

BITS, PILANI-DUBAI  
INTERNATIONAL ACADEMIC CITY, DUBAI

FIRST YEAR - SEMESTER-II (2008-09)

MATHEMATICS-II (MATH C192)

COMPREHENSIVE EXAMINATION (CLOSED BOOK)

Date: 01.06.2009

Time: 3 hours

Max. Marks: 120

Weightage: 40 %

Answer all the questions.

Answer Part A, Part B, Part C and Part D in separate Answer Books.

PART A

1 a) Check for the consistency of the following system of linear equations and solve it.

$$x_1 + 2x_2 - 3x_4 = 2$$

$$x_1 + 2x_2 + x_3 + 3x_4 = 3$$

$$2x_1 + 4x_2 - 6x_4 = 4$$

$$3x_1 + 5x_2 + x_3 - 4x_4 = 4$$

[8]

b) Let  $R^2$  be the set of all ordered pairs of positive real numbers in which the addition of vectors ' $\oplus$ ' and scalar multiplication '\*' are defined as

$$(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2) \text{ and } c * (x, y) = (3cx, y), \quad c > 0.$$

Check whether the closure and associative properties are true in  $R^2$ . Is  $(R^2, \oplus, *)$  a vector space? Justify.

[7]

2 a) Let  $S = \{t^3 + t^2 - 2t + 1, t^2 + 1, t^3 - 2t, 2t^3 + 3t^2 - 4t + 3\}$  be a subset of  $P_3$ . Find a basis from  $S$  for the subspace  $W = \text{Span } S$  and also find  $\dim(W)$ .

[8]

b) Let  $L : R^3 \rightarrow R^3$  be defined by  $L(x, y, z) = (x + 2y + z, 2x - y, 2y + z)$ . Let  $S$  be the natural basis for  $R^3$  and  $T = \{(1, 0, 1), (0, 1, 1), (0, 0, 1)\}$  be any other basis for  $R^3$ . Find the matrix of  $L$  with respect to

(i)  $S$  &  $T$       (ii)  $T$  &  $S$

[7]

PART B

3. a) Prove that the function  $u(x, y) = x^3 - 3xy^2$  is harmonic and find its harmonic conjugate  $v(x, y)$ .

[8]

b) Test whether the complex function  $f(z) = e^{-y} \sin x - ie^{-y} \cos x$  is entire.

[7]

4. a) Find the principal value of  $(1 - i)^{4i}$ .

[5]

b) If  $C$  is the upper half of the circle  $|z| = 4$ , then without integrating find an upper bound for

$$\left| \int_C \frac{(z+1)dz}{(z^3-2)} \right| \quad [10]$$

### PART C

5 a) Find the characteristic equation and eigenvalues of the following matrix.  
Also find the eigenvector corresponding to any one eigenvalue.

$$\begin{pmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{pmatrix} \quad [10]$$

b) Evaluate  $\int_C \frac{dz}{(z^2+2z+2)}$  where  $C$  is given by  $|z|=1$ . [5]

6 a) Apply Cauchy's integral formula to evaluate  $\int_C \frac{\cos(z) dz}{z(z^2+8)}$  where  $C$  denote the positively oriented boundary of the square whose sides lie along the lines  $x = \pm 2$ ,  $y = \pm 2$ . [8]

(b) Expand  $f(z) = \frac{z}{(z+1)(z-3)}$  as a Laurent's series valid in the region  $1 < |z| < 3$ . [7]

### PART D

7. Let  $C$  be the circle  $|z-3|=2$ , described in the positive sense. Evaluate the following integral:

$$\int_C \frac{(z^2+2) dz}{z^2-5z+6} \quad [10]$$

8. Find the residues for the following functions:

(a)  $\frac{e^z}{(z^2+4)^2}$  at the pole in the upper plane      (b)  $z e^{\left(\frac{1}{z-2}\right)}$

Also indicate the nature of the poles. [5+5]

9. Evaluate  $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+16)}$  [10]

All the Best

**MATHEMATICS-II (MATH C192)**  
**Test-II (Open Book)**

Time: 50 minutes Max. Marks: 60

Weightage: 20%

26.04.2009

**Note 1. Only the prescribed text books and handwritten class notes are allowed**

**2. Answer all the questions sequentially.**

1. Find the square roots of  $-1 - i\sqrt{3}$  and represent them geometrically. (5marks)
2. Let  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be defined by  $L(x, y, z) = (x - y + z, 2x + 3y - \frac{z}{2}, x + y - 2z)$   
Let  $S$  be the natural basis and  $T = \{ (1, 1, 0), (1, 2, 3), (-1, 0, 1) \}$  be another basis for  $\mathbb{R}^3$ . Find the matrix of  $L$  with respect to  $S$  and  $T$ . (10 marks)
3. Find bases for the range and kernel of the linear transformation  $L: V_4 \rightarrow V_4$   
defined by  $L(x_1, x_2, x_3, x_4) = (3x_1 + 2x_2, x_1 - x_3, x_1 - x_4, -x_3 + x_4)$  and  
check that  $\dim(V_4) = \text{rank}(L) + \text{nullity}(L)$ . (8 marks)
4. Verify whether the linear transformation  
 $T(x_1, x_2, x_3, x_4) = (x_1 - x_4, x_2 + x_3, x_3 - x_4)$  defined from  $\mathbb{R}^4$  to  $\mathbb{R}^3$  is one to  
one and onto or not. (7 marks)
5. Find the characteristic polynomial and all the eigenvalues of the matrix  
 $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ . Also find an eigenvector corresponding to the largest eigenvalue.  
(10 marks)
6. Prove that the following limit does not exist:  
$$\lim_{z \rightarrow 0} \frac{\text{Re}(z^2)}{|z|^2}.$$
 (5 marks)
7. Show that  $f'(0)$  does not exist if  $f(z) = \sqrt{|xy|}$ ,  $z \neq 0$  and  $f(0) = 0$ .  
(8 marks)
8. Does the function  $f(z) = (x^2 - y^2 + 3x) - i(2xy + 3y)$  have a derivative  
everywhere in the complex plane? Justify. (7 marks)

BITS-PILANI, DUBAI  
International Academic City, Dubai  
First Year-Semester-II (2008-09)

**MATHEMATICS-II (MATH C192)**  
**Test-I (Closed Book)**

TIME: 50 Minutes

Marks: 75

Weightage: 25%

15.03.2009

Answer all the questions sequentially.

1. For what values of  $a, b$  the linear system

$$x + y + 3z = 2, \quad x + 3y + 2z = 4, \quad x + y + (a^2 - 1)z = b$$

has (i) no solution (ii) unique solution (iii) infinitely many solutions (10)

2. Find the inverse of the following matrix using Gauss Jordan procedure:

$$\begin{pmatrix} 1 & 3 & -2 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{pmatrix} \quad (10)$$

3. Find a basis for the subspace  $W = \text{span } S$  of  $R^4$  where

$$S = \{(1, 0, 0, -1), (0, 1, 2, 1), (1, 0, 1, -1), (1, 1, -6, -3), (-1, -5, 1, 0)\}.$$

What is  $\dim W$ ? (10)

4. Check whether the polynomials

$$t^3 + 2t + 1, \quad t^2 - t + 2, \quad t^3 + 2, \quad -t^3 + t^2 - 5t + 2 \text{ are LI?} \quad (10)$$

5. Find a basis and the dimension of the solution space of the system

$$x_1 + 3x_2 - x_3 + x_4 = 0, \quad 2x_1 + 2x_2 - x_3 + x_4 = 0, \quad x_1 + 2x_2 - 3x_3 = 0 \quad (8)$$

6. Let  $S = \{(1, 2, -1), (1, 9, -1), (-3, 8, 3)\}$  and let  $V = \text{span } S$ . Find a basis for  $V$ . What is  $\dim V$ ? (7)

6. Is  $S$  a subspace of the indicated vector space? Justify.

$$S = \{(x, y, z, w) / z = x + 2y \text{ and } w = x - 3y\}, \quad V = R^4 \quad (6)$$

7. Does  $p(t) \in \text{span } S$  if  $S = \{t^2 - t, t^2 - 2t + 1, -t^2 + 1\}$  and  $p(t) = 2t^2 - t - 1$  (7)

8. Let  $V$  be the set of real numbers. Define  $\oplus$  by  $u \oplus v = 3u + 5v - 6$  and  $\otimes$  by  $c \otimes u = c^2 u$ . Is  $V$  a vector space? Justify. (7)

All the Best!

question

BITS, PILANI, DUBAI  
MATHEMATICS-II (MATH C192)  
QUIZ-3 (SECTION-8)

TIME: 20 MINUTES

MAX. MARKS: 15

13.05.2009

1. Find the principal value of  $(-3i)^{-1+i}$
2. Find the Laurent's series of  $f(z) = \frac{z}{(z+4)(z-2)}$  when  $z$  lies inside the circle  $|z|=2$
3. Evaluate  $\int_C \frac{\sin z}{z^2 + 4z + 4} dz$  where  $C$  is given by  $|z+2|=1$

BITS, PILANI, DUBAI  
 MATHEMATICS-II (MATH C192)  
 QUIZ-2 (SECTION-8)  
 MAX. MARKS: 15

26.03.2009

Time: 20 minutes

1. Is  $L(x, y) = (3x + 2y, -x + y + 6)$  a LT? Justify.  
No
2. Check whether  $L(at + b) = (a + b)t^2 + (a - 6b)$  is a LT? Justify.  
YES
3. Find Ker L if  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by  
 $L(x, y, z) = (3x - 7y, y + 3z)$  Ker L =  $\text{span}\{(7, 3, -1)\}$
4. Is the LT  $L(a_1, a_2) = (a_1 + 3a_2, -a_2)$  onto? Justify. YES,  $\text{range } L = \mathbb{R}^2$
5. Find the range of  
 $L(at + b) = (3a + b)t + (5a - b)$ .

no ~

$\text{range } L = \text{span}\{3t + 5, t - 1\}$

BITS, PILANI, DUBAI  
 MATHEMATICS-II (MATH C192)  
 QUIZ-2 (SECTION-8)  
 MAX. MARKS: 15

Time: 20 minutes

26.03.2009

1. Is  $L(x, y) = (2x + 2y, x - y, 1)$  is a LT? Justify.
2. Check  $L(at^2 + bt + c) = (2a - b)t + (b + 4c)$  is a LT?
3. Find the null space of the LT  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^3$   
 by  $L(x, y) = (2x - y, 3x + 4y, x + y)$
4. Is the LT  $L(a_1, a_2, a_3) = (0, a_2, a_1 - a_3)$  is  
 one-one? Justify.
5. What is the range of the LT  
 $L(x) = (x, 2x, -4x)$ .

✓ 0 ✓

BITS, PILANI, DUBAI  
 MATHEMATICS-II (MATH C192)  
 QUIZ-2 (SECTION-8)  
 MAX. MARKS: 15

Time: 20 minutes

26.03.2009

1. Is  $L(x, y) = (3x + 2y, -x + y + 6)$  a LT? Justify.
2. Check whether  $L(at + b) = (a + b)t^2 + (a - 6b)$  is a  
 LT? Justify.
3. Find Ker L if  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by  
 $L(x, y, z) = (3x - 7y, y + 3z)$
4. Is the LT  $L(a_1, a_2) = (a_1 + 3a_2, -a_2)$   
 onto? Justify.
5. Find the range of  
 $L(at + b) = (3a + b)t + (5a - b)$ .

✓ 0 ✓

**A**

5x3 = 15 marks.

① Find the rank of  $\begin{bmatrix} 0 & 0 & 1 & 2 \\ 2 & 3 & 0 & -2 \\ 3 & 3 & 6 & -9 \end{bmatrix}$

② Find B such that  $AB=I$  if  $A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$

③ Is  $(2, -1, 8) \in [S]$  where  $S = \{(1, 2, 1), (-1, 1, 1), (4, 2, -5)\}$ ? Justify.

④ Is  $S = \{(a, b, c) \mid a, b, c \text{ are real and } b = 2a + 1\}$  a subspace of  $\mathbb{R}^3$ ? Justify.

⑤ Is  $S = \{t^2 - 4, 5t^2 - 5t + 6, t + 2\}$  LI? Justify.

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**B**

5x3 = 15 marks.

① Solve  $x - 2y = 4, y + 3z = 1, x - y + z = 3.$

② Find A if  $A^{-1} = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$

③ Is  $x + 2 \in [S]$  if  $S = \{x^2 + 5, x - 1, x + 2\}$ ? Justify.

④ Is  $S = \{(a, b, c) \mid a = 3b \text{ or } b + c = 0\}$  a subspace of  $\mathbb{R}^3$ ? Justify.

⑤ Is  $S = \{(2, 2, 3), (-1, -2, 1), (0, 1, 0)\}$  LI? Justify.

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**BITS, PILANI – DUBAI**  
**DUBAI INTERNATIONAL ACADEMIC CITY**  
**1 YEAR – II SEMESTER 2008-2009**  
**QUIZ-II(CB)**

Course : Mathematics - II

Section - VI

Course No: MATH C192

Max.Marks:15

Weightage:5%

Time:15 Mins. Dt.29-4-09

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Name:

ID. No.

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1. The largest eigen value of the matrix  $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$  is \_\_\_\_\_
2. If  $L : R^3 \rightarrow R^3$  defined by  $L(x_1, x_2, x_3) = (x_1 - x_2 + x_3, x_1 + x_2, x_2 - x_3)$ , then the coordinates of  $(1, 2, -1)$  with respect to the basis  $S = \{(1, 0, 1), (0, 1, -1), (0, 0, 1)\}$  are \_\_\_\_\_
3. Find the inverse of  $\sqrt{2} + i\sqrt{3}$
4. Imaginary component of the function  $f(z) = (z + 2i)(-2z - 3)$  is \_\_\_\_\_
5. The Principal Value of  $i^{2i}$  is \_\_\_\_\_

**BITS, PILANI – DUBAI**  
**DUBAI INTERNATIONAL ACADEMIC CITY**  
**1 YEAR – II SEMESTER 2008-2009**  
**QUIZ-II (CB)**

Course : Mathematics - II

Section - VI

Course No: MATH C192

Max. Marks: 15

Weightage: 5%

Time: 15 Mins. Dt. 26-3-09

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Name:

ID. No.

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1. The basis for the solutions space of the homogeneous system  
 $x_2 + 2x_3 = 0$ ;  $x_1 + 2x_2 + 3x_3 = 0$ ;  $x_1 + 3x_2 + 5x_3 = 0$  is \_\_\_\_\_
  
2. Coordinate vector of  $(1, -1, 2)$  with respect to the ordered basis  
 $S = \{(-1, 1, 0), (0, 1, -1), (1, 0, 1)\}$  is \_\_\_\_\_
  
3. If  $L : P_2 \rightarrow P_3$  is a linear transformation for which  
 $L(1) = 1, L(t) = t^3, L(t^2) = t^2 + t$  then  $L(2t^2 - 5t + 3) =$  \_\_\_\_\_
  
4. If  $L : R^2 \rightarrow R^2$  is defined as  $L(x, y) = (0, y)$ , then  $\ker L =$  \_\_\_\_\_
  
5. Let  $L : R^3 \rightarrow R^2$  be defined by  $L(x, y, z) = (x - y, x + 2y)$  is
  - a) one-to-one
  - b) onto
  - c) one-to-one & onto
  - d) None

**BITS, PILANI – DUBAI**  
**DUBAI INTERNATIONAL ACADEMIC CITY**  
**1 YEAR – II SEMESTER 2008-2009**  
**QUIZ-1 (CB)**

Course : Mathematics - II

Section - VI

Course No: MATH C192

Max.Marks:15

Weightage:5%

Time:15 Mins. Dt.4-3-09

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Name:

ID. No.

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1. The row reduced echelon form of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & -9 \\ 2 & -1 & 1 & -8 \\ 3 & 0 & -1 & -3 \end{bmatrix} \text{ is } \underline{\hspace{4cm}}.$$

2. Let  $V$  be the set of all positive real ordered pairs, with usual addition of vectors '+' and scalar multiplication '\*' is not a vector space. Give one property, which it fails to satisfy (with details).
3. Express the vector  $v = (3,7)$  as a linear combination of the vectors in the basis  $S = \{v_1 = (1,2), v_2 = (0,1)\}$ .

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**1 YEAR – II SEMESTER 2008-2009**  
**QUIZ-1I (CB)**

Course : Mathematics - II

Section - V

Course No: MATH C192

Max.Marks:15

Weightage:5%

Time:15 Mins. Dt.29-3-09

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Name:

ID. No.

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1. The basis and dimension of the null space of homogeneous system of equations  
 $x_1 - x_2 + x_3 - x_4 = 0, 2x_1 + x_2 - x_3 + x_4 = 0$  is \_\_\_\_\_.
2. The coordinate vector of  $5 - 3t - 2t^2$  relative to the ordered basis  $\{t^2 + t + 1, t + 1, 1\}$  is \_\_\_\_\_.
3. If  $L: R^3 \rightarrow R^3$  is a linear transformation for which  
 $L(1, -1, 0) = (3, 0, 1), L(0, 1, -1) = (1, 0, 3), L(-1, 1, 1) = (0, 1, 3)$  then  
 $L(-a, 2b, c) =$  \_\_\_\_\_.
4. If  $L: R^3 \rightarrow R^3$  is a linear transformation defined as  
 $L(x, y, z) = (x + 2y, 2x + y, z)$  then a basis for the Ker L is \_\_\_\_\_.
5. If  $L: R^2 \rightarrow R^3$  is a linear transformation defined as  $L(x, y) = (x, 2x + y, y)$  then L is
  - a)      Onto
  - b)      One to one
  - c)      One to one & onto
  - d)      None

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**DUBAI INTERNATIONAL ACADEMIC CITY**  
**1 YEAR – II SEMESTER 2008-2009**

**QUIZ-1 (CB)**

Course : Mathematics - II

Section - V

Course No: MATH C192

Max.Marks: 15

Weightage: 5%

Time: 15 Mins. Dt. 5.3.09

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Name:

ID. No.

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$$2x + y - z = 0$$

1. The homogeneous system of equations  $x - 2y - 3z = 0$  has a non trivial  
 $-3x - y + az = 0$   
solution if  $a =$  \_\_\_\_\_.

2. Express  $v = (-1, 4, 2, 2)$  as a linear combination of the vectors in  
 $S = \{v_1 = (1, 0, 0, 1), v_2 = (1, -1, 0, 0), v_3 = (0, 1, 2, 1)\}$

3. Let  $V$  be the set of all positive real numbers. The vector addition '+' and  
scalar multiplication '\*' are defined as  $u+v = u + v - uv$ ;  $c*u = u^c$   
Check the associative property of vector addition.

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**DUBAI INTERNATIONAL ACADEMIC CITY**  
**1 YEAR – II SEMESTER 2008-2009**  
**QUIZ-1 (CB)**

Course : Mathematics - II

Section - V

Course No: MATH C192

Max.Marks: 15

Weightage: 5%

Time: 15 Mins. **Dt.5-3-09**

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Name:

ID. No.

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- $2x + y - z = 0$
1. The homogeneous system of equations  $x - 2y - 3z = 0$  has only  
 $-3x - y + (a^2 - 2)z = 0$   
trivial solution if  $a = \underline{\hspace{2cm}}$ .
2. Let  $V$  be the set of all positive real ordered pairs, with addition of vectors '+' and scalar multiplication '\*' defined as  
 $(x_1, y_1) + (x_2, y_2) = (x_1 x_2, y_1 y_2)$  ;  $c * (x, y) = (cx, cy)$   
is **not** a vector space.  
Give one property, which it fails to satisfy (with details).
3. Express the vector  $v = (4, 6, 8, 6)$  as a linear combination of the vectors  
 $v_1 = (1, 1, 2, 1)$ ,  $v_2 = (1, 0, 0, 2)$ ,  $v_3 = (0, 3, 2, 1)$ .

BITS, PILANI, DUBAI  
MATHEMATICS-II (MATH C192)  
QUIZ-3 (SECTION-4)  
MAX. MARKS: 15

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TIME: 20 MINUTES

12.05.2009

1. Find the principal value of  $(1+i)^{2i}$
2. Evaluate  $\int_C (z^2 + 4) dz$  where C is the straight line joining 1 and  $1+i$ .
3. Evaluate  $\int_C \frac{e^{2z}}{z^2 + 9} dz$  where C is given by  $|z - 3i| = 1$

q. b

BITS, PILANI, DUBAI  
 MATHEMATICS-II (MATH C192)  
 QUIZ-2 (SECTION-4)

A

TIME: 20 MINUTES

MAX. MARKS: 15

29.03.2009

1. Check whether  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  by  $L(x, y, z) = (4, 2, -7)$  is a LT? NO
2. Find the coordinate vector of  $v = (6, -2, 1)$  w.r. to the basis  $S = \left\{ (1, 0, 0), (0, 2, 4), (0, 0, 5) \right\}$   
 $[v]_S = \begin{bmatrix} 6 \\ -1 \\ 1 \end{bmatrix}$
3. Find the Kernel of  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  by  $L(x, y) = (x - 2y, 3x + 4y, x)$  Ker  $L = \left\{ (0, 0) \right\}$
4. Find the range of  $L: \mathcal{P}_2 \rightarrow \mathcal{P}_3$  by  $L(at^2 + bt + c) = (2a + b)t - (3a + c)$   
 range =  $\text{Span}\{2t - 3, 1\}$
5. Find  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  if  $L(1, 2) = (4, -1)$ ,  $L(0, 1) = (2, 3)$  given that  $L$  is a LT.  
 $L(x, y) = (2y, 3y - 7x)$

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BITS, PILANI, DUBAI  
 MATHEMATICS-II (MATH C192)  
 QUIZ-2 (SECTION-4)

B

TIME: 20 MINUTES

MAX. MARKS: 15

29.03.2009

- Check whether  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  by  
 $L(x, y, z) = (0, 0, 0)$  is a LT? **YRS**
- Find the coordinate vector of  $2t - 4$  w.r. to  
 the basis  $S = \{t-1, t+1\}$ .  $[v]_S = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$
- Find the null space of  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  by  
 $L(x, y, z) = (2x+3y, x-z)$   
 $\text{Ker } L = \text{span} \left\{ \begin{pmatrix} 1 \\ -2/3 \\ 1 \end{pmatrix} \right\}$
- Find the range of  $L: \mathcal{P}_3 \rightarrow \mathcal{P}_2$  by  
 $L(at^3 + bt^2 + ct + d) = (a-b)t^2 + (2a-c)t$ .  
 $\text{range } L = \text{span} \{t^2 + 2t, t\}$ .
- Find the LT  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  by  
 $L(2, 3) = (1, -1)$ ,  $L(0, 2) = (1, 2)$ .  
 - cos n -  $L(x, y) = \left( \frac{2y-x}{4}, y-2x \right)$

BITS, PILANI, DUBAI  
 MATHEMATICS-II (MATH C192)  
 QUIZ-2 (SECTION-4)

A

TIME: 20 MINUTES

MAX. MARKS: 15

29.03.2009

1. Check whether  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  by  $L(x, y, z) = (4, 2, -7)$  is a LT?
2. Find the coordinate vector of  $v = (6, -2, 1)$  w.r. to the basis  $S = \{(1, 0, 0), (0, 2, 4), (0, 0, 5)\}$ .
3. Find the Kernel of  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  by  $L(x, y) = (x - 2y, 3x + 4y, x)$
4. Find the range of  $L: P_2 \rightarrow P_3$  by  $L(at^2 + bt + c) = (2a + b)t - (3a + c)$
5. Find  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  if  $L(1, 2) = (4, -1)$ ,  $L(0, 1) = (2, 3)$  given that  $L$  is a LT.

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BITS, PILANI, DUBAI  
 MATHEMATICS-II (MATH C192)  
 QUIZ-2 (SECTION-4)

B

TIME: 20 MINUTES

MAX. MARKS: 15

29.03.2009

1. Check whether  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  by  $L(x, y, z) = (0, 0, 0)$  is a LT?
2. Find the coordinate vector of  $2t - 4$  w.r. to the basis  $S = \{t - 1, t + 1\}$ .
3. Find the null space of  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  by  $L(x, y, z) = (2x + 3y, x - z)$ .
4. Find the range of  $L: P_3 \rightarrow P_2$  by  $L(at^3 + bt^2 + ct + d) = (a - b)t^2 + (2a - c)t$ .
5. Find the LT  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  by  $L(2, 3) = (1, -1)$ ,  $L(0, 2) = (1, 2)$ .

- ~ ~ ~ -

A

5x3 = 15 marksQUIZ - ISECTION - 4Time: 20 min.

- ① For what values of 'a', the system  
 $x + 2y - 3z = 0$ ,  $2x + y - 4z = 0$ ,  $x - y + 9z = 0$  will  
 have non-zero solutions? Justify your answer.
- ② Is  $S = \{ p \in P \mid p'(x) = x p(x) + 2 \}$  a subspace  
 of  $P$ ? Justify.
- ③ Is  $S = \{ (1, -2, 4), (4, -1, 5), (3, 1, 0) \}$  LI? Justify.
- ④ If  $A = \begin{bmatrix} 1 & 3 \\ -1 & 6 \end{bmatrix}$ , find  $A^{-1}$ .
- ⑤ Is  $(1, -2, 2) \in [S]$  if  $S = \{ (2, 1, 3), (-1, 1, 4), (2, 3, 1) \}$ .  
 Justify.

-uon-

B

5x3 = 15 marksQUIZ - ISECTION - 4Time: 20 min.

- ① Find the rank of  $\begin{bmatrix} 1 & -1 & 1 \\ 3 & 2 & -4 \\ 2 & 4 & 3 \end{bmatrix}$
- ② Is  $S = \{ (a, b, c) \mid a + b = -2c \}$  a subspace of  
 $\mathbb{R}^3$ ? Justify.
- ③ Is  $S = \{ (3, 2, 7), (1, 1, 0), (2, 4, 6) \}$  LI? Justify.
- ④ If  $A^{-1} = \begin{bmatrix} 1 & 2 \\ -1 & 5 \end{bmatrix}$ , find  $A$ .
- ⑤ Is  $t^2 - 2t + 5 \in [S]$  if  $S = \{ t^2 + 2t - 4, t - 2, t^2 + 5 \}$ ?  
 Justify.

-uon-

BITS, PILANI – DUBAI  
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(I YEAR – II SEMESTER 2008-2009)

Set A

QUIZ – III (CB)

MATHEMATICS-II  
(MATH C192)

Max. Marks: 15

Weightage: 5%

Date: 22-4-2009

Time: 15 Mins.

Name : \_\_\_\_\_

Id. No.: \_\_\_\_\_

Sec.: II

**NOTE:**

*Write your Name and Id. No in the space provided.*

*Attempt all the questions. Total number of questions: 5*

*Each question carries 3 marks. Specific answer is required, not the formula.*

*Overwriting/multiple answers will be treated as incorrect answer.*

**Fill in the blanks with correct answers:**

1. The characteristic roots of the matrix  $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$  are \_\_\_\_\_.
2. If  $L_1 : R^2 \rightarrow R^2$  and  $L_2 : R^2 \rightarrow R^2$  be two linear transformation defined by  $L_1(x, y) = (x + y, x - 2y)$  and  $L_2(x, y) = (y, x - y)$ , then  $(L_2 \circ L_1)(2, 2) =$  \_\_\_\_\_ and  $(L_1 \circ L_2)(2, 2) =$  \_\_\_\_\_.
3. The modulus and the principal argument of  $(1 + i)^2$  are respectively \_\_\_\_\_ and \_\_\_\_\_.
4. If  $f(z) = z + \frac{1}{z} = u(x, y) + iv(x, y)$ , then  $v(x, y) =$  \_\_\_\_\_.
5. The cube roots of 3 will lie on a circle of radius \_\_\_\_\_.

**BITS, PILANI – DUBAI**  
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**(I YEAR – II SEMESTER 2008-2009)**

*Scr B*

**QUIZ – III (CB)**

**MATHEMATICS-II**  
**(MATH C192)**

Max. Marks: 15      Weightage: 5%      Date: 22-4-2009      Time: 15 Mins.

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Name : \_\_\_\_\_ Id. No.: \_\_\_\_\_ Sec.: II

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**NOTE:**

*Write your Name and Id. No in the space provided.*

*Attempt all the questions. Total number of questions: 5*

*Each question carries 3 marks. Specific answer is required, not the formula.*

*Overwriting/multiple answers will be treated as incorrect answer.*

**Fill in the blanks with correct answers:**

1. The characteristic polynomial of the matrix  $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$  is \_\_\_\_\_ .
2. If  $L_1 : R^2 \rightarrow R^2$  and  $L_2 : R^2 \rightarrow R^2$  be two linear transformation defined by  $L_1(x, y) = (x + y, x - 2y)$  and  $L_2(x, y) = (y, x - y)$ , then  $(L_2 \circ L_1)(2, 1) =$  \_\_\_\_\_ and  $(L_1 \circ L_2)(2, 1) =$  \_\_\_\_\_ .
3. The modulus and the principal argument of  $(1 - i)^2$  are respectively \_\_\_\_\_ and \_\_\_\_\_ .
4. If  $f(z) = z + \frac{1}{z} = u(x, y) + iv(x, y)$ , then  $u(x, y) =$  \_\_\_\_\_ .
5. The cube roots of 2 will lie on a circle of radius \_\_\_\_\_ .

(A)

BITS, PILANI – DUBAI  
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(I YEAR – II SEMESTER 2008-2009)

QUIZ – II (CB)

MATHEMATICS-II  
(MATH C192)

Max. Marks: 15      Weightage: 5%      Date: 25-3-2009      Time: 15 Mins.

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Name : \_\_\_\_\_      Id. No.: \_\_\_\_\_      Sec.: II

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**NOTE:**

*Write your Name and Id. No in the space provided.*

*Attempt all the questions. Total number of questions: 5*

*Each question carries 3 marks. Specific answer is required, not the formula.*

*Overwriting/multiple answers will be treated as incorrect answer.*

**Fill in the blanks with correct answers:**

1. A system of linear homogeneous equations with four unknowns  $x, y, z, w$  has the solution:  $x = -2k + 3l, y = -2l, z = k, w = l$  where  $k$  and  $l$  are arbitrary real numbers. The dimension of the solution space is \_\_\_\_\_.
2. For a linear transformation  $L: V \rightarrow W$  if the rank of  $L = 2$  and the  $\dim V = 4$ , then the nullity of  $L =$  \_\_\_\_\_.
3. Let  $S = \{(1, 0), (1, -1)\}$  be an ordered basis of  $R^2$ . Then the coordinates of  $(2, 2)$  with respect to the basis  $S$  are \_\_\_\_\_.

**Tick the correct answer:**

4. The linear transformation  $L: R^2 \rightarrow R^2$  defined by  $L(x, y) = (x + y, x - y)$  is
  - a) onto but not one-to-one
  - b) one-to-one but not onto
  - c) onto and one-to-one
  - d) none of these
5. If  $L: R^2 \rightarrow R^3$  is a linear transformation defined by  $L(x, y) = (x, x + y, y)$ , then which of the following is true?
  - a)  $(1, 2, 3) \in \text{range}L$
  - b)  $(1, 0, -1) \in \text{range}L$
  - c)  $(4, 2, 3) \in \text{range}L$
  - d) none of these

(B)

BITS, PILANI – DUBAI  
INTERNATIONAL ACADEMIC CITY, DUBAI  
(I YEAR – II SEMESTER 2008-2009)

QUIZ – II (CB)

MATHEMATICS-II  
(MATH C192)

Max. Marks: 15      Weightage: 5%      Date: 25-3-2009      Time: 15 Mins.

Name : \_\_\_\_\_ Id. No.: \_\_\_\_\_ Sec.: II

**NOTE:**

*Write your Name and Id. No in the space provided.*

*Attempt all the questions. Total number of questions: 5*

*Each question carries 3 marks. Specific answer is required, not the formula.*

*Overwriting/multiple answers will be treated as incorrect answer.*

**Fill in the blanks with correct answers:**

1. For a linear transformation  $L: V \rightarrow W$  if the rank of  $L = 2$  and the  $\dim V = 5$ , then the nullity of  $L =$  \_\_\_\_\_.
2. A system of linear homogeneous equations with four unknowns  $x, y, z, w$  has the solution:  $x = -2k + 3l, y = -2l, z = k, w = l$  where  $k$  and  $l$  are arbitrary real numbers. The dimension of the solution space is \_\_\_\_\_.
3. Let  $S = \{(1, 0), (1, -1)\}$  be an ordered basis of  $R^2$ . Then the coordinates of  $(1, 2)$  with respect to the basis  $S$  are \_\_\_\_\_.

**Tick the correct answer:**

4. If  $L: R^2 \rightarrow R^3$  is a linear transformation defined by  $L(x, y) = (x, x + y, y)$ , then which of the following is true?  
a)  $(1, 2, 3) \in \text{range}L$       b)  $(1, 0, 1) \in \text{range}L$       c)  $(-1, 2, 3) \in \text{range}L$   
d) none of these
5. The linear transformation  $L: R^2 \rightarrow R^2$  defined by  $L(x, y) = (x + y, x - y)$  is  
a) onto but not one-to-one      b) one-to-one but not onto      c) onto and one-to-one  
d) none of these

(A)

**BITS, PILANI – DUBAI  
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(I YEAR – II SEMESTER 2008-2009)**

**QUIZ – I (CB)**

**MATHEMATICS-II  
(MATH C192)**

Max. Marks: 15      Weightage: 5%      Date: 04-3-2009      Time: 15 Mins.

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Name : \_\_\_\_\_      Id. No.: \_\_\_\_\_      Sec.: II

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**NOTE:**

*Write your Name and Id. No in the space provided.*

*Attempt all the questions. Total number of questions: 15*

*Each question carries 3 marks. Specific answer is required, not the formula.*

*Overwriting/multiple answers will be treated as incorrect answer.*

**Fill in the blanks with correct answers:**

1. Consider the system:  $x + 2y = 0$ ,  $3x + (2 - \lambda)y = 0$ . The value of  $\lambda$  for which the above system has non-trivial solution is \_\_\_\_\_.
2. If  $A$  is an invertible matrix of order 2, then the reduced row echelon form of  $A$  is \_\_\_\_\_.
3. The standard or natural basis of  $P_2$  is \_\_\_\_\_.
4. Which of the following vectors is linearly independent? (Tick the correct answer)  
a)  $\{(1, 2), (2, 2), (1, 1)\}$     b)  $\{(2, 1), (0, 0)\}$     c)  $\{(2, 1), (1, 2)\}$     d) none of these.
5. If  $V$  is a 3-dimensional vector space and  $W$  is a nonzero subspace of  $V$ , then the set of values that  $\dim W$  can take is {\_\_\_\_\_}.



(B)

**BITS, PILANI – DUBAI**  
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**(I YEAR – II SEMESTER 2008-2009)**

**QUIZ – I (CB)**

**MATHEMATICS-II**  
**(MATH C192)**

Max. Marks: 15      Weightage: 5%      Date: 04-3-2009      Time: 15 Mins.

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Name : \_\_\_\_\_      Id. No.: \_\_\_\_\_      Sec.: II

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**NOTE:**

*Write your Name and Id. No in the space provided.*

*Attempt all the questions. Total number of questions: 15*

*Each question carries 3 marks. Specific answer is required, not the formula.*

*Overwriting/multiple answers will be treated as incorrect answer.*

**Fill in the blanks with correct answers:**

1. Consider the system:  $x + 2y = 0$ ,  $3x + (3 - \lambda)y = 0$ . The value of  $\lambda$  for which the above system has non-trivial solution is \_\_\_\_\_.
2. If  $A$  is an invertible matrix of order 3, then the reduced row echelon form of  $A$  is \_\_\_\_\_.
3. The standard or natural basis of  $P_3$  is \_\_\_\_\_.
4. Which of the following vectors is linearly independent? (Tick the correct answer)  
a)  $\{(1, 2), (2, 2), (1, 1)\}$     b)  $\{(2, 1), (0, 0)\}$     c)  $\{(2, 1), (4, 2)\}$     d) none of these.
5. If  $V$  is a 2-dimensional vector space and  $W$  is a nonzero subspace of  $V$ , then the set of values that  $\dim W$  can take is {\_\_\_\_\_}.

TO Exam Branch

**BITS, PILANI – DUBAI**  
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**1 YEAR – II SEMESTER 2008-2009**  
**QUIZ-1II (CB)**

Course : Mathematics - II

Section -I

Course No: MATH C192

Max.Marks: 15

Weightage: 5%

Time: 15 Mins. Dt. 7.5.09

Name:

ID. No.

1. The characteristic polynomial of the matrix

$$\begin{bmatrix} 2 & 3 & 2 \\ 1 & 2 & 0 \\ 2 & -2 & 2 \end{bmatrix} \text{ is } \underline{\hspace{2cm}}$$

2. An eigen vector corresponding to the eigen value  $\lambda = 4$  for the matrix

$$\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} \text{ is } \underline{\hspace{2cm}}$$

3. The  $x + iy$  form of the complex number  $\frac{5i(3-i2)}{2+i}$  is \_\_\_\_\_

4.  $\left(\frac{-1+i\sqrt{3}}{2}\right)^{1/3} = \underline{\hspace{2cm}}$

5. The real component of  $\text{Sin}(z)$  is \_\_\_\_\_

**BITS, PILANI – DUBAI**  
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**1 YEAR – II SEMESTER 2008-2009**  
**QUIZ-II(CB)**

Course : Mathematics - II

Section -I

Course No:MATH C192

Max.Marks:15

Weightage:5%

Time:15 Mins. Dt.29.3.09

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Name:

ID. No.

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1. A basis for the solution space of homogeneous system of equations  $2x + y - z = 0, x - 2y - 3z = 0, -3x - y + 2z = 0$  is \_\_\_\_\_.
2. The coordinate vector of  $3 + 3t + 2t^2$  relative to the ordered basis  $\{1 - t, 1 + t, 1 - t^2\}$  is \_\_\_\_\_.
3. If  $L : R^2 \rightarrow R^2$  is a linear transformation for which  $L(1,2) = (3,0), L(2,1) = (1,2)$  then  $L(4,-5) =$  \_\_\_\_\_.
4. If  $L : R^3 \rightarrow R^3$  is a linear transformation defined as  $L(x, y, z) = (x + y, x - y, z)$  then the Ker L = \_\_\_\_\_.
5. If  $L : R^2 \rightarrow R^3$  is a linear transformation defined as  $L(x, y) = (x - y, x + 2y, x)$  then **L is**
  - a) One to one
  - b) Onto
  - c) One to one & onto
  - d) None

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**1 YEAR – II SEMESTER 2008-2009**  
**QUIZ-1 (CB)**

Course : Mathematics - II

Section - I

Course No: MATH C192

Max.Marks:15

Weightage:5%

Time:15 Mins.

---

Name:

ID. No.

---

1. The row reduced echelon form of the matrix

$$A = \begin{bmatrix} 5 & 9 & 2 & -1 \\ 4 & 4 & 1 & -1 \\ 2 & 4 & 1 & -1 \end{bmatrix} \text{ is } \underline{\hspace{2cm}}$$

2. If the linear system of equations  $x + y - z = 2$ ;  
 $x + 2y + z = 3$  is consistent, then the  
 $x + y + (a^2 - 9)z = a$   
value of 'a' is \_\_\_\_\_.

3. Let  $V$  be the set of all real numbers, the addition of vectors '+' and scalar multiplication '\*' are defined as  
 $\mathbf{u} + \mathbf{v} = \mathbf{u} - 3\mathbf{v}$  and  $c * \mathbf{u} = c\mathbf{u}$  is not a vector space. Give one property, which it fails to satisfy.

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**1 YEAR – II SEMESTER 2008-2009**  
**QUIZ-1 (CB)**

Course : Mathematics - II

Section - I

Course No: MATH C192

Max. Marks: 15

Weightage: 5%

Time: 15 Mins.

---

Name:

ID. No.

---

1. The row reduced echelon form of the matrix

$$A = \begin{bmatrix} 5 & 9 & 2 & -1 \\ 4 & 4 & 1 & -1 \\ 2 & 4 & 1 & -1 \end{bmatrix} \text{ is } \underline{\hspace{4cm}}$$

2. If the linear system of equations  $x + y - z = 2$ ;  
 $x + 2y + z = 3$  is consistent, then the  
 $x + y + (a^2 - 9)z = a$   
value of 'a' is \_\_\_\_\_.

3. Let  $V$  be the set of all real numbers, the addition of vectors '+' and scalar multiplication '\*' are defined as  
 $\mathbf{u} + \mathbf{v} = \mathbf{u} - 3\mathbf{v}$  and  $c * \mathbf{u} = c\mathbf{u}$  is not a vector space. Give one property, which it fails to satisfy.