BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

I - Year - SEMESTER - II (2008-09)

MATHEMATICS - I (MATH C191)

COMPREHENSIVE EXAMINATION

(Closed-Book)

Time: 03 Hours — Max. Marks: 40

Date: June 01, 2009 — Weightage: 40 %

Note:- 1. All questions are compulsory and should be answered sequentially.

2. There are TWO sections (A and B) in the question paper and should be answered in separate answer sheets AND write ***A/B*** *on the top of each answer sheet in CAPITAL BOLD LETTERS.*

SECTION A

1. Find (2)

$$\lim_{x\to 1^+} \frac{\sqrt{2x}(x-1)}{|\,x-1\,|}.$$

2. Find the vertices, foci and centre of the curve (3)

$$r = \frac{1}{2+\cos\theta}.$$

3. Find the area of the region shared by the cardioids $r = 2(1+\cos\theta)$ and $r = 2(1-\cos\theta)$. (3)

4. Find the length of the curve (3)

$$\overrightarrow{r}(t) = 2\cos t\,\overrightarrow{i} + 2\sin t\,\overrightarrow{j} + \sqrt{5}\,t\,\overrightarrow{k},\ 0 \le t \le \pi.$$

5. Find $\hat{T}, \hat{N}$ and κ for the curve (4)

$$\overrightarrow{r}(t) = (e^t\cos t)\,\overrightarrow{i} + (e^t\sin t)\,\overrightarrow{j} + 2\,\overrightarrow{k}.$$

6. Discuss the convergence of the series (3+2)

(a) $$\sum_{n=1}^{\infty} \frac{n2^n(n+1)!}{3^n n!}.$$

(b) $$\sum_{n=2}^{\infty} \frac{n}{(\ln n)^{(n/2)}}.$$

SECTION B

Note:- Each question in this section carry 2.5 MARKS.

1. (a) Evaluate dw/dt at $t = 0$, by using chain rule $w = x^2 + y^2$, $x = \cos t + \sin t$, $y = \cos t + \sin t$.

   (b) Find the direction in which the function $f(x, y) = x^2y + e^{xy} \sin y$ increases most rapidly at $P_0(1, 0)$. Then find the derivative of the function in that direction.

2. (a) Sketch the curve $x^2 - y = 1$ together with ∇f and the tangent line at $(\sqrt{2}, 1)$.

   (b) Find all the local maxima, local minima, and saddle points of the function $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$.

3. (a) Sketch the region and evaluate the integral (the st-plane)

$$\int_0^1 \int_0^{\sqrt{1-s^2}} 8t \; dtds.$$

   (b) Change the following integral to an equivalent polar integral and evaluate.

$$\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2 + y^2).\; dxdy$$

4. (a) Integrate $f(x, y) = x^3/y$ over the given curve $y = x^2/2$, $0 \leq x \leq 2$.

   (b) Use Green's theorem to find outward flux for the field $\mathbf{F} = (y^2 - x^2)\mathbf{i} + (x^2 + y^2)\mathbf{j}$ over the triangle bounded by $y = 0$, $x = 3$, and $y = x$.

Good Luck . . .

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TEST 2 - OPEN-BOOK

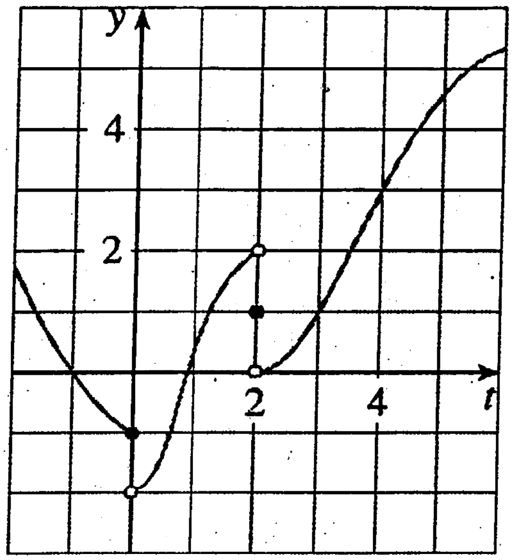
Time: 50 Minutes Max. Marks: 20

Date: April 26, 2009 Weighage: 20 %

Note: (i) Solve all the questions. (ii) All questions carry equal marks

1. (a) For the function whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

   $\lim_{t\to 0} g(t)$, $\lim_{t\to 2} g(t)$, $\lim_{t\to 4} g(t)$



   (b) Find the area of the region enclosed by the limacon $r = 2 - \cos\theta$.

2. (a) Find the velocity, acceleration, and speed of a particle with the given position function. $r(t) = t^2 i + \ln t j + tk$ at $t = 2$.

   (b) Find the curve's unit tangent vector. Also, find the length of the indicated portion of the curve: $r(t) = (3\cos t)i + (3\sin t)j + 2t^{3/2}$, $0 \le t \le 3$.

3. (a) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x^2 - y^2$ subject to the given constraint: $x^2 + y^2 = 1$.

(b) Sketch the region of integration and evaluate the integral

$$\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x} \right) dydx.$$

4. (a) Integrate the function $f(x, y) = x^2 + y^2$ over the region bounded by the line $y = 2x$ and the parabola $y = x^2$.

(b) Evaluate using polar coordinate

$$\int\int e^{-x^2-y^2} dydx,$$

where R is the region bounded by the semicircle $x = \sqrt{4 - y^2}$ and the y-axis.

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MATHEMATICS - I (MATH C191)

TEST - I (Closed-Book)

Time: 50 Minutes — Max. Marks: 25

Date: March 15, 2009 — Weighage: 25 %

Note: Solve all the questions in sequence.

1. (a) Find all the *polar representations* of $(-4,\ \pi/4)$.

   (b) Find the *vertices and center* of

$$r = \frac{8}{4 - \cos\theta}. \qquad (3+3)$$

2. (a) Plot the graph of $r^2 = 4\sin\theta$.

   (b) Convert to the *cartesian* form: $r = 1 + \cos\theta$. (3+3)

3. (a) Find $\frac{dw}{dt}$, at $t = 3$, if $w = \ln(x^2 + y^2 + z^2)$, where $x = \cos t,\ y = \sin t,\ z = 4\sqrt{t}.$

   (b) *Sketch* the curve $x^2 + y^2 = 4$ together with ∇f and *tangent line* at the point $(\sqrt{2}, \sqrt{2})$. Also write an *equation of tangent line.* (3+3)

4. (a) Find the direction in which the function $f(x, y, z) = x^3 - xy^2 - z$ *increases and decreases more rapidly* at $(1, 1, 0)$. Also find the derivative of the function in these directions.

   (b) Find all the *local maxima, local minima, and saddle points* of the function $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$. (4+3)

Good Luck . . .

Q'P

BITS, PILANI, DUBAI
MATHEMATICS-I (MATH C191)
QUIZ-3 (I Biotech)

TIME: 15 MINUTES MAX. MARKS: 5 11.5.2009

1. What is the tangential component of the acceleration if

$$\bar{r}(t) = (t+1)\bar{i} + 2t\bar{j} + t^2\bar{k}, \quad t = 1.$$

(2 marks)

2. Find the sum of the series

$$\sum_{n=0}^{\infty} \left[\frac{5}{2^n} + \frac{1}{3^n} \right]$$ if it exists.

(2 marks)

3. Test the convergence of $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.

(1 mark)

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BITS, PILANI, DUBAI
MATHEMATICS-I (MATH C191)
QUIZ-2 (I Biotech)

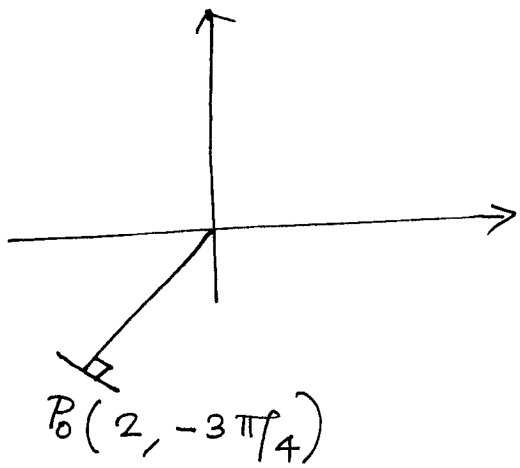
TIME: 15 MINUTES **MAX. MARKS: 15** **30.03.2009**

1. Find the general form of $P(-2, -\pi/3)$.

2. Find the centre and vertices of

$$r = \frac{2}{4 + \sin\theta}$$

3. Write the equation of the st. line



4. Find the area between the circles

$r = 4\cos\theta$, $r = 4\sin\theta$.

5. Find the length of the spiral

$r = \sqrt{2}\, e^{\theta/2}$, $0 \le \theta \le \pi/2$

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MATHEMATICS (MATH C191)

SURPRISE QUIZ I (Closed-Book)

Time: 15 Minutes　　　　　　　　　　　　　　　　　　Max. Marks: 5

ID No:　　　　　　　　　Section No:　　　　　　　　Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. Find the partial derivatives f_x and f_y of the following functions $f(x,\ y) = (3x^2 + 2y^3 + 2xy)^5$.

2. Find the derivative of $w = xy$ along the path $x = \cos t,\ y = \sin t$ at $t = \pi/2$.

3. The equation of tangent line to the curve $x^2/4 + y^2 = 2$ at the point (2,-1) is

4. What is the direction of zero change in $f(x, y) = x^2/2 + y^2/$ at the point (1,1)?

5. Find the local extreme or saddle point(s) of the function $f(x, y) = y \sin x$