Repeater

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

I - Year - SEMESTER - II (2005-06)

MATHEMATICS - II (MATH UC192)

COMPREHENSIVE EXAMINATION

(Closed-Book)

Time: 03 Hours | Max. Marks: 120

Date: May 28, 2006 | Weightage: 40 %

Note:- 1. All questions are compulsory and should be answered sequentially.

*2. There are TWO sections (A and B) in the question paper and should be answered in separate answer sheets AND write **A/B** on the top of each answer sheet in CAPITAL BOLD LETTERS.*

SECTION A

1. (a) Determine whether the following system of linear equations is consistent.

$$\begin{aligned} x_1 - x_2 + 2x_3 + 3x_4 &= 1 \\ 2x_1 + 2x_2 + 2x_4 &= 1 \\ 4x_1 + x_2 - x_3 - x_4 &= 1 \\ x_1 + 2x_2 + 3x_3 &= 1 \end{aligned}$$

If so, discuss completely the solution.

(b) Let U and W be two distinct $(n-1)-$dimensional subspace of an $n-$dimensional vector space V. Then Prove that $\dim(U \cap W) = n - 2$. (10+6)

2. (a) Prove that a non-empty subset of a vector-space V is subspace of V iff $\alpha u + \beta v \in S$, whenever $u, v \in S$ and α, β, are scalars.

(b) Verify rank-nullity theorem for the linear transformation $T : V_4 \to V_3$ such that $T(e_1) = (1,\ 1,\ 1)$, $T(e_2) = (1,\ -1,\ 1)$, $T(e_3) = (1,\ 0,\ 0)$, $T(e_4) = (1,\ 0,\ 1)$. (6+8)

3. (a) Determine the matrix $(T: B_1, B_2)$ for the linear transformation $T: P_3 \to P_2$ defined by $T(\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^2) = \alpha_3 + (\alpha_2 + \alpha_3)\, x + (\alpha_0 + \alpha_1)\, x^2$, relative to the bases $B_1 = \{1, (x-1), (x-1)^2, (x-1)^3\}$ and $B_2 = \{1, x, x^2\}$.

(b) Find an ordered basis of V_4 relative which the vector $(-1, 3, 2, 1)$ has coordinates $4, 1, -2$, and 7. (9+6)

4. (a) Prove that the linear transformation $T: V_3 \to V_3$ such that $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 - e_2 + e_3$, $T(e_3) = 3e_1 + 4e_3$, is non-singular and find its inverse.

(b) Determine eigenvalues and the corresponding eigenvectors for the following matrix

$$A = \begin{pmatrix} 0 & i & i \\ i & 0 & i \\ i & i & 0 \end{pmatrix}.$$

(7+8)

SECTION B

1. (a) Let

$$f(z) = \begin{cases} \frac{(1+i)x^3 - (1-i)y^3}{x^2+y^2}, & (x,y) \neq (0,0), \\ 0 & (x,y) = (0,0). \end{cases} \qquad (1)$$

Are Cauchy-Riemann equations satisfied at (0,0)? Justify.

(b) Show that $u(x,y) = e^x \cos y$ is harmonic in some domain D and find a function $v(x,y)$ such that $f(z) = u + iv$ is analytic in the domain D. (7+8)

2. (a) Show that (i) $\text{Log}(1+i)^2 = 2\text{Log}(1+i)$;

(ii) $\text{Log}(-1+i)^2 \neq 2\text{Log}(-1+i)$.

(b) Show that

$$\int_C z^i dz = \frac{1+e^{-\pi}}{2}(1-i),$$

where z^i denotes the principal branch

$z^i = \exp(i\text{Log}z) \quad (|z| > 0, -\pi < Arg z < \pi)$ and where the path of the integartion is any contour from $z = -1$ to $z = 1$. (7+8)

3. (a) Let C be positively oriented unit circle $|z| = 1$. Then evaluate

$$(i) \int_C (z + \frac{1}{z})^2\, dz, \qquad (ii) \int_C (|z|^2 + e^z)\, dz.$$

(b) Give the two *Laurent series* expansions for the function

$$f(z) = \frac{1}{z(z^2+1)},$$

and specify the region in which those expansions are valid. (7+8)

4. (a) Use residues to evaluate the improper integral

$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+1)} dx.$$

(b) Use residues to evaluate

$$\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta}. \qquad (-1 < a < 1)$$

(7+8)

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

I - Year - SEMESTER - II (2005-06)

MATHEMATICS - II (MATH UC192)

COMPREHENSIVE EXAMINATION

(Closed-Book)

Time: 03 Hours | Max. Marks: 120

Date: May 28, 2006 | Weightage: 40 %

Note:- 1. All questions are compulsory and should be answered sequentially.

*2. There are TWO sections (A and B) in the question paper and should be answered in separate answer sheets AND write **A/B** on the top of each answer sheet in CAPITAL BOLD LETTERS.*

SECTION A

1. (a) Determine whether the following system of linear equations is consistent:

$$x_1 + 2x_2 + 4x_3 + x_4 = 4$$
$$2x_1 - x_3 + 3x_4 = 4$$
$$x_1 - 2x_2 - x_3 = 0$$
$$3x_1 + x_2 - x_3 - 5x_4 = 7.$$

If so, discuss completely the solution.

(b) Find whether the polynomial $3x^2+x+5 \in [S]$, if $S = \{x^3,\ x^2+2x,\ x^2+2,\ 1-x\}$.

(c) Is

$$S = \{f \in \mathcal{C}(a,b) \mid f(\frac{a+b}{2}) = 1\},$$

a subspace of $\mathcal{C}(a,b)$? Justify. (9+4+3)

2. (a) Find out whether the subset $S = \{(1,2,3),\ (3,1,0),\ (-2,1,3)\}$ forms a basis for V_3? If not, find a basis for $[S]$.

(b) Find the coordinate vector of the vector $(-\sqrt{2},\ \pi,\ e)$ of V_3 relative to ordered basis $B = \{(2,1,0),\ (2,1,1),\ (2,2,1)\}$. (7+7)

3. (a) Find range, rank, kernel and nullity of the linear transformation $T : V_3 \to V_3$ such that $T(x_1, x_2, x_3) = (x_1 + 2x_2 - x_3, x_2 + x_3, x_1 + x_2 - 2x_3)$, also verify rank-nullity theorem.

(b) Prove that the linear transformation $T : P_2 \to P_2$ defined by
$T(\alpha_0 + \alpha_1 x + \alpha_2 x^2) = (\alpha_0 + \alpha_1) + (\alpha_1 + 2\alpha_2)x + (\alpha_0 + \alpha_1 + 3\alpha_2)x^2$,
is non-singular and find its inverse. (8+7)

4. (a) Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$.

Determine a linear transformation T such that $A = (T : B_1, B_2)$, where,
$B_1 = \{(1,1), (-1,1)\}$, $B_2 = \{(1,1,1), (1,-1,1), (0,0,1)\}$.

(b) Determine the eigenvalues and corresponding eigenvectors of the following matrix:

$$\begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$

(7+8)

SECTION B

1. (a) Prove that the function $f(z) = e^{-\theta}\cos(\ln r) + ie^{-\theta}\sin(\ln r)$ is differentiable in the domain $r > 0,\ 0 < \theta < 2\pi$, and find $f'(z)$.

(b) Show that $u(x, y) = \log\sqrt{x^2 + y^2}$ is harmonic in some domain D and find a function $v(x, y)$ such that $f(z) = u + iv$ is analytic in the domain D. (7+8)

2. (a) Evaluate

$$\int_C f(z)dz, \text{ where } f(z) = \pi \exp(\pi \bar{z}),$$

and C is the boundary of the positively oriented square with vertices at $0,\ 1,\ 1 + i$, and i.

(b) Let C be the circle $|z - z_0| = R$, taken in the positive sense. Show that

$$\int_C (z - z_0)^{a-1} dz = i\frac{2R^a}{a}\sin(a\pi),$$

where a is non-zero real number. Here the *principal branch* of the integration is taken $(-\pi < \theta \leq \pi)$. (8+7)

3. (a) Let C be any simple closed contour described in the positive sense in the z plane, and

$$g(w) = \int_C \frac{z^3 + 2z}{(z-w)^3} dz.$$

Show that $g(w) = 6\pi i w$, where w is inside C and find $g(w)$ when w is outside C.

(b) Give the two *Laurent series* expansions for the function

$$f(z) = \frac{1}{z^2(1-z)},$$

and specify the region in which those expansions are valid. (7+8)

4. Use *residues* to evaluate the improper integrals

(a)

$$\int_0^\infty \frac{1}{x^4+1} dx.$$

(b)

$$\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}.$$

(8+7)

Good Luck . . .

A

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

I - Year - SEMESTER - II MATHEMATICS - II (MATH UC192)

QUIZ II - (Closed-Book)

Time: 30 Minutes | Dated: May 11, 2006 | Max. Marks: 30

ID No: | Section No: | Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. The Laurent series of the function $f(z) = \frac{z+1}{z-1}$ for the domain $1 <| z |< \infty$ is ....

2. Let C be positively oriented boundary of the square with sides $x = \pm 2$ and $y = \pm 2$ then

$$\int_C \frac{z}{2z+1}\, dz = ...$$

3. By using the branch $\log z = \ln r + i\theta$ $(r > 0, 0 < \theta < 2\pi)$, the value of the $\log(-1-i)$ is calculated as ....

4. The value of the integral

$$\int_C \text{Log}[(z+2)^2]dz = ...,$$

where C is the positively oriented circle $| z |= 1$.

5. Let C be the positively oriented circle $|z| = 2$, then

$$\int_C \frac{1}{z^2+1} dz = \text{................},$$

6. Let C be the positively oriented circle $|z| = 2$ from $z = 2$ to $z = 2i$ then

$$\left| \int_C \frac{1}{z^2+1} dz \right| \leq \text{................},$$

7. If C is straight line from $(0, 0)$ to $(2, 1)$ then

$$\int_C z^2 dz = \text{................},$$

8. The principal value of the

$$\frac{1}{i^i} = \text{................}$$

9. Give the domain in which the function $f(z) = (x^3 + 3xy^2) + i(y^3 + 3x^2y)$ is differentiable ....

10. If C is the positively oriented circle $|z| = 1$, then the value of the integral is

$$\int_C (z + \frac{1}{z})^2 \, dz = \text{................}$$

Good Luck . . .

B

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

I - Year - SEMESTER - II MATHEMATICS - II (MATH UC192)

QUIZ II - (Closed-Book)

Time: 30 Minutes | Dated: May 11, 2006 | Max. Marks: 30

ID No: | Section No: | Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. The value of the integral

$$\int_C \text{Log}[(z+2)^2]dz = \ldots\ldots\ldots\ldots\ldots,$$

where C is the positively oriented circle $|z| = 1$.

2. Let C be the positively oriented circle $|z| = 2$, then

$$\int_C \frac{1}{z^2+1}dz = \ldots\ldots\ldots\ldots\ldots,$$

3. The principal value of the

$$\frac{1}{i^i} = \ldots\ldots\ldots\ldots\ldots$$

4. Give the domain in which the function $f(z) = (x^3 + 3xy^2) + i(y^3 + 3x^2y)$ is differentiable ....

5. If C is the positively oriented circle $|z|=1$, then the value of the integral is

$$\int_C (z+\frac{1}{z})^2\, dz = \text{................}$$

6. The Laurent series of the function $f(z) = \frac{z+1}{z-1}$ for the domain $1 < |z| < \infty$ is ....

7. Let C be positively oriented boundary of the square with sides $x = \pm 2$ and $y = \pm 2$ then

$$\int_C \frac{z}{2z+1}\, dz = \text{................}$$

8. By using the branch $\log z = \ln r + i\theta$ $(r > 0, 0 < \theta < 2\pi)$, the value of the $\log(-1-i)$ is calculated as ....

9. Let C be the positively oriented circle $|z| = 2$ from $z = 2$ to $z = 2i$ then

$$\left| \int_C \frac{1}{z^2+1} dz \right| \leq \text{................},$$

10. If C is straight line from $(0,0)$ to $(2,1)$ then

$$\int_C z^2 dz = \text{................},$$

Good Luck . . .

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai
I Year- Semester-II (2005-2006)
Mathematics-II (MATH UC 192)
(Open Book)-MAKE- UP

Max. Marks: 60
Weightage: 20 %

Date: 2006
Duration: 50 Minutes

Note: 1. Only the text book and class notes are allowed.
2. All questions must be answered in serial order.

Q1. If $T: V_3 \to V_3$ is a linear map defined by

$T(x, y, z) = (x + y - 2z, x + 2y + z, 2x + 2y - 3z)$. Verify the rank-nullity theorem.

(8)

Q2. Find the matrix of $T: V_3 \to V_2$ by $T(x, y, z) = (2x + 3y - z, 4x - y + 2z,)$ relative to the basis $S_1 = \{(1,1,0), (1,2,3), (1,3,5)\}$, $S_2 = \{(1,2), (2,3)\}$
Is T non singular?

(8)

Q3. Find the eigenvalues and eigenvectors of A^{-1} *if* $A = \begin{bmatrix} 3 & -1 & 1 \\ 7 & -5 & 1 \\ 6 & -6 & 2 \end{bmatrix}$

(8)

Q4. Find the range, rank, null space and nullity of the linear map $T: V_2 \to P_3$ defined by $T(a, b) = a + ax + ax^2$.

(6)

Q5. (a) Find the value of z if $e^{3z} = -\sqrt{6} - \sqrt{2}i$

(5)

(b) Is $Log(1 + \sqrt{3}\,i)^2 = 2Log(1 + \sqrt{3}\,i)$? Justify.

(5)

Q6. Is $f(z) = e^{z^2}$ analytic? Justify.

(8)

Q7. Show that $u(x, y) = xy$ *and* $v(x, y) = x^3 - 3xy^2$ are both harmonic functions but they are not harmonic conjugates.

(6)

Q8. Using polar form of the C-R equation, prove that

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} = 0$$

(6)

BITS, PILANI-DUBAI CAMPUS
Knowledge Village, DUBAI
I Year- Semester-II (2005-06)

MATHEMATICS-II (MATH UC192)

MAKE Make-Up TEST-I (Closed Book)

Time: 50 Minutes
April 20, 2006

MM: 60
Weight age: 20%

NOTE: All questions are compulsory.

1. (a) Investigate for what values of λ *and* μ, the system of equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ have:
   (i) No solution, (ii) Unique Solution and (iii) Infinite number of solutions.

   (b)Prove that a non empty subset S of a vector space V is a subspace of V if and only if $\alpha u+\beta v \in S,\ \forall\ u,v \in S$ and all scalars α,β.

   (9+6)

2. (a) In any vector space prove that $\alpha u = 0$ *iff* $\alpha = 0$ *or* $u = 0$.
   (b) Let $v_1, v_2, \ldots\ldots, v_n$ be n-elements in vector space V. If $v_k \in [v_1, v_2, \ldots\ldots v_{k-1}]$, then prove that $[v_1, v_2, \ldots\ldots, v_{k-1}, v_k, v_{k+1}, \ldots\ldots, v_n] = [v_1, v_2, \ldots\ldots, v_{k-1}, v_{k+1}, \ldots\ldots, v_n]$.

   (7+8)

3. (a) If a set is linearly independent, then any subset of it is also linearly independent.
   (b) Is the set $S=\{\sin x, \sin 2x, \sin 3x, \ldots\ldots \sin nx\}$ linearly independent subset of $C[-\pi,\pi]$ for every positive integer n? Justify.

   (7+8)

4. (a) Given $S_1=\{(1,2,3),(0,1,2),(3,2,1)\}$ *and* $S_2=\{(1,-2,3),(-1,1,-2),(1,-3,4)\}$, determine the dimension and basis for $[S_1]\cap[S_2]$.
   (b) Determine whether there exists a linear map in the following case and if exists give the general formula $T:V_2 \to V_2$ such that
   $T(0,1)=(3,4),\quad T(3,1)=(2,2)\quad$ *and* $\quad T(3,2)=(5,7)$

   (7+8)

BITS-PILANI, DUBAI CAMPUS

MATHEMATICS-II (MATHUC 192)

I- Year-Semester-II (2005-06)

TEST-II (OPEN BOOK)

TIME: 50 Minutes APRIL 23, 2006 **Max. Marks: 60**

Note: 1. Answer the questions in serial order.
2. Only Text Book and Class Notes are allowed.

1. Show that the linear transformation $T: V_2 \to V_2$ defined by $T(x,y) = (5x + y, 3x - 2y)$ is nonsingular and find T^{-1}. Determine the matrix $(T^{-1} : B_{1,} \; B_2)$ where $B_{1,} \; B_2$ are the standard bases. (9)

2. Find the eigenvalues and eigenvectors of A^3 if $A = \begin{pmatrix} 0 & 1 & -2 \\ -1 & 2 & -4 \\ -1 & 1 & 4 \end{pmatrix}$ (8)

3. Find the linear transformation $T: V_3 \to V_2$ such that $A = (T \; : B_{1,} \; B_2)$ where $A = \begin{pmatrix} 3 & 2 & -1 \\ 0 & 1 & 4 \end{pmatrix}$, $B_1 = \{(1,0,1),(1,1,0),(1,0,0)\}$, $B_2 = \{(1,2),(2,3)\}$. (9)

4. Find the matrix A of the differential operator D on a vector space V relative to the basis $S = \{\sin\theta, \cos\theta\}$ for V. (4)

5. (a) Find all the values of z if $e^{5z} = 1 + i$. (4)

(b) Find $\arg z$ if $z = \dfrac{1 - i\sqrt{3}}{4 + i4\sqrt{3}}$. (3)

(c) Is $Log(i^3) = 3Log(i)$? (3)

6. Let $f(z) = \dfrac{x^2 y^2 (x+iy)}{x^4+y^4}, \quad z \neq 0$
$\quad\quad\quad = 0, \quad z = 0$

$$f(z) = \begin{cases} \dfrac{x^2 y^2 (x+iy)}{x^4+y^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

Prove that the CR equations are satisfied at $(0,0)$ but $f'(0)$ does not exist. (8)

7. Are the polar form of the CR equations satisfied for the function $f(z) = z + \dfrac{1}{z}$ where $z \neq 0$? Justify. Is $f(z)$ analytic for $z \neq 0$? (6)

8. If $u(x,y) = x^2 - y^2 + x - y$, show that $u(x,y)$ is harmonic and find its harmonic conjugate and construct $f(z)$. (6)

BITS, PILANI-DUBAI CAMPUS
KNOWLEDGE VILLAGE, DUBAI

I-Year-SEMESTER-II (2005-06)
MATHEMATICS-II (MATH UC192)
MAKE UP TEST -1 (Closed Book)

Time: 50 Minutes | Max Marks: 60
March 22, 2006 | Weightage: 20%

NOTE: - Answer all FOUR questions in serial order.

1. (i) Check the consistency of the following system of equations:
$x+2y-z=3, \quad 3x-y+2z=1, \quad 2x-2y+3z=2, \quad x-y+z=-1.$
If consistent, solve completely.

(ii) Is the subset $S=\left\{(x_1,x_2,x_3)\,\middle|\,x_1-2x_2=x_3-\frac{3}{2}x_2\right\}$ of V_3 a subspace? Justify. (9+6)

2. (i) If S is a non-empty subset of a vector space V, prove that $[S]=S$ iff S is a subspace of V.

(ii) If
$U=\{(x_1,x_2,x_3,x_4)\in V_4 \mid x_1+x_2+x_3=0\}$,
$V=\{(x_1,x_2,x_3,x_4)\in V \mid 3x_1-x_4=0\}$; $U,V\in V_4$.
Find the dimension and basis for subspace $U\cap V$ of V_4. (6+9)

3. (i) Is the subset $S=\{1,x,(x-1)x,(x-1)(x-2)x\}$ form a basis for $V=P_3$? Justify.

(ii) Prove that the vectors (a, b), (c, d) are linearly dependent iff $ad=bc$. (8+7)

4. (i) Determine whether there exists a linear map in the following case $T:V_3\to V_3$ such that $T(0,1,2)=(3,1,2)$ and $T(1,1,1)=(2,2,2)$.
If it does exist, give the general formula.

(ii) Determine a linear transformation $T:V_2\to V_2$ which maps all the vectors on the line $x+y=0$ onto themselves. $(T\neq I)$. (9+6)

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

I - Year - SEMESTER - II (2005-06)

MATHEMATICS - II (MATH UC192)

Test - I (Closed-Book)

Time: 50 Minutes

March 12, 2006

Max. Marks: 60

Weighage: 20 %

Note:- Answer all four questions in serial order.

1. (i) Determine whether the following system of linear equations is consistent.

$$2x_1 + x_2 + x_3 + x_4 = 2$$
$$3x_1 - x_2 + x_3 - x_4 = 2$$
$$x_1 + 2x_2 - x_3 + x_4 = 1$$
$$6x_1 + 2x_2 + x_3 + x_4 = 5$$

If so, discuss completely the solution.

(ii) Determine whether the subset $S = \{(x, y, z) \in V_3 \mid xx = yy\}$ of vector space V_3 is a subspace. Justify your answer. (10+5)

2. (i) Let v_1 and v_2 be two elements of a vector space V, prove that $[v_1, v_2] = [v_1 - v_2, v_1 + v_2]$

(ii) Which of the following subset S of a given vector space V are L.I.?

(a) $S = \{2,\ 4\sin^2 x,\ \cos^2 x,\ x\}$; $V = C[-\pi, \pi]$.

(b) $S = \{2x^2 - x + 7,\ x^2 + 4x + 2,\ x^2 - 2x + 4\}$; $V = \mathcal{P}_2$. (7+8)

3. (i) If $U = \{p \in \mathcal{P}_3 \mid p(1) = 0\}$ and $W = \{p \in \mathcal{P}_3 \mid p'(1) = 0\}$ are subspace of $\mathcal{P}_3$. Then find the dimension and basis for $U \cap W$.

(ii) Find an ordered basis for V_4 relative to which the vector $(-1,\ 3,\ 2,\ 1)$ has the coordinates 4, 1, -2, and 7. (9+6)

4. (i) $T : \mathcal{P}_4 \rightarrow \mathcal{P}_3$ such that $T(1 + x) = 1$, $T(x) = 3$ and $T(x^2) = 4$. Find a formula for T, if it exists.

(ii) Prove that a linear map on a *1-dimensional* vector space is nothing but multiplication by fixed scalar.

(iii) Is the *zero map* linear ? Justify. (6+5+4)

BITS-PILANI, DUBAI CAMPUS

MATHEMATICS-II (MATH UC192)

B

I- Year-Semester-II (2005-06)

QUIZ – I (CLOSED BOOK)

TIME: 30 Minutes February 21, 2006 **Max. Marks: 30**

ID No. **Name:** **Section No.:**

Note: 1. Write ID No., Name and Sec. No. in the space provided.
2. Overwriting will be treated as wrong answer.

1. Is the system

$$y - 2z = 3$$
$$3x + z = 4$$
$$x + y + z = 1$$

inconsistent? ----------------------

2. Is an empty subset of a vector space V is a subspace of V? --------------------------

3. A homogeneous system of linear equations is always consistent, since ------------------

----

4. The rank of the matrix $A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 3 & 0 \\ 2 & 1 & 3 \end{pmatrix}$ is --------------------------

5. A non-homogeneous system of linear equations has a unique solution iff ---------------

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6. Can $\{f \in C / f(0) = f(2)\}$ be a subspace of C? ---------------------------

7. Is $\left\{ p \in \mathrm{P} \Big/ p\left(\frac{1}{2}\right) = 0 \right\}$ a subspace of P_n? ---------------------------

8. Is $\{(x_1, x_2, x_3)/2x_1 + x_2 + 3x_3 = 1\}$ a subspace of V_3? Justify.

9. Is $A = \{\alpha(0,0,0)/\alpha \; is \; a \; scalar\}$ a subspace of V_3? Justify.

10. Is the matrix $A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ a singular matrix? ---------------------------

BITS-PILANI, DUBAI CAMPUS

MATHEMATICS-II (MATHUC 192)

I- Year-Semester-II (2005-06)

QUIZ – I (CLOSED BOOK) MAKE-UP

TIME: 30 Minutes | March 06, 2006 | **Max. Marks: 30**

ID No. | **Section No.:** | **Name:**

Note: 1. Answer all the questions. All carry equal Marks.
2. Overwriting will be treated as wrong Answer.

Q.1. Can the set $S = \{(x_1, x_2, x_3, x_4) \mid x_1 + \frac{2}{3}x_2 - 3x_3 + x_4 = 0\}$ be a vector space with respect to co-ordinate wise addition and scalar Multiplication? . . . .

Q.2. Show that if S is a subspace of V then $\alpha u + \beta v \in S$ for all $u, v \in S$ and all scalar α and β

Q.3. Is $S = \{ p \in P \mid \deg \text{ of } p \leq 4 \text{ and } p'(0) = 0 \}$ a subspace of P? . . . .

Q.4. In the vector space V_3 $\alpha \bar{0} = 0$ for every scalar α. State True or False . . . .

Q.5. Can $P_3[a,b]$ be a subspace of $C[a,b]$? State True or False. . . . .

Q.6. The row reduced echelon form of the Matrix

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 1 & -1 & 2 \\ 0 & 2 & 1 & -1 \end{bmatrix}$$ is . . . .

Q.7. Define column rank of a matrix.

Q.8. Tick the correct answer. The system of equation

$$x_1 + 2x_2 - x_3 - 2x_4 = 0$$
$$2x_1 + 4x_2 + 2x_3 + 4x_4 = 4$$
$$3x_1 + 6x_2 + 3x_3 + 6x_4 = 6$$

has

(a) infinite solution (b) No solution (c) Unique solution

Q.9. The Homogeneous system has a nontrivial solution iff rank(A) < n, where A is the coefficient matrix, state True or False . . . .

Q.10. The row rank of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$

is . . . .