BITS, PILANI-DUBAI CAMPUS
Knowledge Village, DUBAI

FIRST YEAR - SEMESTER-II (2006-07)

MATHEMATICS-II (MATH UC192)

COMPREHENSIVE EXAMINATION (CLOSED BOOK)

Date: 29.05.2007 **Max. Marks: 120**
Time: 3 hours **Weightage: 40 %**

<u>Answer all the questions.</u>

<u>Answer Part A, Part B and Part C in separate Answer Books.</u>

PART A

1. If S is a non-empty subset of a vector space V, prove that $[S]=S$ if and only if S is a subspace of V. (8)

2. Determine a non-zero linear transformation $T:V_2 \to V_2$ which maps all the vectors on the line $x=y$ onto the origin. (8)

3. Show that the set $S=\{\sin x, \sin 2x, \sin 3x, ..., \sin nx\}$ is a linearly independent subset of $C[-\pi,\pi]$ for every positive integer n. (8)

4. Find an ordered basis for V_4 relative to which the vector $(-1,3,2,1)$ has the coordinates $4,1,-2$ and 7. (6)

5. Determine whether the following system of linear equations is consistent. If consistent, solve completely.

$$x_1+3x_2-3x_3+2x_4=1,$$
$$4x_1+x_2-2x_3+x_4=1,$$
$$6x_1+5x_2+10x_3+3x_4=15,$$
$$x_1+2x_2+3x_3+x_4=6.$$

(10)

PART B

6. Check whether the linear transformation $T: V_3 \to V_3$ defined by $T(x, y, z) = (x - y + 2z,\ x + z,\ y - 2z)$ is nonsingular and if so, find T^{-1}. (8)

7. Determine the matrix $(T; B_1, B_2)$ for the given linear transformation $T: V_3 \to V_2$ and the bases B_1 and B_2:

$T(x, y, z) = (x + z,\ y + z)$, $B_1 = \left\{\left(1, 1, \frac{2}{3}\right), (-1, 2, -1), \left(2, 3, \frac{1}{2}\right)\right\}$ and

$B_2 = \left\{(1, 3), \left(\frac{1}{2}, 1\right)\right\}$. (8)

8. Find the eigenvalues and eigenspaces of $A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$. (8)

9 Show that the function $u(x, y) = e^{-x}(x\cos y + y\sin y)$ is harmonic and hence find its harmonic conjugate $v(x, y)$. (10)

10. Show that $\log(-1 + i\sqrt{3}) = \ln 2 + 2\left(n + \frac{1}{3}\right)\pi i,\ n = 0, \pm 1, \pm 2, \ldots$ (6)

PART C

11. Use residues to evaluate the following improper integrals:

$$\int_0^\infty \frac{x^2\,dx}{(x^2+1)(x^2+4)}.$$ (10)

12. Use residues to evaluate the following definite integrals:

$$\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta}, (-1 < a < 1).$$ (10)

13. Let C be the circle $|z| = 5$, described in the positive sense. Evaluate the following integral:

$$\int_C \frac{\cot z\, dz}{z}$$ (10)

14. Find the Laurent's series of the function $\frac{z}{(z-1)(z-3)}$ in the interval $0 < |z - 1| < 2$. (10)

A

BITS, PILANI – DUBAI CAMPUS
DUBAI KNOWLEDGE VILLAGE
(I YEAR – II SEMESTER 2006-2007)

QUIZ – II (CB)

MATHEMATICS-II
(MATH UC192)

Max. Marks: 30 Weightage: 10% Date:08-05-2007 Time: 30 Mins.

Name : Id. No.: Sec.:

NOTE:
Write your Name, Id. No. and Sec. in the space provided.
Attempt all the questions.
Each question carries 3 marks.
No marks for overwriting/multiple answers/partly correct answers.

1. In cartesian form, $(1+i)^7 =$ ____________________________ .

2. If $\exp(3z+4) = -2$, then $z =$ ____________________________ .

3. If C: $|z| = 2$ (in positive sense), then $\int_C \left(\frac{2}{z(z+2i)}\right)^{-2} dz =$ ______________ .

4. If the Laurent series expansion of $\frac{1}{z^2(1-z)}$ in the domain $0 < |z| < 1$ contains n non-zero terms in its *principal part*, then the value of n is ________________ .

5. The number of singular points of the function $f(z) = \tan z$ interior to the circle $|z| = 6$ is __________ .

6. If the modulus and the argument of a complex number are 2 and $\frac{11\pi}{6}$ respectively, then this number in the form $x+iy$ is ______________________ .

7. The 4^{th} roots of 4 lie on a circle with radius ________________________ .

8. If the function $2x + iV(x, y)$ is analytic everywhere on the Z-plane, then $V(x, y) =$

______________________________.

9. The principal value of $\left[\frac{e}{2}(-1 - i\sqrt{3})\right]^{3i\pi}$ is ______________________________.

10. The value of the integral $\int_C \tan z \, dz$, where $C : |z| = 1$ (in positive sense), is

____________________________________.

BITS-PILANI, DUBAI CAMPUS

I- Year-Semester-II (2006-07)

MATHEMATICS-II (MATH UC192)

TEST-II (OPEN BOOK)

TIME: 50 Minutes **22th April, 2007** **Max. Marks: 60**

Weightage: 20%

Note: 1. Only the Prescribed Text Books and Class Notes are allowed.
2. Answer the questions in serial order.

1. Determine whether T is non-singular and if so, find T^{-1}, given
$T(x,y,z)=(x+3y-2z,\ x-z,\ y+z)$. **(10 M)**

2. Let T be a linear transformation from $V_3 \rightarrow V_3$ given by
$T(x,y,z)=(2x+3y-8z,\ x+y+z,\ 4x-5z)$. Find the matrix $(T:B_1,B_2)$
when $B_1=\{(1,1,1),(1,1,0),(1,0,0)\}$, $B_2=\{(1,2,3),(0,4,-1),(0,0,1)\}$ **(8 M)**

3. Verify the rank-nullity theorem by finding $R(T)$ and $N(T)$, given $T:V_3 \rightarrow V_3$ by
$T(x,y,z)=(x+2y-z,\ y+z,\ x+y-2z)$. **(8 M)**

4. Determine the eigenvalues and corresponding eigenspaces for the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **(7 M)**

5. Find the cube roots of $z=i$. **(7 M)**

6. Test whether the function $u(x,y)=x^4-6x^2y^2+y^4+x^3y-xy^3$ is harmonic or not. If it is harmonic, find the harmonic conjugate $v(x,y)$ and hence the analytic function $f(z)=u+iv$. **(10 M)**

7. Give an example of a function of complex variable $z\ (=x+iy)$ which is analytic on the entire complex plane except at $z=i$. Prove why it is analytic everywhere except $z=i$. **(10 M)**

BITS-PILANI, DUBAI CAMPUS

MATHEMATICS-II (MATH UC192)

I- Year-Semester-II (2006-07)

TEST-I (CLOSED BOOK)

TIME: 50 Minutes 11^{th} March, 2007   **Max. Marks: 60**

Weightage: 20%

Note: Answer the questions in serial order.

1. Reduce the matrix $\begin{bmatrix} 0 & 6 & 6 & 1 \\ -8 & 7 & 2 & 3 \\ -3 & 2 & 1 & 1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$ to the **row -reduced echelon form** and hence find the rank.

2. Determine whether the system of equations
$x_1 + x_2 - x_3 - 6x_4 + 6x_5 = -19$,
$x_1 + 7x_4 - 7x_5 = 28$,
$2x_2 - 3x_3 + 18x_4 - 4x_5 = 24$,
is consistent. If consistent, write the complete solution.

3. Find whether the set $\{f \in C(a,b) | f'(x) = x^2 f(x)\}$ a subspace of $C(a,b)$?

4. Let $S = \{x^3, x^2 + 2x, x^2 + 2, 1 - x\}$. Check whether $3x^2 + x + 5 \in [S]$?

(P.T.O)

5. Is the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ invertible? If yes, find its inverse using elementary row transformations.

6. Check whether the set $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is linearly independent in V_3.

7. Check whether the set $S = \{xe^x, \sin x, \sin 2x\}$ is linearly dependent in $C(0, \infty)$.

BITS-PILANI, DUBAI CAMPUS

MATHEMATICS-II (MATH UC192)

A

I- Year-Semester-II (2006-07)

QUIZ – I (CLOSED BOOK)

TIME: 30 Minutes   February 20, 2007   **Max. Marks:30**

ID No.   **Section No.:**   **Name:**

Note: 1.Write ID No., Name and Sec.No. in the space provided.
2. Overwriting and multiple answers will be treated as wrong answer.

Tick the correct answer:

1. Let $\mathcal{P}_5$ denote the set of all polynomials of degree ≤ 5 and $P \subset \mathcal{P}_5$. Which of the following P is not a vector space under 'addition of polynomials' and 'scalar multiplication'?

a) $p \in P$ iff degree of $p \leq 2$.
b) $p \in P$ iff degree of $p \geq 2$.
c) $p \in P$ iff $p(2) = 0$.
d) None of these.

2. The row equivalent of the augmented matrix of a system of linear equations is

$$\begin{pmatrix} 1 & 2 & 2 & 5 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & a-5 & b+12 \end{pmatrix}$$

The system has no solution if
a) $a = 5$ and $b = -12$;
b) $a = 5$ and $b \neq -12$;
c) $a \neq 5$ and $b = -12$;
d) $a \neq 5$ and $b \neq -12$.

3. The echelon form of the augmented matrix of a system of linear equations is

$$\begin{pmatrix} 1 & 4 & 0 & 4 \\ 0 & 1 & 8 & 5 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The number of solutions of the system is ______________.

4. Let a binary operation * be defined on the set of real numbers $\Re$ by $a*b=a+b-1$ for all $a,b\in \Re$. The identity element w.r.t. the operation * is

__________________________.

5. The set $A=\{\bar{0}\}$ is a subset of a vector space V. Can A be a vector space?

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6. Is $S=\left\{(x,y,z)\Big/ \dfrac{x}{3y}=1\right\}$ a vector space with the usual addition and scalar multiplication? Justify.

7. The inverse of $A=\begin{pmatrix}1 & 2\\ -1 & 0\end{pmatrix}$ is -----------------------------------

8. Let $V=\{x \mid x\in\Re, x>0\}$, and define addition and scalar multiplication as follows:

(i) Addition. Let $x\in V, y\in V$. Define $x+y=x+y$.

(ii) Scalar multiplication. Let $\alpha\in\Re, x\in V$. Define $\alpha x=\alpha . x$.

Is V a vector space? -----------------------------------

9. Under the point wise addition and scalar multiplication, is the set $V=\{(x,y,z) \mid x^2+y^2+z^2=1, x\in\Re, y\in\Re, z\in\Re\}$ a vector space?

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10. The rank of $A=\begin{pmatrix}0 & 2 & -1 & 0\\ 1 & 0 & -1 & 4\\ 0 & 0 & 0 & 2\\ 0 & 0 & 1 & -7\end{pmatrix}$ is -----------------------------------