BITS, PILANI-DUBAI
International academic City, Dubai

First Year - Semester-II (2007- 08)

Mathematics-I (MATH UC191)

Comprehensive Examination (Closed Book)

Date: 02.06.2008 **Max. Marks: 80**
Time: 3 hours **Weightage: 40 %**

Answer all the questions sequentially in Blue colour Answer book.

1. Find the area shared by the cardioids $\mathbf{r} = 2(1+\cos(\theta))$ and $\mathbf{r} = 2(1-\cos(\theta))$.
(5 marks)

2. Solve the initial value problem: $d\mathbf{r}/dt = (3/2)\,(t+1)^{(1/2)}\,\mathbf{i} + e^{-t}\,\mathbf{j} + 1/(t+1)\,\mathbf{k}$
with the initial condition $\mathbf{r}(0) = \mathbf{k}$. (5 marks)

3. Express $\hat{\mathbf{a}}$ in the form $\hat{\mathbf{a}} = a_T\,\hat{\mathbf{T}} + a_N\,\hat{\mathbf{N}}$ without finding $\hat{\mathbf{T}}$ and $\hat{\mathbf{N}}$
at t=0 for $\mathbf{r} = t^2\,\mathbf{i} + (t+(1/3)t^3)\,\mathbf{j} + (t-(1/3)t^3)\,\mathbf{k}$. (7 marks)

4. Find an equation for the level surface of the function

$$g(x,y,z) = \sum_{n=0}^{\infty} (x+y)^n / (n!\, z^n) \text{ at } (\ln(2), \ln(4), 3).$$ (7 marks)

5. Use the chain rule to find $\delta z/\delta u$ and $\delta z/\delta v$ when u = 1 and v = -2 if
$z = \ln(q)$ and $q = \sqrt{v+3}\,\tan^{-1}(u)$. (7 marks)

6. Find the directions in which the function $f(x,y,z) = \ln(xy) + \ln(yz) + \ln(xz)$ increase and decrease most rapidly at the point $P_0(1,1,1)$. Also find the derivatives of the function in those directions. (7 marks)

(P.T.O)

7. Find the tangent plane and normal line to the surface

$f(x,y,z) = \cos(\pi x) - x^2 y + e^{xz} + yz - 4 = 0$ at the point (0,1,2). (7 marks)

8. Discuss the convergence of the following series:

(a) $\sum_{n=1}^{\infty} (2n)! / (n!\, n!)$ (b) $\sum_{n=1}^{\infty} 6 / ((2n-1)(2n+1))$ (7 marks)

9. Sketch the region of integration and write an equivalent double integral with the order of integration reversed.

$$\int_{y=0}^{1} \int_{x=-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y \, dx \, dy$$

(7 marks)

10. Change the following Cartesian integral into an equivalent polar integral and evaluate

$$\int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} (x^2 + y^2) \, dy \, dx$$

(7 marks)

11. Solve the system $u = x - y$ and $v = 2x + y$ for x and y in terms of u and v. Find the Jacobian $\delta(x,y)/\delta(u,v)$ and use it to evaluate

$\iint_R (2x^2 - xy - y^2) \, dx \, dy$ for the region bounded by the lines $y = -2x + 4$, $y = -2x + 7$

$y = x - 2$, $y = x + 1$

(7 marks)

12. Verify whether $\mathbf{F} = y\sin(z)\,\mathbf{i} + x\sin(z)\,\mathbf{j} + xy\cos(z)\,\mathbf{k}$ is a conservative field.

(7 marks)

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INTERNATIONAL ACADEMIC CITY, DUBAI
First Year – Semester II (2007-08)

MATHEMATICS-I (MATH UC191)

TEST –II (Open Book)

Date : 08.05.08.
Time: 50 minutes

Max. Marks : 40
Weightage : 20%

Answer all the questions

Note: Text book and handwritten Class notes are allowed

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1. If $f(x,y) = 1/\text{sqrt}(64 - x^2 - y^2)$ then find
   a. function's domain.
   b. function's range.
   c. boundary of the function's domain.
   d. determine whether the domain is open or closed.
   e. determine whether the domain is bounded or unbounded . (6 marks)

2. Find the linearization of $f(x,y,z) = e^x + \cos(y+z)$ at $(0, \pi/2, 0)$ (5 marks)

3. Find dw/dt at t = 3 using the chain rule for the following
   $w = x/y + y/z$, $x = \cos^2(t)$; $y = \sin^2(t)$; $z = 1/t$ (6 marks)

4. Find the directions in which the following functions increase and decrease most rapidly at the point P_0. Then find the directional derivative of the function in that direction.
   $f(x,y) = x^2y + e^{xy}\sin(xy)$ $P_0 = (1,0)$ (6 marks)

5. Find the parametric equations for the line of tangent to the curve of intersection of the surfaces
   $x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0$; $x^2 + y^2 + z^2 = 11$ at the point $(1,1,3)$. (6 marks)

6. Find the local maxima, local minima and saddle points , if any for the function $x^3 - y^3 - 2xy + 6 = 0$. (5 marks)

7. Sketch the region of integration for the following and write an equivalent double integral with the order of integration reversed

$$\int_{y=1}^{2}\int_{x=y}^{y^2} dx\, dy$$

(6 marks)

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First Year – Semester II (2007-08)
MATHEMATICS-I (MATH UC191)

TEST –I (Closed Book)

Date : 30.03.08.
Time: 50 minutes

Max. Marks : 50
Weightage : 25%

Answer all the questions

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1. Solve the initial value problem for **r** as a vector function of t:

   Differential equation : $d\mathbf{r}/dt = t\,\mathbf{i} - t\,\mathbf{j} - t\,\mathbf{k}$
   Initial condition : $\mathbf{r}(0) = \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$ 8 marks

2. Find the unit tangent vector and also the length of the indicated portion of the curve
   $\mathbf{r}(t) = (6\sin 2t)\,\mathbf{i} + (6\cos 2t)\,\mathbf{j} + 5t\,\mathbf{k}$, $0 \le t \le \pi$. 8 marks

3. Find δ algebraically for the following $f(x) = \sqrt{x-7}$, $L=4$, $x_0 = 23$ and $\varepsilon = 1$.
   8 marks

4. Find $\lim_{x \to (-2)^-} (x+3) \cdot \dfrac{|x+2|}{(x+2)}$. 8 marks

5. Find the area inside the lemniscate $r^2 = 6\cos(2\theta)$ and outside the circle $r = \sqrt{3}$.
   8 marks

6. Find the length of the curve $r = \cos^3(\theta/3)$, $0 \le \theta \le \pi/4$ 5 marks

7. Label the vertices and the centre with polar coordinates for the ellipse
   $r = 25 / (10 - 5\cos(2\theta))$ 5 marks