BITS, PILANI-DUBAI
INTERNATIONAL ACADEMIC CITY, DUBAI

FIRST YEAR - SEMESTER-II (2007-08)

MATHEMATICS-II (MATH C192)

COMPREHENSIVE EXAMINATION (CLOSED BOOK)

Date: 02.06.2008 **Max. Marks: 120**

Time: 3 hours **Weightage: 40 %**

Answer all the questions.

Answer Part A, Part B and Part C in separate Answer Books.

Part-A

1. Test for consistency of the following linear system of equations and if so solve completely

$$x+y+2z=-1,\ x-2y+z=-5,\ 3x+y+z=3.$$ **(10)**

2. Verify whether the following set of vectors in R^4 is linearly dependent. If it is so, express one vector as a linear combination of rest

$$\{(1,1,2,10),(1,0,0,2),(4,6,8,6),(0,3,2,1)\}.$$ **(10)**

3. Let $L:R^3 \to R^3$ be defined by $L(x,y,z)=(x-y,x+2y,z)$. Find $\ker L$, *range* L and verify Rank-Nullity theorem. **(10)**

4. Let $L:R^3 \to R^3$ be defined by $L(x,y,z)=(x+2y+z,2x-y,2y+z)$. Let S be the natural basis for R^3 and let $T=\{(1,0,1),(0,1,1),(0,0,1)\}$ be another basis for R^3. Find the matrix of L with respect to S and T. **(10)**

Part- B

5. Check whether the linear transformation $T:R^3 \to R^3$ defined by $T(x,y,z)=(x+z,\ y+z,\ x+2z)$ is nonsingular and if so, find T^{-1} **(10)**

6. Find the eigenvalues and eigenvectors of $A=\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ **(10)**

7. Show that the function $f(z) = z^2 e^{-z}$ is entire and hence find its derivative.

(10)

8. Show that $Log(1+i)^2 = 2\,Log(1+i)$ but $Log(-1+i)^2 \neq 2\,Log(-1+i)$. **(10)**

Part-C

9. Evaluate $\int_C f(z)dz$ where $f(z)$ is defined by the equations $f(z) = \begin{cases} 1, \text{ when } y < 0 \\ 4y, \text{ when } y > 0 \end{cases}$ and C is the arc from $z = -1 - i$ to $z = 1 + i$ along the curve $y = x^3$. **(8)**

10. Find the Laurent's series that represents the function $f(z) = \dfrac{z+2}{z^2 - 5z + 6}$ in the domain $0 < |z-2| < 1$

(8)

11 Evaluate the integral $\int_C \tan z\, dz$, where C the positively oriented circle $|z| = 2$.

(8)

12. Use residues to evaluate the improper integral $\int_0^\infty \dfrac{dx}{(x^2+1)^2}$. **(8)**

13. Use residues to evaluate the definite integral $\int_{-\pi}^{+\pi} \dfrac{d\theta}{1 + \sin^2\theta}$ **(8)**

BITS-PILANI, DUBAI

First Year-Semester-II (2007-08)

MATHEMATICS-II (MATH UC192)

Test-2 (Open Book)

Time: 50 Minutes	Marks:60	Weightage: 20%	8^{th} May, 2008

NOTE: 1.Only the Prescribed Text books and Class Notes are allowed.
2. Answer the questions in serial order.

1. Determine the eigenvalues and eigenvectors of A^{-1} if $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ **(10)**

2. If $f(z) = \begin{cases} \dfrac{x^3 y(y-ix)}{x^6+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$, show that the Cauchy-Riemann equations are satisfied at the origin but $f'(0)$ does not exist. **(10)**

3. Find all the roots of $(-8-i8\sqrt{3})^{\frac{1}{4}}$ and exhibit them geometrically. **(8)**

4. Find the real and imaginary parts of $Log\left[(1+i)Log\, i\right]$ **(8)**

5. Find the constants a,b,c such that the function
$$f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + cy^2)$$
is analytic. Express $f(z)$ in terms of z. **(8)**

6. Evaluate the integral $\int_C \text{Re}(z^2)dz$ from 0 to $2+4i$ along the
   (i) x-axis from 0 to 2, and then vertically to $2+4i$,
   (ii) Parabola $y = x^2$. **(8)**

7. Show that the function $u(r,\theta) = r^2 \cos 2\theta$ is harmonic. Find the conjugate harmonic function and the corresponding analytic function $f(z)$. **(8)**

BITS-PILANI, DUBAI
First Year-Semester-II (2007-08)

MATHEMATICS-II (MATH UC192)
Test-I (Closed Book)

TIME: 50 Minutes **Marks: 75** **Weightage:25%** **30.03.2008**

1. Find an equation relating a, b, c, so that the linear system

$$x+2y-3z=a,\ 2x+3y+3z=b,\ 5x+9y-6z=c$$

is consistent for any values of a, b and c that satisfy the equations. (9)

2. Find the inverse of the following matrix using Gauss Jordan procedure:

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$$ (9)

3. Let $S=\{v_1, v_2, v_3, v_4, v_5\}$, where $v_1=(1,1,0,-1)$, $v_2=(0,1,2,1)$, $v_3=(1,0,1,-1)$ $v_4=(1,1,-6,-3)$ and $v_5=(-1,-5,1,0)$. Find a basis for the subspace $W=\text{span } S$ of R^4. What is $\dim W$? (10)

4. Let $S=\{x^3+2,\ x^2+2x+1,\ x^2-x+2,\ -x^3+x^2-5x+2\}$ Check whether S spans P_3. (9)

5. Check whether the set $S=\{(1,1,2,1), (1,1,1,0), (1,0,0,2), (0,3,2,1)\}$ is linearly independent in R^4 ? (9)

6. Which of the following are subspaces of the indicated vector spaces ?

   (a) $S=\left\{(x,y,z)\big/ x-2y=z-\frac{3y}{2}\right\},\ V=R^3$

   (b) $S=\{p\in P/\deg p\le 4\ \&\ p(0)=2\},\ V=P$ (10)

7. Let V be P_1, the vector space of all polynomials of degree ≤ 1, and let $S=\{v_1, v_2\}$ and $T=\{w_1, w_2\}$ be bases for P_1, where $v_1=t$, $v_2=1$, $w_1=t+1$, $w_2=t-1$. Let $v=p(t)=5t-2$. Compute $[v]_s$ and $[v]_T$. (10)

8. Let $L: P_2 \to P_2$ be a linear transformation for which $L(1)=1$, $L(t)=t^2$, and $\mathrm{L}(t^2)=t^3+t$.
   Compute $L(2t^2-5t+3)$ and $L(at^2+bt+c)$ (10)