2 \neq 2\text{Log}(-1-i)$

(5)

$$\begin{aligned}\text{LHS: } \text{Log}(-1-i)^2 &= \text{Log}(2i) \\ &= \ln 2 + i\pi/2\end{aligned}$$

$$\begin{aligned}\text{RHS: } 2\text{Log}(-1-i) &= 2[\ln\sqrt{2} - i(3\pi/4)] \\ &= \ln 2 - i(3\pi/2)\end{aligned}$$

$\therefore \text{LHS} \neq \text{RHS}$ .

PTO

4. Find all the values of

$$\left( \frac{ie\sqrt{3}-e}{4} \right)^i. \quad (5)$$

$$= e^{i \log \left( \frac{ie\sqrt{3}-e}{4} \right)}$$

$$= e^{i [\ln(e/2) + i(\frac{2\pi}{3} + 2n\pi)]}, \quad n=0, \pm 1, \pm 2, \dots$$

$$= e^{i \ln(\frac{e}{2}) - (\frac{2\pi}{3} + 2n\pi)} \cdot e$$

$$= e^{-\frac{(2\pi+2n\pi)}{3}} [ \cos(1-\ln 2) + i \sin(1-\ln 2) ]$$

$$n=0, \pm 1, \pm 2, \dots$$

BITS PILANI, DUBAI CAMPUS  
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FIRST YEAR – SECOND SEMESTER 2012- 2013

A

Quiz 2

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Course Title: MATHEMATICS II  
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Answer the following questions:

1. Check whether  $f(z) = \sqrt[3]{r} e^{i\theta/3}$ , ( $r \neq 0$ ) is analytic. If so, find  $f'(z)$  (6)

$$u = \sqrt[3]{r} \cos(\theta/3) \quad v = \sqrt[3]{r} \sin(\theta/3)$$

$$u_r = \frac{1}{3} r^{-2/3} \cos(\theta/3) \quad v_r = \frac{1}{3} r^{-2/3} \sin(\theta/3)$$

$$u_\theta = -\frac{1}{3} r^{1/3} \sin(\theta/3) \quad v_\theta = \frac{1}{3} r^{1/3} \cos(\theta/3)$$

Since the partial derivatives are continuous and satisfies the CR eqs  $u_r = v_\theta$  &  $u_\theta = -v_r$ ,  $f(z)$  is analytic

$$f'(z) = e^{-i\theta} (u_r + i v_r)$$

$$= \frac{1}{3} r^{-2/3} e^{-i\theta/3}$$

$$= \frac{1}{3} z^{-2/3}$$

PTO

2. If  $u(x, y) = e^y \cos x$  is harmonic, find its harmonic conjugate  $v(x, y)$ . (5)

$$u_x = -e^y \sin x$$

$$u_y = e^y \cos x$$

$u$  &  $v$  satisfies the CR eqs.

$$\begin{aligned} v_x &= -u_y \\ &= -e^y \cos x. \end{aligned}$$

Sing partially wrt to  $x$

$$v(x, y) = -e^y \sin x + \phi_1(y) \quad (1)$$

$$\begin{aligned} v_y &= u_x \\ &= -e^y \sin x \end{aligned}$$

Sing partially wrt to  $y$

$$v(x, y) = -e^y \sin x + \phi_2(x) \quad (2)$$

From (1) & (2)

$$v(x, y) = -e^y \sin x + C$$

3. Show that  $\log(-1+i)^2 \neq 2\log(-1+i)$  (5)

$$\text{LHS} : \log(-1+i)^2 = \log(-2i) \\ = \ln 2 - i\pi/2$$

$$\text{RHS} : 2\log(-1+i) = 2[\ln 5z + i3\pi/4] \\ = \ln 2 + i3\pi/2$$

$\therefore \text{LHS} \neq \text{RHS}$ .

PTO

4. Find all the values of  $\left(\frac{e - ie\sqrt{3}}{4}\right)^i$ . (5)

$$= e^{i \log\left(\frac{e - ie\sqrt{3}}{4}\right)}$$

$$= e^{i [\ln(e/2) + i(-\pi/3 + 2n\pi)]}, \quad n=0, \pm 1, \pm 2, \dots$$

$$= e^{i \ln(e/2)} \cdot e^{i(-\pi/3 + 2n\pi)}$$

$$= e^{i(-\pi/3 + 2n\pi)} [\cos(1 - \ln 2) + i \sin(1 - \ln 2)]$$

$$n=0, \pm 1, \pm 2, \dots$$

B

BITS PILANI, DUBAI CAMPUS  
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FIRST YEAR – SECOND SEMESTER 2012- 2013

Quiz 2

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Course Title: MATHEMATICS II  
Duration: 20 minutes

Date: 16.5.2013  
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Weightage: 7%

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Answer the following questions:

1. Check whether  $f(z) = \sqrt{r} e^{i\theta/2}$ , ( $r \neq 0$ ) is analytic. If so, find  $f'(z)$  (6)

PTO

2. If  $u(x, y) = e^x \cos y$  is harmonic, find its harmonic conjugate  $v(x, y)$  (5)

PTO

3. Show that  $\text{Log}(-1-i)^2 \neq 2 \text{Log}(-1-i)$

(5)

PTO

4. Find all the values of

$$\left( \frac{ie\sqrt{3} - e}{4} \right)^i \quad (5)$$

A

BITS PILANI, DUBAI CAMPUS  
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FIRST YEAR – SECOND SEMESTER 2012- 2013

Quiz 2

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PTO

2. If  $u(x, y) = e^y \cos x$  is harmonic, find its harmonic conjugate  $v(x, y)$ . (5)

PTO

3. Show that  $\text{Log}(-1+i)^2 \neq 2 \text{ Log}(-1+i)$

(5)

PTO

4. Find all the values of

$$\left( \frac{e - ie\sqrt{3}}{4} \right)^i. \quad (5)$$

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A

Quiz 1

Course Code: MATH F112/MATH C192  
Course Title: MATHEMATICS II  
Duration: 20 minutes

Date: 04.4.2013  
Max Marks: 24  
Weightage: 8%

Name: ..... ID No: ..... Name of the Faculty: .....

Answer the following questions:

1. If  $S = \{(1, -2, 3), (4, 2, 1), (2, a, b)\}$ , find the values of constants  $a$  and  $b$  for which the following set of vectors  $\mathbb{R}^3$  are linearly dependent. (5)
  
2. Find the basis and dimension of the subspace of  $P_3$  consisting of all vectors of the form  $ax^3 + bx^2 + cx + d$  where  $b = 3a - 5d$  and  $c = d + 4a$  (5)

3. Let  $L : R^2 \rightarrow R^2$  be the linear transformation such that  $L(1, 1) = (0, 2)$  and  $L(1, -1) = (2, 0)$ . Find  $L(1, 4)$ . (5)

4. Check whether  $L : R^2 \rightarrow R$  defined by  $L(x, y) = |x - y|$  is a linear transformation. Justify your answer. (4)

5. If  $L : R^2 \rightarrow R^2$  is a linear transformation defined by  $L(x, y) = (x + y, x + y)$ , check whether L is one-one and onto. Justify your answer. (5)

BITS PILANI, DUBAI CAMPUS  
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FIRST YEAR - SECOND SEMESTER - 2012- 2013

B

Quiz 1

Course Code: MATH F112/MATH C192  
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Date: 04.4.2013  
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Weightage: 8%

Name: ..... ID No: ..... Name of the Faculty: .....

Answer the following questions:

1. If  $S = \{(1, -2, 3), (2, 6, -5), (4, a, b)\}$ , find the values of constants  $a$  and  $b$  for which the following set of vectors  $\mathbb{R}^3$  are linearly dependent. (5)
2. Find the basis and dimension of the subspace of  $P_3$  consisting of all vectors of the form  $ax^3 + bx^2 + cx + d$  where  $b = 3a + 5d$  and  $c = d - 4a$  (5)

3. Let  $L : R^2 \rightarrow R^2$  be the linear transformation such that  $L(1, 1) = (0, 2)$  and  $L(1, -1) = (2, 0)$ . Find  $L(4, -1)$ . (5)

4. Check whether  $L : R^2 \rightarrow R$  defined by  $L(x, y) = |x + y|$  is a linear transformation. Justify your answer. (4)

5. If  $L : R^2 \rightarrow R^2$  is a linear transformation defined by  $L(x, y) = (x - y, x - y)$ , check whether L is one-one and onto. Justify your answer. (5)

BITS PILANI, DUBAI CAMPUS  
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FIRST YEAR - SECOND SEMESTER - 2012- 2013

A

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Answer the following questions:

1. If  $S = \{(1, -2, 3), (4, 2, 1), (2, a, b)\}$ , find the values of constants  $a$  and  $b$  for which the following set of vectors  $\mathbb{R}^3$  are linearly dependent. (5)

$$c_1(1, -2, 3) + c_2(4, 2, 1) + c_3(2, a, b) = 0$$

$$\begin{bmatrix} 1 & 4 & 2 \\ -2 & 2 & a \\ 3 & 1 & b \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 2 \\ 0 & 10 & a+4 \\ 0 & 0 & 11a-10b+104 \end{bmatrix} \quad (4)$$

$S$  is linearly dependent if  $11a - 10b + 104 = 0$ .

$$\text{If } a = k, \quad b = \frac{11k + 104}{10} \quad (1)$$

2. Find the basis and dimension of the subspace of  $P_3$  consisting of all vectors of the form  $ax^3 + bx^2 + cx + d$  where  $b = 3a - 5d$  and  $c = d + 4a$  (5)

$$\begin{aligned} ax^3 + bx^2 + cx + d &= ax^3 + (3a - 5d)x^2 + (d + 4a)x + d \\ &= a(x^3 + 3x^2 + 4x) + d(-5x^2 + x + 1) \end{aligned}$$

$\therefore \text{Range } L = \text{Span } S$  (2)

$$\text{where } S = \{x^3 + 3x^2 + 4x, -5x^2 + x + 1\}$$

Since  $S$  is linearly independent,

$S$  forms the basis for Range  $L$

$$\dim(\text{Range } L) = 2.$$

3. Let  $L: R^2 \rightarrow R^2$  be the linear transformation such that  $L(1, 1) = (0, 2)$  and  $L(1, -1) = (2, 0)$ . Find  $L(1, 4)$ . (5)

$$\begin{aligned} (1, 4) &= \frac{5}{2}(1, 1) - \frac{3}{2}(1, -1) \quad \text{--- (2)} \\ L(1, 4) &= \frac{5}{2}L(1, 1) - \frac{3}{2}L(1, -1) \\ &= \frac{5}{2}(0, 2) - \frac{3}{2}(2, 0) \\ &= (-3, 5) \quad \text{--- (3)} \end{aligned}$$

4. Check whether  $L: R^2 \rightarrow R$  defined by  $L(x, y) = |x - y|$  is a linear transformation. Justify your answer. (4)

L is not a linear transformation. This can be proved by taking any axiom or by particular example. (1) (3)

5. If  $L: R^2 \rightarrow R^2$  is a linear transformation defined by  $L(x, y) = (x+y, x+y)$ , check whether L is one-one and onto. Justify your answer. (5)

Ker L :  $L(x, y) = (x+y, x+y) = (0, 0)$   
 $x = -y$ ,  $y = K$ .

$$\text{Ker } L = K(-1, 1)$$

Since  $\dim(\text{Ker } L) \neq 0$ , L is not 1-1. (3)

By dimension theorem,  $\dim(\text{Range } L) = \dim(R^2) - \dim(\text{Ker } L)$   
 $= 2 - 1 = 1$

Since  $\dim(\text{Range } L) \neq \dim(\text{codomain})$ ,

L is not onto (2)

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Quiz 1

B

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$$c_1(1, -2, 3) + c_2(2, 6, -5) + c_3(4, a, b) = 0$$

$$\begin{bmatrix} 1 & 2 & 4 \\ -2 & 6 & 9 \\ 3 & -5 & b \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & 10 & a+8 \\ 0 & 0 & 10b+11a-32 \end{bmatrix} \quad (4)$$

$S$  is linearly dependent if  $10b+11a-32=0$

$$\text{If } a=k, b = \frac{32-11k}{10} \quad (1)$$

2. Find the basis and dimension of the subspace of  $P_3$  consisting of all vectors of the form  $ax^3 + bx^2 + cx + d$  where  $b = 3a + 5d$  and  $c = d - 4a$  (5)

$$\begin{aligned} ax^3 + bx^2 + cx + d &= ax^3 + (3a + 5d)x^2 + (d - 4a)x + d \\ &= a(x^3 + 3x^2 - 4x) + d(5x^2 + x + 1) \end{aligned} \quad (2)$$

Range L = Span S where

$$S = \{x^3 + 3x^2 - 4x, 5x^2 + x + 1\}$$

Since S is linearly independent, S forms the basis for Range L and  $\dim(\text{Range L}) = 2$ . (3)

3. Let  $L: R^2 \rightarrow R^2$  be the linear transformation such that  $L(1, 1) = (0, 2)$  and  $L(1, -1) = (2, 0)$ . Find  $L(4, -1)$ . (5)

$$(4, -1) = \frac{3}{2}(1, 1) + \frac{5}{2}(1, -1) \quad \text{--- (2)}$$

$$\begin{aligned} L(4, -1) &= \frac{3}{2}L(1, 1) + \frac{5}{2}L(1, -1) \\ &= \frac{3}{2}(0, 2) + \frac{5}{2}(2, 0) \\ &= (3, 5) \end{aligned} \quad \text{--- (3)}$$

4. Check whether  $L: R^2 \rightarrow R$  defined by  $L(x, y) = |x + y|$  is a linear transformation. Justify your answer. (4)

L is not a Linear transformation. This can be proved by taking any axiom or by particular example. (3)

5. If  $L: R^2 \rightarrow R^2$  is a linear transformation defined by  $L(x, y) = (x - y, x + y)$ , check whether L is one-one and onto. Justify your answer. (5)

$$\begin{aligned} \text{Ker } L : L(x, y) &= (x - y, x + y) = (0, 0) \\ x &= k, y = k. \end{aligned}$$

$\text{Ker } L = K(1, 1)$  or any equivalent vector.

Since  $\dim(\text{Ker } L) \neq 0$ , L is not 1-1. (3)

By dimension theorem,

$$\dim(\text{Range } L) = \dim(R^2) - \dim(\text{Ker } L)$$

$$= 1$$

Since  $\dim(\text{Range } L) \neq \dim(\text{co-domain})$ ,

L is not onto (2)