BITS, PILANI-DUBAI
INTERNATIONAL ACADEMIC CITY, DUBAI

FIRST YEAR - SEMESTER-II (2009-10)

MATHEMATICS-II (MATH C192)

COMPREHENSIVE EXAMINATION (CLOSED BOOK)

Date: 23.05.2010 **Max. Marks: 120**
Time: 3 hours **Weightage: 40 %**

Answer Part A, Part B and Part C in separate Answer Books.

Answer all the questions.

PART A

1 Check for the consistency of the following system of linear equations and solve it.

$$x + y + z + w = 6,\ 2x + y - z = 3,\ 3x + y + 2w = 6$$ [10]

2. (a) Is $S = \{(a,b,c,d) / a,b,c,d \text{ are all real and } c = a + 2b \ \&\ d = a - 3b\}$ a subspace of R^4? Justify your answer.

(b) Is $S = \{t^3 + 2t + 1,\ t^2 - t + 2,\ t^3 + 2,\ -t^3 + t^2 - 5t + 2\}$ linearly independent? Justify your answer. [5+5]

3. Find a subset of $S = \{(-2,4,6,4), (0,1,2,0), (-1,2,3,2), (-3,2,5,6), (-2,-1,0,4)\}$ which forms a basis for $W = Span\ S$. Also find the dimension of *W*. [10]

4. Let $L : R^3 \to R^3$ be defined by $L(x,y,z) = (x + y - z, x + y, y + z)$. Find bases for $Ker\ L$ and $Range\ L$. Also verify the rank-nullity theorem. [10]

PART B

5. Find the eigenvalues and eigenvectors of the following matrix. [10]

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{pmatrix}$$

6. a) Find all the roots of the equation $z^4 + i = 0$. [5]

b) If $\log z = \ln r + i\theta,\ \left(r > 0, \frac{\pi}{4} < \theta < \frac{9\pi}{4}\right)$ then prove that $\log\left(i^2\right) = 2 \log i$ [5]

[PTO]

7. Show that the function $f(z) = e^{-\theta}\cos(\ln r) + i\, e^{-\theta}\sin(\ln r)$, $(r > 0, \quad 0 < \theta < 2\pi)$ is differentiable in the indicated domain. Also find $f'(z)$. [8]

8. (a) Find an analytic function whose real part is $u(x, y) = 2xy + 2x$.

   (b) If C is the upper half of the unit circle $|z| = 1$, then find an upper bound for the integral:

$$\left| \int_C \frac{e^z \, dz}{z} \right|$$

[6+6]

PART C

9. Find the value of the integral of $f(z) = \dfrac{1}{(z^2+9)^2}$ around the circle $|z - i| = 3$ [10]

10. Find the Laurent series expansions for $f(z) = \dfrac{z}{(z-1)(z-3)}$ in the regions

    (a) $|z| < 1$ (b) $1 < |z| < 3$ [10]

11. Find the value of the integral $\int_C \dfrac{dz}{z^3(z+2)^2}$ where C is taken counterclockwise around the circle $|z| = 3$. [10]

12. Use residues to evaluate the improper integral $\int_0^\infty \dfrac{x^2 \, dx}{(x^2+1)(x^2+4)}$ [10]

All the Best

BITS-PILANI, DUBAI
International Academic City, Dubai
First Year-Semester-II (2009-10)

MATHEMATICS-II (MATH C192)
Test-II (Open Book)

TIME: 50 Minutes **Marks: 60** **Weightage: 20%** **18.04.2010**

Answer all the questions sequentially.

Only the handwritten class notes and the prescribed text books are permitted.

1. Find the eigenvalues and eigenvectors of the $A=\begin{pmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{pmatrix}$ (10M)

2. Let $L:R^3 \to R^4$ be a linear transformation defined as $L(x, y, z)=(x+y+z,\ x+y-z,\ x-y-z,\ -x-y+z)$. Find the range and the kernel of L and hence state whether L is one-one and onto. (9M)

3. Show that the linear transformation $L:R^3 \to R^3$ defined by $L(x, y, z)=(2x+2z,\ 3y+3z,\ 2x+4z)$ is invertible and hence find the inverse linear transformation. (9M)

4. Show that the function $f(z)=\begin{cases} e^{-\frac{1}{z^4}}, z\neq 0 \\ 0, \quad z=0 \end{cases}$ is not differentiable at the origin using two paths method. (9M)

5. Find the fourth roots of $\left(-1+i\sqrt{3}\right)$ (8M)

6. Find the upper bound of the function $\left|\frac{z-2}{z^2+2iz-3}\right|$ when $|z|\leq\frac{1}{2}$. (7M)

7. Prove that $f(z)=(\bar{z})^2$ is not differentiable anywhere in the complex plane using the two path's method. (8M)

All the Best!

BITS-PILANI, DUBAI
International Academic City, Dubai
First Year-Semester-II (2009-10)

MATHEMATICS-II (MATH C192)
Test-I (Closed Book)

TIME: 50 Minutes **Marks: 75** **Weightage: 25%** **07. 03. 2010**

Answer all the questions sequentially.

1. Solve the linear system
$$2x - y + z = 3,\ x - 3y + z = 4,\ -5x - 2z = -5$$
by reducing into RREF. (9)

2. Find the inverse of the following matrix using Gauss Jordan procedure:
$$\begin{pmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$ (9)

3. V is the set of all polynomials of the form $at^2 + bt + c$ where a, b, c are real numbers with $b = a + 1$. Determine whether the given set V is a vector space under the operations + and * defined as $(at^2 + bt + c) + (a't^2 + b't + c') = (a + a')t^2 + (b + b')t + (c + c')$ and $\alpha * (at^2 + bt + c) = \alpha at^2 + \alpha bt + \alpha c$. Verify in detail. (10)

4. Check whether the set of all vectors of the form (a, b, c, d) where $c = a + 2b$ *and* $d = a - 3b$ is a subspace of R^4. (9)

5. Determine $S = \{(1, 2, 1), (3, 0, 1), (1, 8, 4)\}$ spans R^3. If it spans, express the vector $v = (-2, 14, 4)$ as a linear combination of these vectors. (10)

6. Check whether the set is LD and if so, find a relation between the vectors :
$$S = \{(1, 2, -1, 4), (2, 4, 3, 5), (-1, -2, 6, -7)\}$$ (9)

7. Let $S = \{t - 2, 2t - 1, 4t - 2, t^2 - t + 1, t^2 + 2t + 1\}$. Find the basis for spans S. Also write its dimension? (9)

8. Check whether the set is a basis for P_3:
$$S = \{t^3 + t^2 + t + 1,\ t^3 + 2t^2 + t + 3,\ 2t^3 + t^2 + 3t + 2,\ t^3 + t^2 + 2t + 2\}$$
(10)

All the Best!

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MATHEMATICS-II (MATH C192)
QUIZ-2

A

TIME: 20 MINUTES MAX. MARKS: 21 06.05.2010

Name: ID No.: Section:

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1. **Verify the CR equations for** $f(z) = \sin z$ **(6 marks)**

(P.T.O)

3. **Show that** $Log\,(-1+i\sqrt{3})^2 \neq 2\,Log\,(-1+i\sqrt{3})$ **(5 marks)**

(P.T.O)

4. **Find all the values of** $(1-i)^{(-1+i\sqrt{3})}$ **(5 marks)**

BITS, PILANI, DUBAI
MATHEMATICS-II (MATH C192)
QUIZ-2

B

TIME: 20 MINUTES MAX. MARKS: 21 06.05.2010

Name: ID No.: Section:

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1. **Verify the CR equations for** $f(z) = \cos z$ **(6 marks)**

(P.T.O)

3. **Show that** $Log\,(1+i\sqrt{3})^2 = 2\,Log\,(1+i\sqrt{3})$ **(5 marks)**

(P.T.O)

4. Find all the values of $(1+i)^{(-1+i\sqrt{3})}$ (5 marks)

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First year – Second semester 2009 – 2010
Mathematics II (MATH C192)

A

Quiz - 1

Time: 20 Minutes	Max Marks: 24	Weightage: 8%	25.3.2010
Name:		ID:	Section:

Answer all the questions: (Each question carries equal marks)

1. Is the set of all polynomials of the form $a_2\ t^2 + a_1 t + a_0$, where $a_1 = 2\ a_0$ is a subspace of P_2. Justify your answer.

2. Is the set of vectors $\{(4, 2, 1), (2, 6, -5), (1, -2, 3)\}$ linearly dependent? If so express it as a linear combination.

3. Find a basis and dimension of the solution space of the homogeneous system:

$$x_1 + x_2 + x_3 + x_4 = 0$$
$$2\ x_1 + x_2 - x_3 + x_4 = 0$$

4. Let S = $\{t^2+1, t-2, t+3\}$ be the basis for P_2. If v = $8t^2-4t+6$, find the co-ordinate vector of v with respect to S.

5. Let L: $R^3 \to R^2$ be defined as $L(x, y, z) = (x+1, y-z)$. Is L a linear transformation? Justify your answer?

6. Let L: $P_2 \to P_2$ be a linear transformation for which $L(2t^2+4t-1) = 2t^2+3t-2$, $L(-t+1) = 2t+1$, $L(t+1) = 5$. Find $L(3t^2+5t+1)$?

ALL THE BEST!

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B

Quiz - 1

Time: 20 Minutes | Max Marks: 24 | Weightage: 8% | 25.3.2010

Name: ID: Section:

Answer all the questions: (Each question carries equal marks)

1. The set of all polynomials of the form $a_2 t^2 + a_1 t + a_0$, where $a_2 + a_1 + a_0 = 2$ is not a subspace of P_2. Justify?

2. Is the set of vectors $\{(1, 2, 3), (1, 1, 1), (1, 0, 1)\}$ linearly dependent? If so express it as a linear combination.

3. Find a basis and dimension of the solution space of the homogeneous system:

$$x_1 - x_2 + x_3 + x_4 = 0$$
$$3x_1 - 3x_2 + 2x_3 + 2x_4 = 0$$

4. Let S = $\{2t^2 + t, t^2 + 3, t\}$ be the basis for P_2. If v = $7t^2 - t + 9$, find the co-ordinate vector of v with respect to S

5. Let L: $R^3 \to R^2$ be defined as $L(x, y, z) = (x^2 + y, y - z)$. Is L a linear transformation? Justify your answer?

6. Let L: $P_2 \to P_2$ be a linear transformation for which $L(t^2 - 2t - 1) = 3t^2 + 2t - 3$, $L(t+1) = 2t + 1$, $L(1) = 6$. Find $L(2t^2 - 7t + 3)$

ALL THE BEST!