BITS, PILANI-DUBAI
INTERNATIONAL ACADEMIC CITY, DUBAI

FIRST YEAR - SEMESTER-II (2010-11)
MATHEMATICS-II (MATH C192)
COMPREHENSIVE EXAMINATION (CLOSED BOOK)

Date: 06.06.2011
Time: 3 hours

Max. Marks: 120
Weightage: 40 %

Answer Part A, Part B and Part C in separate Answer Books.

Answer all the questions.

PART A

1. Solve the system of linear equations using Gauss Jordan Method.
    $$x + 2y + z = 3,\; 2x + 5y - z = -4,\; 3x - 2y - z = 5$$
    [10]

2. (i) Determine whether the vectors $\{(1, 1, 2), (2, 3, 1), (4, 5, 5)\}$ in R^3 are linearly independent? Justify your answer.
    [5]

    (ii) Consider the subspace $W = \{(a, b, c, d) : a = d, b = 2c\}$ of R^4. Find a basis and dimension of W.
    [5]

3. Determine whether $S = \{t^3 + t^2 - 2t + 1,\; t^2 + 1,\; t^3 - 2t,\; 2t^3 + 3t^2 - 4t + 3\}$ forms a basis for P_3. If not, find the basis and dimension of the subspace they span.   [10]

4. Let $L : R^4 \to R^3$ be defined by $L(x, y, z, t) = (x - y + z + t,\, x + 2z - t,\, x + y + 3z - 3t)$. Find bases for $Ker\, L$ and $Range\, L$. Also verify the rank-nullity theorem.   [10]

PART B

1. Find the eigenvalues and eigenvectors of the following matrix.

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix}$$
[9]

2. a) Find all the roots of the equation $z^4 + 16 = 0$ [5]

   b) Find all the values of $\log(-1 + i\sqrt{3})$ [4]

[PTO]

3. Show that the function $f(z) = z \sin z$ is an entire function. Also find $f'(z)$. [10]

4. Find an analytic function whose real part is $u(x, y) = e^x \cos y$. [6]

5. If C is the upper half of the circle $|z| = R$, then find an upper bound for the integral in terms of R:

$$\left| \int_C \frac{(2z^2 - 1)\, dz}{z^4 + 5z^2 + 4} \right|$$

[6]

PART C

1. Evaluate $\oint_C f(z)dz$ where $f(z) = \pi \exp(\pi \overline{z})$ and C is the boundary of the square with vertices $0, 1, (1 + i),\ i$ and the orientation is in the positive direction. [10]

2. Find the value of the integral $\int_C \frac{\cosh(\pi z)dz}{z(z^2 + 1)}$ where C is the circle $|z| = 2$. [10]

3. Find the Laurent's series expansions for $\dfrac{z}{(z-1)(z-3)}$ in the region $0 < |z-1| < 2$. [10]

4. Find the value of the improper integral $\int_0^\infty \frac{\cos x\, dx}{(x^2 + 9)(x^2 + 4)}$

[10]

All the Best

Mathematics II (MATH C 192)

Test - 2 (Open Book)

Date: 24.04.2011   **Max. Marks: 60**
Time: 50 Minutes   **Weightage: 20%**
Answer ALL the Questions

1. Find the eigenvalues and the corresponding eigenvectors for the following matrix:

$$A = \begin{pmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{pmatrix}$$ (9M)

2. Consider the linear transformation defined by $L(x, y, z) = (2x, 4x - y, 2x + 3y - z)$.
    Show that L is invertible and find L^{-1}. Also find the matrix representing L^{-1} with
    respect to the natural basis of R^3.   (9M)

3. Verify the rank-nullity theorem for the linear transformation $L : P_2 \rightarrow P_2$ defined by
    $L(at^2 + bt + c) = (a + 2c)t^2 + (b - c)t + (a - c)$, finding bases for the range and null
    space.   (9M)

4. Find the fourth root of $\left(\dfrac{i}{-\sqrt{3} - i} \right)$ (8M)

5. Show by two path method that $f'(0)$ does not exist for

    $$f(z) = \frac{z^5}{|z|^4} \qquad \text{when } z \neq 0$$
    $$= 0 \qquad \text{when } z = 0$$ (9M)

6. Find p such that $f(z) = r^3(\cos 3\theta + i \sin p\theta)$ is differentiable everywhere and also
    find $f'(z)$?   (8M)

7. Show that $f(z) = z e^z$ is analytic everywhere and hence find its derivative.   (8M)

ALL THE BEST!

Mathematics II (MATH C 192)

Test - 1 (Closed Book)

Date: 06.03.2011   Max. Marks: 75
Time: 50 Minutes   Weightage: 25%

Answer ALL the Questions

1. Solve the following system of linear equations by using Gauss elimination method.   (9)
$$2x - 3y + z = -1$$
$$3x + z = 6$$
$$x + 2y - 2z = -1$$

2. Find all the values of a for which the resulting system has (a) no solution, (b) a unique solution, and (c) infinitely many solutions.   (9)
$$x + y - z = 3$$
$$x - y + 3z = 4$$
$$x + y + (a^2 - 10)z = a$$

3. Find the inverse of the matrix A=$\begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{pmatrix}$ using Gauss Jordan Method.   (10)

4. (a) Is the set of all real numbers with the operations $u \oplus v = uv$, $c \otimes v = cu$ a vector space? Justify your answer.   (5)

   (b) Consider the set W of all vectors in R^3 of the form (a, b, c) where $c = a + 2b$ or $b - 3c = 0$. Is W a subspace of R^3? Justify your answer.   (5)

5. Let $S = \{t - 1, t + 1, t^2 + t + 1\}$. Determine whether $v = t^2 + 2t + 4$ belongs to $Span\ S$?   If so express v as a linear combination of vectors of S.   (9)

6. Check whether the set $S = \{t^2 e^t, t\sin t, \cos t\}$ is linearly independent.   Justify your answer.   (9)

7. Check whether the set $S = \{(2, 2, 3), (-1, -2, 1), (0, 1, 0)\}$ spans R^3.   (9)

8. Suppose the vectors u, v, w are linearly independent. Prove that the vectors $u + v$, $u - v$, $u - 2v + w$ are also linearly independent.   (10)

Good luck!

B

Course Code: MATH C192
Course Title: MATHEMATICS II
Duration: 20 minutes

Date: 09.05.2011
Max Marks: 21
Weightage: 7%

Name: . . . .   **ID No:** . . . .   **Sec:** . . .

Answer the following questions:

1. Find a harmonic conjugate of $u(x, y) = e^x \cos y$ (5)

2. Find all the solutions of the equation $e^z = 1 - i\sqrt{3}$ (5)

3. Find the principal value of $(1+i)^{3+i}$ (5)

4. Evaluate $\int_C |z|^2 \, dz$ where C is the line segment from 1 to i (6)

A

Course Code: MATH C192
Course Title: MATHEMATICS II
Duration: 20 minutes

Date: 09.05.2011
Max Marks: 21
Weightage: 7%

Name: .... **ID No:** .... **Sec:** ...

Answer the following questions:

1. Find a harmonic conjugate of $u(x,y) = e^{-x}\cos y$ (5)

2. Find all the solutions of the equation $\quad e^{z} = 1 + i\sqrt{3}$ (5)

3. Find the principal value of $(1+i)^{3-i}$ (5)

4. Evaluate $\int_C |z|^2 \, dz$ where C is the line segment from -1 to i (6)

A

Course Code: MATH C192   Date:28.3.2011
Course Title: MATHEMATICS II   Max Marks: 24
Duration: 20 minutes   Weightage: 8%

Name: ....   ID No: ....   Sec: ...

Answer the following questions:

1. Consider the subspace $W = \{(a,b,c,d) \,/\, a = 2c, b = d\}$ of R^4. Find a basis and dimension of W.   (5)

2. Let $L: R^3 \rightarrow R^2$ be defined by $L(x,y,z) = (|x|, y+z)$. Check whether L is a linear transformation.   (5)

3. Let $L : P_1 \rightarrow P_3$ be defined by $L(p(t)) = t^2(p(t))$. Let $S = \{t+1, t-1\}$ and $T = \{t^3, t^2 - 1, t, t+1\}$ be bases of P_1 and P_3 respectively. Find the matrix of L with respect to S and T. (5)

4. Let $L : R^3 \rightarrow R^4$ be defined by $L\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a-b \\ a+b \\ b+c \\ b-a \end{pmatrix}$. Find the basis and dimension for

   Kernel (L). (5)

5. Let $L : V \rightarrow R^5$ be a linear transformation. (4)
   (a) If L is one to one and dim (Range L) = 3, what is the dim (V)?

   (b) If L is onto, what can you say about dim (V)?

B

Course Code: MATH C192
Course Title: MATHEMATICS II
Duration: 20 minutes

Date:28.3.2011
Max Marks: 24
Weightage: 8%

Name: ....   ID No: ....   Sec: ...

Answer the following questions:

1. Consider the subspace $W = \{(a,b,c,d) / a = d, b = 2c\}$ of R^4. Find a basis and
   dimension of W. (5)

2. Let $L : R^3 \to R^2$ be defined by $L(x, y, z) = (x + y, |z|)$. Check whether L is a linear
   transformation. (5)

PTO

3. Let $L: P_1 \rightarrow P_3$ be defined by $L(p(t)) = t^2(p(t))$. Let $S = \{t+1, t\}$ and $T = \{t^3, t^2 - 1, t, t+1\}$ be bases of P_1 and P_3 respectively. Find the matrix of L with respect to S and T. (5)

4. Let $L: R^3 \rightarrow R^4$ be defined by $L\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a-b \\ a+b \\ b+c \\ b-a \end{pmatrix}$. Find the basis and dimension for

Range (L). (5)

5. Let $L: V \rightarrow R^5$ be a linear transformation. (4)
   (a) If dim (V) = 3 and dim (Ker L) = 1, what is the dim (Range L)?

   (b) If L is one to one, what can you say about dim (V)?