

**BITS PILANI, DUBAI CAMPUS**  
**INTERNATIONAL ACADEMIC CITY, DUBAI**

FIRST YEAR - SEMESTER-II (2011-12)

**MATHEMATICS-II (MATH F112/MATH C192)**

**COMPREHENSIVE EXAMINATION (CLOSED BOOK)**

Date: 05.06.2012

Time: 3 hours

Max. Marks: 120

Weightage : 40 %

Answer Part A, Part B and Part C in separate Answer Books.

Answer all the questions.

**PART A**

1. Solve the system of linear equations by Gauss Jordan Method:

$$x_1 + 2x_2 - x_3 + 3x_4 = 0, \quad 2x_1 + 2x_2 - x_3 + 2x_4 = 0, \quad x_1 + 3x_3 + 3x_4 = 0 \quad [10]$$

2. Let  $L: R^2 \rightarrow R^3$  be defined by  $L(a, b) = (a - 2b, 2a + b, a + b)$  and let  $S = \{(1, -1), (0, 1)\}$  and  $T = \{(1, 1, 0), (0, 1, 1), (1, -1, 1)\}$  be the bases for  $R^2$  and  $R^3$  respectively. Find the matrix representing  $L$  with respect to  $S$  &  $T$ . [10]

3. Determine whether  $S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \right\}$  forms a basis for  $M_{22}$ . If not, find a basis and dimension of the subspace spanned by  $S$ . [10]

4. Determine whether the linear transformation  $L \left( \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$  is invertible. If so, find  $L^{-1}$ . [10]

**PART B**

5. Find the eigenvalues and eigenvectors of the following matrix: [12]

$$A = \begin{pmatrix} 3 & 1 & 1 \\ -4 & -2 & -5 \\ 2 & 2 & 5 \end{pmatrix}$$

6. Find the fourth roots of  $(-8)$ . [8]
7. Check whether  $u(x, y) = e^{2x}(x \cos 2y - y \sin 2y)$  is harmonic and hence find its harmonic conjugate. [10]
8. (a) Find  $\log(-1 - i\sqrt{3})$  and its principal value. [4]
- (b) Find all the values of  $(1 - i)^{-i}$ . [6]

**PART C**

9. Find the upper bound of (without evaluating the integral)  $\left| \int_C \frac{(z^6 + 6)\text{Log}z}{z^2 - 1} dz \right|$  where

$$C = \left\{ z : z = 2e^{i\theta}, 0 \leq \theta \leq \frac{\pi}{2} \right\}. \quad [10]$$

10. Find the value of the integral  $\int_C \frac{e^{3iz}}{(z - \pi)^2(z - 7)} dz$  where C is taken counter clockwise around the circle  $|z - \pi| = 3.14$ . [8]

11. Find the Laurent's series expansions for  $f(z) = \frac{7z - 2}{z^3 - z^2 - 2z}$  valid in the regions

a)  $0 < |z + 1| < 1$       (b)  $|z + 1| > 3$ . [10]

12. Use the calculus of residues to evaluate  $\int_{-\infty}^{\infty} \frac{x \sin x}{(x^2 + 9)(x^2 + 16)} dx$ . [12]

**All the Best**

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**First year – Second Semester 2011 – 2012**

**Mathematics II (MATH F112/MATH C192)**

**Test - 2 (Open Book)**

**Date: 22.04.2012**  
**Time: 50 Minutes**

**Max. Marks: 60**  
**Weightage: 20%**

**Answer ALL the Questions**

1. Let  $L : R^3 \rightarrow R^3$  defined by  $L[(x, y, z)] = (x + y + z, 2x - y + 3z, z - 3y)$ . Find the kernel L, range L and verify the dimension theorem. (10)
2. If  $L : R^3 \rightarrow M_{22}$  defined by  $L[(a, b, c)] = \begin{pmatrix} 2a+b & 3b-c \\ a+c & a+b+c \end{pmatrix}$ , check whether L is 1-1 and onto. Justify your answer. (10)
3. Find the matrix of the linear transformation  $L(ax^3 + bx^2 + cx + d) = (a+b)x^2 + (b+c)x + (c+d)$  with respect to  $S = \{x^3, x^2 + x^3, x + x^2, 1 + x\}$  and  $T = \{x^2 + 4x + 3, 2x^2 + x - 5, 2\}$ . (11)
4. Check whether  $L : P_2 \rightarrow R^3$  defined by  $L(ax^2 + bx + c) = (a - c, 2b, a + c)$  is invertible. If so find  $L^{-1}$ . (11)
5. Find the cube roots of  $\frac{1}{\sqrt{3} - i}$  (10)
6. Find the upper bound of  $\left| \frac{z-3}{2z^2 - z - 3} \right|$  if z lies on the circle  $|z| = 2$ . (8)

**Good luck!**

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First year – Second Semester 2011 – 2012

Mathematics II (MATH F112/MATH C 192)

Test - 1 (Closed Book)

Date: 08.03.2012  
Time: 50 Minutes

Max. Marks: 75  
Weightage: 25%

Answer ALL the Questions

1. Solve the following system of linear equations by using Gauss Elimination method. (10)  
$$\begin{aligned}x + 2y - 3z &= 4 \\x + 3y + z &= 11 \\2x + 5y - 4z &= 13 \\2x + 6y + 2z &= 22\end{aligned}$$
  
2. Find all values of  $k$  for which the given system has (a) no solution (b) infinitely many solutions. (10)  
$$\begin{aligned}2x + 2y + 3z &= 2 \\3x - y + 5z &= 2 \\x - 3y + 2z &= k - 2\end{aligned}$$
  
3. Find the inverse of the matrix  $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$  using Gauss Jordan Method. (11)
  
4. (a) Let  $V$  be the set of all ordered pairs  $(a, b)$  defined by  $(a, b) \oplus (c, d) = (ac, bd)$  and  $k \otimes (a, b) = (ka, kb)$ . Is  $V$  a vector space? (7)  
(b) Let  $S = \{(a, b, c) \in \mathbb{R}^3 / a = 2b = 3c\}$ . Is  $S$  a subspace of  $\mathbb{R}^3$  under the usual operations of vector addition and scalar multiplication. Justify your answer. (7)
  
5. Let  $S = \{(1, 3, -1), (2, 7, -3), (4, 8, -7)\}$ . Determine whether  $v = (1, 4, 2)$  belongs to  $\text{Span } S$ ? If so, express  $v$  as a linear combination of vectors of  $S$ . (10)
  
6. Check whether the set  $S = \{x^2 + 2x - 1, 2x^2 + x + 4, 5x + 3\}$  spans  $P_2$ . (10)
  
7. Find the eigenvalues and eigenvectors for the matrix (10)  
$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{pmatrix}$$

**Good luck!**

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FIRST YEAR SECOND SEMESTER 2011- 2012

A

Quiz 2

Course Code: MATH F112/MATHC192  
Course Title: MATHEMATICS II  
Duration: 20 minutes

Date: 16.5.2012  
Max Marks: 21  
Weightage: 7%

Name: ..... ID No: ..... Sec: ...

Answer the following questions:

1. Check whether  $f(z) = \sqrt[3]{r} e^{i\theta/3}$  is analytic. If so find  $f'(z)$  (6)

PTO

2. Show that  $u(x, y) = \sin x \cosh y$  is harmonic and then find its harmonic conjugate. (6)

3. Find all the values of  $\log\left(\frac{e + ie\sqrt{3}}{4}\right)$ . (4)

PTO

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**B**

Quiz 2

Course Code: MATH F112/MATHC192  
Course Title: MATHEMATICS II  
Duration: 20 minutes

Date:16.5.2012  
Max Marks: 21  
Weightage:7%

Name: ..... ID No: ..... Sec: ...

Answer the following questions:

1. Check whether  $f(z) = \sqrt[4]{r} e^{i\theta/4}$  is analytic. If so find  $f'(z)$  (6)

PTO



2. Show that  $u(x, y) = \cos x \cosh y$  is harmonic and then find its harmonic conjugate. (6)

3. Find all the values of  $\log\left(\frac{e - ie\sqrt{3}}{4}\right)$ . (4)

PTO