BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

I - Year - SEMESTER - II (2004-05)

MATHEMATICS - II (MATH UC 192)

Test - I (Closed-Book)

Time: 50 Minutes
March 13, 2005

Max. Marks: 20
Weighage: 20 %

Note: Answer all four questions in serial order.

1. Determine whether the following system of linear equations is consistent? If consistent, solve the system completely.

$$2x_1 + x_3 - x_4 + x_5 = 2$$
$$x_1 + x_3 - x_4 + x_5 = 1$$
$$12x_1 + 2x_2 + 8x_3 + 2x_5 = 12$$

(5)

2. (a) Which of the following sets are subspaces? Justify.

(i) $S = \{f \in C[a, b] \mid f(x) = \sqrt{2}\}$

(ii) $S = \{(x_1, x_2, x_3) \in V_3 \mid x_3 \text{ is integer}\}$

(b) Let $S = \{x^3,\ x^2 + 2x,\ x^2 + 2,\ 1 - x\}$. Determine whether the polynomial $x^3 - \frac{3}{2}x^2 + \frac{x}{2} \in [S]$. (3+2)

3. (a) Find a basis and dimension for a subspace

$$U = \{p \in \mathcal{P}_3 \mid p(1) = 0 \text{ and } p'(1) = 0\}$$

of $\mathcal{P}_3$.

(b) Find the coordinate vector of $(2,\ 3,\ 4,\ -1)$ relative to the ordered basis $B = \{(1,\ 1,\ 0,\ 0), (0,\ 1,\ 1,\ 0), (0,\ 0,\ 1,\ 1)(1,\ 0,\ 0,\ 0)\}$ for V_4. (3+2)

4. (a) If a set is linearly dependent, then prove that any superset of it is also linearly dependent.

(b) Determine whether
$S = \{1,\ x,\ x(1-x),\ x(x-1)(x-2)\}$ forms a basis for $\mathcal{P}_3$. (3+2)

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

I - Year - SEMESTER - II (2004-05)

MATHEMATICS - II (MATH UC 192)

Test - II (Open-Book)

Time: 50 Minutes
April 24, 2005

Max. Marks: 20
Weighage: 20 %

Note: 1. Answer all four questions in serial order. 2. The textbooks and class-notes are allowed.

1. (a) Find the matrix for the linear transformation $T : V_3 \to V_2$, such that,
$T(x, y, z) = (2x + y - z,\ 3x - 2y + 4z)$
relative to the bases
$B_1 = \{(1,1,1), (1,1,0), (1,0,0)\}$, $B_2 = \{(1,3), (1,4)\}$.
(b) Show that $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and A^T have the same eigenvalues, but different eigenvectors for an eigenvalue.
(2.5+2.5)

2. (a) Find the square root of $3i$ and express in the cartesian form.
(b) Sketch the region $|z - 1 + 2i| \leq 1$ and state whether it is a domain.
(c) Find Arg(z) if $z = -1 - \sqrt{3}i$
(2+2+1)

3. (a) Is $f(z) = \sqrt{|xy|}$ differentiable at (0,0)? Justify.
(b) Let

$$f(z) = \begin{cases} \frac{(1+i)x^3-(1-i)y^3}{x^2+y^2}, & (x,y) \neq (0,0), \\ 0 & (x,y) = (0,0). \end{cases} \quad (1)$$

Are Cauchy-Riemann equations satisfied at (0,0)? Justify.
(2+3)

4. (a) Locate the singularities of the function:

$$f(z) = \frac{\tan z}{z^3 + 1}.$$

(b) Determine whether the function $u(x, y) = \log \sqrt{x^2 + y^2}$ *harmonic*. If so, find an analytic function $f(z) = u + iv$.
(2+3)

A

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

I - Year - SEMESTER - II MATHEMATICS - II (MATH UC 192)

QUIZ II - (Closed-Book)

Time: 30 Minutes　　　Dated: April 12, 2005　　　Max. Marks: 10

ID No:　　　Section No:　　　Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. Let $T : V_3 \to V_3$ be a linear map given by $T(x_1, x_2, x_3) = (x_1 + x_3, x_1 - x_3, x_2)$, then $T^{-1}(y_1,\ y_2,\ y_3) =$ ....

2. Let $T : V_3 \to V_3$ given by $T(e_1) = e_1 - e_2$, $T(e_2) = e_2$, $T(e_3) = e_1 + e_2 - 7e_3$. Is T non-singular? ....

3. (a) A linear transformation T is invertible if and only if ....
   (b) Every linear map $T : V_3 \to V_2$ has inverse. State True or False. ....

4. Given that $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$. A linear transformation $T : V_3 \to V_2$ such that $A = (T : B_1, B_2)$, where B_1 and B_2 are standard bases for V_3 and V_2 respectively, is ....

5. The matrix for the linear transformation $T : P_3 \to P_2$, such that, $T(p) = p'$, relative to standard bases B_1 and B_2 for P_3 and P_2 respectively is, ...

6. Given that $\lambda = 5$ is an eigenvalue of matrix $A = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$. The corresponding eigenvector is ....

7. If eigenvalues of an invertible matrix A are $1, -2, 2$, then (a) the eigenvalues of A^{-1} are ....

   (b) the eigenvalues of the fifth power of A are ....

8. The all possible values of $\sqrt[4]{1}$ are ....

9. (a) What does $|z - i| < i$ represent? ....

   (b) Is $z = i$ an accumulation point of the set $|z| < 1$ (Yes/No) ....

10. State true/false with reasons: (a) $3 + 2i > 1 + i$. ....

    (b) The annulus $1 \leq z < 2$ a domain. ....

B

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

I - Year - SEMESTER - II MATHEMATICS - II (MATH UC 192)

QUIZ II - (Closed-Book)

Time: 30 Minutes   Dated: April 12, 2005   Max. Marks: 10

ID No: Section No: Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. If eigenvalues of an invertible matrix A are $1, -2, 2$, then (a) the eigenvalues of A^{-1} are ....

   (b) the eigenvalues of the fifth power of A are ....

2. The all possible values of $\sqrt[4]{1}$ are ....

3. (a) What does $|z - i| < i$ represent? ....
   (b) Is $z = i$ an accumulation point of the set $|z| < 1$ (Yes/No) ....

4. (a) A linear transformation T is invertible if and only if ....
   (b) Every linear map $T : V_3 \to V_2$ has inverse. State True or False. ....

5. Given that $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$. A linear transformation $T : V_3 \to V_2$ such that $A = (T : B_1, B_2)$, where B_1 and B_2 are standard bases for V_3 and V_2 respectively, is ....

6. The matrix for the linear transformation $T : P_3 \rightarrow P_2$, such that, $T(p) = p'$, relative to standard bases B_1 and B_2 for P_3 and P_2 respectively is, ...

7. Given that $\lambda = 5$ is an eigenvalue of matrix $A = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$. The corresponding eigenvector is ....

8. State true/false with reasons: (a) $3 + 2i > 1 + i$. ....

   (b) The annulus $1 \leq z < 2$ a domain. ....

9. Let $T : V_3 \rightarrow V_3$ be a linear map given by $T(x_1, x_2, x_3) = (x_1 + x_3, x_1 - x_3, x_2)$, then $T^{-1}(y_1,\ y_2,\ y_3) =$ ....

10. Let $T : V_3 \rightarrow V_3$ given by $T(e_1) = e_1 - e_2$, $T(e_2) = e_2$, $T(e_3) = e_1 + e_2 - 7e_3$. Is T non-singular? ....

A

BITS, PILANI-DUBAI CAMPUS

Knowledge village, DUBAI

First Year - Semester-II (2004-05)

MATHEMATICS-II (MATH UC192)

QUIZ – I (CLOSED BOOK)

Time: 30 Minutes
Date: March 22, 2005

Max.Marks: 10

Name: **ID No. :** **Section:**

N.B: All questions carry equal marks. Overwriting will be treated as wrong answer.

1. [{(-2, 0), (0, 5)}] = ---------------.

2. [{1, x, x^2 }] = ---------------.

3. Is the set $S = \{t, t^2, e^{2t}\}$ *in* $C(-\infty, \infty)$ linearly independent? ---------------.

4. Is the set {(0,0,1), (1,0,1), (1,-1,1), (3,0,1)} linearly independent? ---------------.

5. If $U = \{(x, y, z) \in V_3 \mid x + y + z = 0\}$, then $\dim U =$ ---------------.

6. If U and W are subspaces of a finite dimensional vector space V with their intersection as the zero subspace of V, then $\dim(U+W) = \dim U + \dim W$. State True or False? ---------------.

7. $[(1,-2,1), (2,1,5)] = [\,[(1,-2,1), (2,1,5)]\,]$. State True or False?---------------.

8. If V has a basis of 10 vectors, then any set of 12 vectors in V is linearly independent. State True or False? ---------------.

9. If U and W are two subspaces of a vector space V. Then $U+W$ is a subspace of V containing both U and W. State True or False? ---------------.

10. Can the set $S = \{x^2 - 4, x + 2, x - 2, \frac{x^2}{3}\}$ form a basis for P_2? ---------------.

11. A linear transformation T is completely determined by its values on the elements of a basis. State True or False? ---------------.

12. Let U be a vector space, then a linear map $T: U \to U$ is called ---------------..

13. Let $T: U \to V$ be a linear map, then $N(T)$ is a subspace of V. State True or False? -----------.

14. Let $T: U \to V$ be a linear map, then $T(0_U) =$ ---------------.

15. The nullity of a linear map $T: U \to V$ is defined as ---------------.

16. The range of a linear map $T: U \to V$ is defined as ---------------.

17. Let $T: V_3 \to V_3$ be a linear map defined by $T(x_1, x_2, x_3) = (x_1, x_2, 0)$, then $N(T) =$ --------------.

18. Let $T: P \to P$ be a map defined by $T(p)(x) = xp(x) + p(1)$, then $T(0)(x) =$ ---------------.

19. Let $T: V_2 \to V_2$ be a linear map defined by $T(e_1) = e_1 - e_2, T(e_2) = e_1 + e_2$ then $T(x_1, x_2) =$ -----------.

20. Let $T: P \to P$ be defined by $T(p)(x) = 2 + 3x + 7x^2 p(x)$. Is this map linear? ---------------.

B

BITS, PILANI-DUBAI CAMPUS

Knowledge village, DUBAI

First Year - Semester-II (2004-05)

MATHEMATICS-II (MATH UC192)

QUIZ – I (CLOSED BOOK)

Time: 30 Minutes
Date: March 22, 2005

Max.Marks: 10

Name: **ID No. :** **Section:**

N.B: All questions carry equal marks. Overwriting will be treated as wrong answer.

1. If $U = \{(x, y, z) \in V_3 \mid x + y + z = 0\}$, then $\dim U =$ ---------------.

2. Is the set {(0,0,1), (1,0,1), (1,-1,1), (3,0,1)} linearly independent? ---------------.

3. If U and W are subspaces of a finite dimensional vector space V with their intersection as the zero subspace of V, then $\dim(U + W) = \dim U + \dim W$. State True or False? ---------------.

4. [{(-2, 0), (0, 5)}] = ---------------.

5. $[\{1, x, x^2\}]$ = ---------------.

6. $[(1,-2,1), (2,1,5)] = [\,[(1,-2,1), (2,1,5)]\,]$. State True or False?---------------.

7. If V has a basis of 10 vectors, then any set of 12 vectors in V is linearly independent. State True or False? ---------------.

8. Is the set $S = \{t, t^2, e^{2t}\}$ in $C(-\infty, \infty)$ linearly independent? ---------------.

9. If U and W are two subspaces of a vector space V. Then $U+W$ is a subspace of V containing both U and W. State True or False? ---------------.

10. Can the set $S = \{x^2 - 4, x+2, x-2, \frac{x^2}{3}\}$ form a basis for P_2? ---------------.

11. Let U be a vector space, then a linear map $T: U \to U$ is called ---------------..

12. Let $T: U \to V$ be a linear map, then $T(0_U) =$ ---------------.

13. The nullity of a linear map $T: U \to V$ is defined as ---------------.

14. Let $T: V_3 \to V_3$ be a linear map defined by $T(x_1, x_2, x_3) = (x_1, x_2, 0)$, then $N(T) =$ ---------------.

15. Let $T: P \to P$ be a map defined by $T(p)(x) = xp(x) + p(1)$, then $T(0)(x) =$ ---------------.

16. Let $T: V_2 \to V_2$ be a linear map defined by $T(e_1) = e_1 - e_2, T(e_2) = e_1 + e_2$ then $T(x_1, x_2) =$ -----------.

17. Let $T: P \to P$ be defined by $T(p)(x) = 2 + 3x + 7x^2 p(x)$. Is this map linear? ---------------.

18. A linear transformation T is completely determined by its values on the elements of a basis. State True or False? ---------------.

19. Let $T: U \to V$ be a linear map, then $N(T)$ is a subspace of V. State True or False? ------------.

20. The range of a linear map $T: U \to V$ is defined as ---------------.

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

I - Year - SEMESTER - II (2004-05)

MATHEMATICS - II (MATH UC 192)

COMPREHENSIVE EXAMINATION

(Closed-Book)

Time: 03 Hours | Max. Marks: 80

May 24, 2005 | Weighage: 40 %

Note:- 1. All questions are compulsory and should be answered sequentially.

2. There are three sections (A, B, and C) in the question paper and should be answered in three separate answer sheets.

3. Write ***A/B/C*** *on the top of each answer sheet in* ***CAPITAL BOLD LETTERS.***

SECTION A

1. Determine whether the following system of linear equations is consistent.

$$\begin{aligned} x_1 - x_2 + 2x_3 + 3x_4 &= 1 \\ 2x_1 + 2x_2 + \quad 2x_4 &= 1 \\ 4x_1 + x_2 - x_3 - x_4 &= 1 \\ x_1 + 2x_2 + 3x_3 \quad &= 1 \end{aligned}$$

   If so, discuss completely the solution. (8)

2. (a) Is the set $S = \{p \in \mathcal{P} |\text{ degree of } p \leq 3\}$ subspace of vector space $\mathcal{P}$? Justify your answer.
   (b) Show that the set $S = \{\sin x, \sin 2x, \sin 3x, \ldots, \sin nx\}$ is a linearly independent subset of $C[-\pi, \pi]$. (4+4)

3. (a) Can the set $S = \{(1, 2, 3), (3, 1, 0), (-2, 1, 3)\}$ form a basis for V_3? In case S is not basis for V_3, determine a basis for $[S]$.
   (b) Determine the range, kernel, rank and nullity of a linear transformation $T : V_3 \to V_4$ such that $T(x_1, x_2, x_3) = (x_1, x_1 + x_2, x_1 + x_2 + x_3, x_3)$. (4+5)

SECTION B

1. (a) If λ is an eigenvalue of the matrix, prove that

   (i) λ is also eigenvalue of A^T.

   (ii) $1/\lambda$ is an eigenvalue of A^{-1}, if A is nonsingular.

   (b) Let $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{pmatrix}$.

   Determine a linear transformation T such that $A = (T : B_1, B_2)$ when

   $B_1 = \{(1,\ 1,\ 1),\ (1,\ 2,\ 3),\ (1,\ 0,\ 0),\}$

   $B_2 = \{(1,\ 1),\ (1,\ -1)\}$. (5+5)

2. Let a function $f(z)$ be analytic in a domain D. Prove that $f(z)$ is constant in D if

   (a) $f(z)$ is real valued for all z in D.

   (b) $\overline{f(z)}$ is analytic in D.

   (c) $|f(z)|$ is constant in D. (6)

3. (a) Verify that the function $f(z) = e^{-y}\sin x - ie^{-y}\cos x$ is entire.

   (b) Find the principal value of

   $$\left[\frac{e}{2}(-1-\sqrt{3}i)\right]^{3\pi i}.$$

   (c) Show that

   $$\log(i^2) \neq 2\log i \qquad \text{when } \log z = \ln r + i\theta \ \left(r > 0, \frac{3\pi}{4} < \theta < \frac{11\pi}{4}\right).$$

   (3+3+3)

SECTION C

1. (a) Evaluate $\int_C f(z)dz$, where $f(z)$ is defined by

   $$f(z) = \begin{cases} 1, & y < 0, \\ 4y & y > 0. \end{cases}$$ (1)

   and C is the arc from $z = -1 - i$ to $z = 1 + i$ along the curve $y = x^3$.

   (b) Use antiderivative to show that

   $$\int_{-2i}^{2i} \frac{dz}{z} = \pi i,$$

when the path of the integration from $-2i$ to $2i$ is the right hand half of the circle $|z| = 2$.

(c) Show that if $f(z)$ is analytic within and on simple closed contour C and z_0 is not on C then

$$\int_C \frac{f'(z)}{z - z_0} dz = \int_C \frac{f(z)}{(z - z_0)^2} dz.$$

(5+3+4)

2. (a) Show that the singular point of the following function is a pole.

$$f(z) = \frac{1 - \exp(2z)}{z^4}.$$

Determine the order m of that pole and the corresponding residue B.

(b) Write the two Laurent series in powers of z that represent the function

$$f(z) = \frac{1}{z(z^2 + 1)}$$

in certain domains, specify domains. (3+5)

3. (a) Use residues to evaluate the improper integral

$$\int_0^\infty \frac{\cos x}{(x^2 + 1)^2} dx.$$

(b) Use residues to evaluate

$$\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta}, \qquad \text{where} \qquad (-1 < a < 1).$$

(5+5)

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

I - Year - SEMESTER - II (2004-05)

MATHEMATICS - II (MATH UC 192)

COMPREHENSIVE EXAMINATION

(Closed-Book)

MAKE-UP

Time: 03 Hours
Date: , 2005

Max. Marks: 80
Weighage: 40 %

Note:- 1. All questions are compulsory and should be answered sequentially.

2. There are three sections (A, B, and C) in the question paper and should be answered in three separate answer sheets.

3. Write A/B/C on the top of each answer sheet in CAPITAL BOLD LETTERS.

SECTION A

1. Determine whether the following system of linear equations is consistent.

$$2x_1 + x_2 + x_3 + x_4 = 2$$
$$3x_1 - x_2 + x_3 - x_4 = 2$$
$$x_1 + 2x_2 - x_3 + x_4 = 1$$
$$6x_1 + 2x_2 + x_3 + x_4 = 5$$

If so, discuss completely the solution. (7)

2. (a) Prove that a non-empty subset of a vector-space V is subspace of V if $u + v \in S$ and $\alpha u \in S$, whenever $u, v \in S$ and α, a scalar.
(b) Verify rank-nullity theorem for the linear transformation $T : V_4 \to V_3$ such that $T(e_1) = (1,\ 1,\ 2)$, $T(e_2) = (1,\ -1,\ 1)$, $T(e_3) = (1,\ 0,\ 0)$, $T(e_4) = (1,\ 0,\ 1)$. (4+5)

3. (a) Prove that the vectors (a, b), (c, d) are LD iff $ad = bc$.
(b) Find the coordinates of the polynomial $3 + 7x + 2x^2$ relative to the ordered basis $\{1-x, 1+x, 1-x^2\}$ of $\mathcal{P}_2$. (4+5)

SECTION B

1. Prove that the linear transformation
$T : V_3 \to V_3$ such that $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 - e_2 + e_3$, $T(e_3) = 3e_1 + 4e_3$, is non-singular and find its inverse.
(b) Determine eigenvalues and the corresponding eigenvectors for the following matrix

$$A = \begin{pmatrix} 0 & i & i \\ i & 0 & i \\ i & i & 0 \end{pmatrix}$$

(5+5)

2. (a) Verify that the function

$$g(z) = \ln r + i\theta \qquad (r > 0, 0 < \theta < 2\pi)$$

is analytic in the indicated domain, with derivative $g'(z) = 1/z$
(b) Show that $u(x, y) = y/(x^2+y^2)$ is harmonic in some domain D and find a function $v(x, y)$ such that $f(z) = u + iv$ is analytic in the domain D. (3+5)

3. (a) Find the principal value of $(1 - i)^{4i}$.
(b) Show that (i) $\text{Log}(1 + i)^2 = 2\text{Log}(1 + i)$;
(ii) $\text{Log}(-1 + i)^2 \neq 2\text{Log}(-1 + i)$ (3+5)

SECTION C

1. (a) Show that

$$\int_C z^i dz = \frac{1 + e^{-\pi}}{2}(1 - i),$$

where z^i denotes the principal branch

$$z^i = \exp(i\text{Log}z) \qquad (|z| > 0, -\pi < Arg z < \pi)$$

and where the path of the integartion is any contour from $z = -1$ to $z = 1$.
(b) Let C be any simple closed contour, described in the positive sense in the z plane, and

$$g(w) = \int_C \frac{z^3 + 2z}{(z - w)^3} dz.$$

Show that if $g(w) = 6\pi i w$ where w is inside C and that $g(w)$ when w is outside

(c) Define a smooth arc and a contour.

(4+4+2)

2. (a) Find the value of the integral

$$\int_c \frac{\cosh \pi z}{z(z^2+1)} dz,$$

taken counterclockwise around the circle $|z| = 2$.

(b) Determine whether that $z_0 = 0$ is removable singular point of the

$$f(z) = \frac{1 - \cos z}{z^2}$$

Hence or otherwise find $f(0)$.

(c) Find the residue of the function

$$f(z) = \frac{z^2 - 2z + 3}{z - 2},$$

when $0 < |z - 2| < \infty$.

(5+3+2)

3. (a) Use residues to evaluate the improper integral

$$\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)} dx.$$

(b) Use residues to evaluate

$$\int_0^{2\pi} \frac{d\theta}{1 + a \sin\theta}. \qquad (-1 < a < 1)$$

(5+5)