BITS Pilani
Dubai Campus

Second Semester 2011-2012
COMPREHENSIVE EXAMINATION

Course: Probability & Statistics **Course No. MATH F113** **Year: I**
Max marks: 120 **Weight age: 40%**
Date: 12-06-2012 **Time: 3 hours**

ANSWER ALL QUESTIONS.

1. The following are measurements of the air velocity and evaporation coefficient of burning fuel droplets in an impulse engine:

Airvelocity(cm/sec) x:	20	60	100	140	180	220	260	300	340	380
Evaporation coefficient (mm^2/sec) y	0.18	0.37	0.35	0.78	0.56	0.75	1.18	1.36	1.17	1.65

a) Estimate a linear regression equation.
b) Estimate the evaporation coefficient of a droplet when the air velocity is 190cm/sec. [12]

2. The following are the numbers of minutes it took 10 mechanics to assemble a piece of machinery in the morning, x and in the late afternoon, y:

X	11.1	10.3	12.0	15.1	13.7	18.5	17.3	14.2	14.8	15.3
Y	10.9	14.2	13.8	21.5	13.2	21.1	16.4	19.3	17.4	19.0

Estimate the correlation between X & Y. [12]

3. As society becomes dependent on computers, data must be communicated via public communication networks such as satellites, microwave systems and telephones. When a message is received, it must be authenticated. This is done by using a secret enciphering key. Even though the key is secret, there is always the possibility that it will fall into the wrong hands, thus allowing an unauthentic message to appear to be authentic. Assume that 95% of all messages received are authentic. Furthermore, assume that only 0.1% of all unauthentic messages are sent using the correct key and that all authentic messages are sent using the correct key. Find the probability that a message is authentic given that the correct key is used. [12]

4. It has been found that 80% of all printers used on home computers operate correctly at the time of installation. The rest require some adjustment. A particular dealer sells 10 units during a given month.
a) Find the probability that at least 9 of the printers operate correctly upon installation.
b) Consider 5 months in which 10 units are sold per month. What is the probability that at least 9 units operate correctly in each of the 5 months? [12]

5. Let X denotes the length in minutes of a long distance telephone conversation. Assume that the density for X is given by

   $f(x) = (1/10)\, e^{-x/10}$; $x>0$

   a) Verify f is a density for a continuous random variable.

   b) Find the probability that a randomly selected call will last exactly 7 minutes. [12]

6. Let X denote the time in hours needed to locate and correct a problem in the software that governs the timing of traffic lights in the downtown area of a large city. Assume that X is normally distributed with mean 10 hours and variance 9.

   a) Find the probability that the next problem will require at most 16 hours to find and correct.

   b) Find $P(7<X<13)$. [12]

7. Let X denote the temperature (°C) and let Y denote the time in minutes that it takes for the diesel engine on an automobile to get ready to start. Assume that the joint density for (X,Y) is given by

   $$f(x,y) = c\,(4x + 2y + 1), \quad 0 \le x \le 40, \quad 0 \le y \le 2$$

   a) Find the value of c that makes this a density.

   b) Find the probability that on a randomly selected day the air temperature will exceed 20 °C and it will take at least 1 minute for the car to be ready to start. [12]

8. Recent research indicates that heating and cooling commercial buildings with ground water source heat pumps is economically sound. The crucial random variable being studied is the water temperature. A sample of 15 wells in the state of California yields a sample standard deviation of 7.5° F. Find a 95% confidence interval on the standard deviation in temperature of wells in California. [12]

9. A new computer network is being designed. The makers claim that it is compatible with more than 99% of the equipment already in use. [12]

   a) Set up the null and alternative hypotheses needed to get evidence to support this claim..

   b) A sample of 300 programs is run, and 298 of these run with no changes necessary. That is, they are compatible with the new network. Can H_o be rejected at 5% level of significance?

10. A random sample from a company's very extensive files shows that orders for a certain piece of machinery were filled, respectively, in 10, 12,19,14,15,18, 11 and 13 days.

a) Write the alternative hypothesis so that rejection of the null hypothesis $\mu = 10.5$ implies that it takes longer than indicated.

b) At level of significance 0.01 can H_o (i.e on the average such orders are filled in 10.5 days) be rejected. Assume normality. [12]

TABLE VALUES: (As per the standard notation) (where α is the right tailed probability)

$P(Z\le2) = 0.9772$, $P(Z\le1) = 0.8413$, $Z_{0.05} = 1.645$

$X^2_{14,\,0.975} = 5.63$, $X^2_{14,\,0.025} = 26.1$, $X^2_{15,\,0.975} = 6.26$, $X^2_{14,\,0.025} = 27.5$

$t_{8,0.01} = 3.545$, $t_{7,0.01} = 2.998$

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Second Semester 2011-2012
TEST -II (OB)

Course: Probability & Statistics | **Course No. MATH F113** | **Year: I**
Max marks: 60 | **Weight age: 20%**
Date: ~~15-04-2012~~ 13-4-12 | **Time: 50 Minutes**

ANSWER ALL QUESTIONS.

1. XYZ have brought a new chemical processing facility into production, top management wants to know the dependable long term capabilities of the system. The processes are delicate, and no matter how carefully things are controlled, there is still some variation from day to day and even from hour to hour in the amount produced. Construct a 90% confidence interval for the average long term yield based on measured yields for a sample of time periods as given below:

71.7, 46.0, 103.9, 54.4, 43.3, 68.1, 73.4, 45.1, 45.6, 44.9, 77.8, 50.5. [10]

2. A soft drink dispensing machine is said to be out of control if the variance of the contents exceeds 1.15 deciliters. If a random sample of 25 drinks from this machine has a variance of 2.03 deciliters, does this indicate at the 0.05 level of significance that the machine is out of control? Assume that the contents are approximately normally distributed.

a) Formulate appropriate null and alternative hypothesis.
b) Can null hypothesis be rejected at 5% level of significance.
c) What is the practical implication of the result? [10]

3. If the cholesterol level of healthy men is normally distributed with a mean of 180 and a standard deviation of 20, at what level (in excess of 180) should men be diagnosed as not healthy from a sample of 25 men, if you want the probability of a type one error to be 2%? [10]

4. A random variable X has the density function $f(x)=\frac{1}{2}e^{-|x|}, -\infty < x < \infty$.

Find an upper bound of $P\{|X-\mu|>2\}$ using the Chebychev's theorem and compare with the actual probability. [12]

5. Find the simulated value of the random variable having Exponential distribution with mean=2 corresponding to the value 0.39 of a uniform random variable. [6]

6. If two random variables X and Y have the joint density

$$f(x,y)=\begin{cases}2(x+2y)/3 & for \quad 0<x<1, \quad 0<y<1\\ 0 & otherwise\end{cases}$$

find a) the marginal densities of X and Y;
b) the conditional density of X when $Y=0.5$. [12]

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Second Semester 2011-2012
TEST -I (CB)

Course: Probability & Statistics **Course No. MATH F113**
Max marks: 75 **Weight age: 25%**
Date: 22-03-2012 **Time: 50 Minutes** **Year: I**

ANSWER ALL QUESTIONS

1. Twenty microprocessor chips are in stock. Three have itching errors that cannot be detected by naked eye. Five chips are selected and installed in field equipment.
   a. Find E(X) and Var X.
   b. Find the probability that no chips with etching errors will be selected.
   c. Find the probability that at least one chip with an etching error will be chosen.

   [15]
2. It has been found that 80% of all printers used on home computers operate correctly at the time of installation. The rest require some adjustment. A particular sells 10 units during a given month.
   a. Find the probability that at least nine of the printers operate correctly upon installation.
   b. Consider 5 months in which 10 units are sold per month. What is the probability that at least 9 units operate correctly in each of the 5 months? [15]

3. Consider the random variable X with the density.

$$f(x) = \left(\frac{1}{6}\, x\right) \quad 2 \leq x \leq 4$$

   a. Verify f is probability density function
   b. Find F(x)
   c. Find E(x)

   [15]

4. Rock noise in an underground mine occurs at an average rate of three per hour. Find the probability that no rock noise will be recorded for at least 30 minutes. [15]

5. A consulting firm rents cars from three agencies, 20% from agency D, 20% from agency E and 60% from agency F. If 10% of the cars from D, 12% of the cars from E and 4% of the cars from F have bad tires.
   (i) What is the probability that affirm will get a car with bad tires?
   (ii) What is the probability that a car with bad tires rented by the firm came from agency F?

   [15]

BITS PILANI – DUBAI CAMPUS
Dubai International Academic City
I Year – II Semester
QUIZ-II (CB)

Course: Probability & Statistics Course No. MATH F113
Max marks: 21 Weightage: 7% Date: 18-04-2012 Time: 20 Minutes

Name: **Id No.:** **Sec:**

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1. The joint distribution of X and Y is defined as follows:

X→ -------- Y↓	0	1	2
0	0.21	0.10	0.32
1	0.01	__	0.1
2	0.10	0.1	0.04

(i) The missing value is _____
(ii) f (x=1) is _____
(iii) Are X and Y independent _____(Yes, No)

2. *X* and *Y* have the joint probability density function

$$f(x,y)=\frac{8x^2}{7y^3}, \quad x\geq 1,\ y\leq 2$$

0, elsewhere

The P (x <1, y>2) is

(a) 0.2 (b) 0 (c) 0.53 (d) 0.88

3. If E(X) = 2 , E(Y) = 8 and E(XY) = 20, then Cov(X,Y) = _____

4. For a normal random variable, P[|X-μ| <2σ] = 0.95. The value assigned to this probability via Chebychev's inequality is _____

5. Suppose X~N(5,3^2) . What is P(X≤8) in terms of the standard normal variable Z?
(a) P(Z≤1) (b) P(Z≤-1) (c) P(Z≤0.6) (d) P(Z≤-0.6) (e) P(Z≤1.67)

BITS PILANI – DUBAI CAMPUS
Dubai International Academic City
I Year – II Semester
QUIZ-I (CB)

Course: Probability & Statistics Course No. MATH F113
Max marks: 24 Weightage: 8% Date: 29-02-2012 Time: 20 Minutes

Name: **Id No.:** **Sec:**

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1. If X has mean 4 and variance 6 while Y has mean -3 and variance 6 and the two are independent then Var (2X + 3Y – 4) is _____

2. You are throwing a ball again and again to hit a spot on a nearby wall. It is known that 20% of the throws fail to hit the spot. The probability that you will hit the spot the first time in fifth throw is _____

3. A lot consists of 10 good articles, 6 with minor defects and 4 with major defects. If two articles are chosen at random, the probability that neither has major defects is _____

4. A box contains 4 bad and 6 good tubes. Two are drawn out from the box at a time. One of them is tested and found to be bad. The probability that the other one is also bad is_____.

5. One die and three coins are thrown. Then the total number of sample points in the sample space is _____

6. Seven white balls, five blue balls and four red balls are arranged in a row. If balls of same colour are indistinguishable from each other then the number of different arrangements possible is_____

7. Bag I contains two white and three black marbles and bag II contains four white and one black marble. A marble is chosen at random from one of the bags is black. The probability that it has come from bag I is_____

8. A card is drawn from a well-shuffled pack of playing cards. The probability that it is neither a spade nor an ace is _____

BITS PILANI – DUBAI CAMPUS
Dubai International Academic City
I Year – II Semester
QUIZ-I (CB)

Course: Probability & Statistics Course No. MATH F113
Max marks: 24 Weightage: 8% Date: 29-02-2012 Time: 20 Minutes

Name: **Id No.:** **Sec:**

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1. If X has mean 5 and variance 7 while Y has mean -4 and variance 7 and the two are independent then Var(2X + 3Y – 4) is _____

2. You are throwing a ball again and again to hit a spot on a nearby wall. It is known that 25% of the throws fail to hit the spot. The probability that you will hit the spot the first time in fifth throw is _____

3. A lot consists of 10 good articles, 4 with minor defects and 2 with major defects. If two articles are chosen at random, the probability that neither has major defects is _____

4. A box contains 4 bad and 6 good tubes. Two are drawn out from the box at a time. One of them is tested and found to be good. The probability that the other one is also good is_____.

5. One die and two coins are thrown. Then the total number of sample points in the sample space is _____

6. Six white balls, four blue balls and three red balls are arranged in a row. If balls of same colour are indistinguishable from each other then the number of different arrangements possible is_____

7. Bag I contains two white and three black marbles and bag II contains four white and one black marble. A marble is chosen at random from one of the bags is white. The probability that it has come from bag I is_____

8. A card is drawn from a well-shuffled pack of playing cards. The probability that it is either a spade or an ace is _____