BITS PILANI, DUBAI CAMPUS

INTERNATIONAL ACADEMIC CITY, DUBAI

FIRST YEAR – I SEMESTER (2013-14)

MATHEMATICS-I (MATH F111)

COMPREHENSIVE EXAMINATION

Date: 02.01.2014	**Max. Marks: 120**
Time: 3 hours	**Weightage: 40 %**

Answer Part A, Part B and Part C in separate Answer Books.
Answer all the questions.

PART A

1. Find the polar coordinates of the centre, foci and vertices of the conic $r = \dfrac{4}{2 - \sin\theta}$ (8)

2. Find the area of the region inside $r = 1 + \cos 2\theta$ and outside the circle $r = 1$. (8)

3. Find $\hat{T}$, $\hat{N}$ and the curvature for $\vec{r}(t) = \sin t\ \hat{i} + \sqrt{2}\cos t\ \hat{j} + \sin t\ \hat{k}$ (8)

4. Find the length of the indicated portion of the curve $\vec{r}(t) = t\ \hat{i} + \dfrac{2}{3}t^{3/2}\ \hat{k},\ 0 \le t \le 8$ (8)

5. Find the parametric equation for the line that is tangent to the given curve at $t = 0$:
$\vec{r}(t) = \sin t\ \hat{i} + (t^2 - \cos t)\ \hat{j} + e^t\ \hat{k}$ (8)

PART B

6. Find $\dfrac{df}{dt}$ at $t = 0$, if $f(x, y, z) = x^3 + xz^2 + y^3 + xyz,\ x = e^t,\ y = \cos t,\ z = t^3$. (7)

7. (a) Find the directional derivative of the function $f(x, y, z) = \ln(2x^2z + 3xy^3 + 6z)$ at the point (-1, -1, 1) in the direction of $2\hat{i} + 3\hat{j} + 6\hat{k}$. (7)

   b) Find the equations of the tangent plane and the normal line to the level surface $x^2 + y^2 + z = 4$ at the point (1, 1, 2). (6)

8. Maximize $f(x, y, z) = x^2 + 2y - z^2$ subject to the constraints $2x - y = 0$ and $y + z = 0$. (6)

9. Evaluate the given cartesian integral by changing it into an equivalent polar integral

$$\int_0^6 \int_0^y x\,dx\,dy\,.$$ (7)

10. Let R be the region in the first quadrant of XY plane bounded by hyperbola $xy=1$; $xy=9$ and the lines $y=x$, $y=4x$. Use the transformation $x=\frac{u}{v}$; $y=uv$ with $u,v>0$ to evaluate the integral

$$\iint_R \left(\sqrt{\frac{y}{x}}+\sqrt{xy}\right)dxdy \qquad (7)$$

PART C

11. Determine if the following series are convergent or divergent (5+5)

i) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\tan^{-1}n}{n^2+1}$

ii) $\sum_{n=1}^{\infty}\frac{(n+1)(n+2)}{n!}$

12. Use Stoke's theorem to find $\int_C \vec{F}.d\vec{r}$, if $\vec{F}=(x+2y)\hat{i}+(x-z)\hat{j}+(y-z)\hat{k}$ where C is the boundary of triangle cut from the plane $3x+2y+z=6$ by the first octant oriented in counterclockwise direction. (8)

13. Show that $\vec{F}=(e^x\cos y+yz)\hat{i}+(xz-e^x\sin y)\hat{j}+(xy+z)\hat{k}$ is conservative field and hence find the scalar potential function for it. (8)

14. Find line integral of $f(x,y)=\frac{x^3}{y}$ over $C: y=\frac{x^2}{2}$, $0\le x\le 2$. (6)

15. Verify Green's theorem for the field $\vec{F}=(x^2-y)\hat{i}+x\hat{j}$ where C: $\vec{r}(t)=(\cos t)\hat{i}+(\sin t)\hat{j}$ for $0\le t\le 2\pi$. (8)

ALL THE BEST!

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INTERNATIONAL ACADEMIC CITY, DUBAI
First Year – Semester I (2013-14)

MATHEMATICS- I (MATH F111)

TEST – II (Open Book)

Date: 17.11.2013 **Max. Marks: 60**
Time: 50 minutes **Weightage: 20%**

Answer all the questions

1. By different path test approach method check whether function $f(x,y)=\dfrac{x^3+y^2}{\sqrt{x^3y^2}}$ is continuous or not at $(0,0)$. [5]

2. Find $\dfrac{\partial w}{\partial p}$ and $\dfrac{\partial w}{\partial q}$ using the chain rule if, $w=x^2+\dfrac{y}{x}$, $x=2p+q-2$ and $y=p-2q+1$ at the point $p=1,\ q=2$. [9]

3. Find the functions's domain, range and boundary if it exists? Also find whether domain is open, closed bounded or unbounded? [10]

   (i) $f(x,y)=\dfrac{x^2+y^2}{\sin(x-y)}$

   (ii) $f(x,y)=\dfrac{x+y}{|x|+|y|}$

4. Show that the following function satisfies Laplace equation. [10]

$$f(x,y,z)=\frac{\cos(3x+4y)+\sin(3x+4y)}{e^{5z}}$$

5. For the function $f(x,y,z)=\sin yz+\log x^2$ [8]

   (i) Find directional derivative in the direction of $\hat{i}+\hat{j}-\hat{k}$ at $(1,1,\pi)$.

   (ii) Find also directional derivative of f in the direction where it is increasing most rapidly at $(1,1,\pi)$.

6. Find the point on the curve $\vec{r}=(t^2\sin t)\hat{i}+(t^2\cos t)\hat{j}+(2t)\hat{k}$ which is at a distance 15 units along the curve from the origin in the direction of increasing arc length. [9]

7. Find unit tangent vector $(\vec{T})$, unit normal vector $(\vec{N})$, and curvature(κ) of [9]
   $\vec{r}=(2t)\hat{i}+(\sin 3t)\hat{j}+(\cos 3t)\hat{k}$

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BITS PILANI, DUBAI CAMPUS
INTERNATIONAL ACADEMIC CITY, DUBAI
First Year – Semester I (2013-14)

MATHEMATICS- I (MATH F111)

TEST – II (Open Book)

Date: 27.11.2013 **Max. Marks: 60**
Time: 50 minutes **Weightage: 20%**

Answer all the questions

1. For the function $f(x,y,z)=2x^3-\tan^{-1}\frac{y}{x}+e^{yz}$. Find directional derivative in the direction of $\hat{i}+2\hat{j}+3\hat{k}$ at $(1,1,2)$ also the direction in which it is increasing most rapidly at $(1,1,2)$. [9]

2. Find unit tangent vector$(\vec{T})$ and arc length parameter for $\vec{r}=(2\cos t)\hat{i}+(2\sin t)\hat{j}+(t^2)\hat{k}$. [9]

3. By different path test approach method check whether function $f(x,y)=\frac{x^7+y^5}{\sqrt{x^7y^5}}$ is continuous or not at $(0,0)$. [5]

4. Find $\frac{dw}{dp}$ using the chain rule if, $w=x^2e^{-3y}+y\sin z-\cos^2 2z$, $x=2\sqrt{p}$ and $y=p-1+\ln p$. [9]

5. Write $\vec{a}$ in the form $\vec{a}=a_T\vec{T}+a_N\vec{N}$ for $\vec{r}(t)=\frac{4}{9}(1+t^2)^{\frac{3}{2}}\hat{i}+\frac{4}{9}(1-t^2)^{\frac{3}{2}}\hat{j}+\frac{1}{3}t\hat{k}$ at $t=0$, without finding $\vec{T}$ and $\vec{N}$. [9]

6. Check whether the following function satisfies Laplace equation. [9]
$$f(x,y)=e^x[(x^2-y^2)\cos y-2xy\sin y]$$

7. Find the functions's domain, range and boundary if it exists? [10]
   (i) $f(x,y)=\ln(9-x^2-y^2)+\ln(4-x^2-y^2)$
   (ii) $f(x,y,z)=\ln(x^2+y^2)-\ln z$

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BITS PILANI, DUBAI CAMPUS
INTERNATIONAL ACADEMIC CITY, DUBAI
First Year – Semester I (2013-14)

MATHEMATICS- I (MATH F111/ MATH C191)

TEST – I (Closed Book)

Date: 29.09.2013 **Max. Marks: 75**
Time: 50 minutes **Weightage: 25%**

Answer all the questions

1. Graph the following inequalities:
   a. $|r| \geq 2, \quad -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$ (6)
   b. $1 \leq r \leq 1 + 2\cos\theta$ (7)
2. Graph the curve $r = 6\sin 2\theta$. (10)
3. Sketch the conic sections whose polar equation is given by $r = \frac{12}{3 + \sin\theta}$. Give the polar coordinates for the vertices, foci and center. (13)
4. Find the area of the region inside the circle $r = 6\sin\theta$ and above the line $r = \frac{3}{2}\cos ec\theta$. (13)
5. Find the area of the region inside the polar curve $r = 3 + 2\sin\theta$ and outside $r = 2$. (13)
6. Find the length of the curve $r = 2\cos\theta + 2\sin\theta, \ 0 \leq \theta \leq \frac{\pi}{2}$ (13)

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First year – I Semester 2013 – 2014
Mathematics I (MATH F111)

A

Quiz – II **10.12.2013**

Time: 20 Minutes Max Marks: 21 Weightage: 7%

Name: **ID:** **Faculty name:**

Answer all the questions

1) Find equation of tangent plane at the point $(2,1,0)$ on given surface $\sin \pi x - x^2 y + 4e^{xz} + yz = 0$ [3]

2) By using Lagrange multiplier method find the points on the curve $xy^2 = 2$ nearest to origin. [5]

3) Change the order of integration $\int_0^1 \int_{2x}^{\sqrt{4x}} (x+y)dydx$. (Do not evaluate, write only limits of integration) [3]

4) Convert into polar integral and evaluate $\int_{-2}^{0} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} (x^2+y^2)dxdy$ [5]

5) Find maxima , minima and saddle points of the function $f(x,y) = y\tan x$ where $\frac{\pi}{2} \le x \le \pi$ and $\infty < y < \infty$. [5]

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Mathematics I (MATH F111)

B

Quiz – II **10.12.2013**

Time: 20 Minutes Max Marks: 21 Weightage: 7%

Name: **ID:** **Faculty name:**

Answer all the questions

1) Find equation of tangent plane at the point $(1,1,0)$ on given surface $\cos \pi x - x^2 y + e^{xz} + xy = 0$. [3]

2) By using Lagrange multiplier method find the points on the curve $x^2 y = 2$ nearest to origin. [5]

3) Convert into polar integral and evaluate $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{0} (x^2+y^2)dydx$ [5]

4) Change the order of integration $\int_{0}^{2} \int_{\frac{y^2}{4}}^{\frac{y}{2}} (x+y)dxdy$ (Do not evaluate, write only limits of integration) [3]

5) Find maxima , minima and saddle points of the function $f(x,y) = y\cos x$, where $0 \le x \le \frac{\pi}{2}$ and $\infty < y < \infty$ [5]

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Mathematics I (MATH F111)

A

Quiz - 1 **22.10.2013**

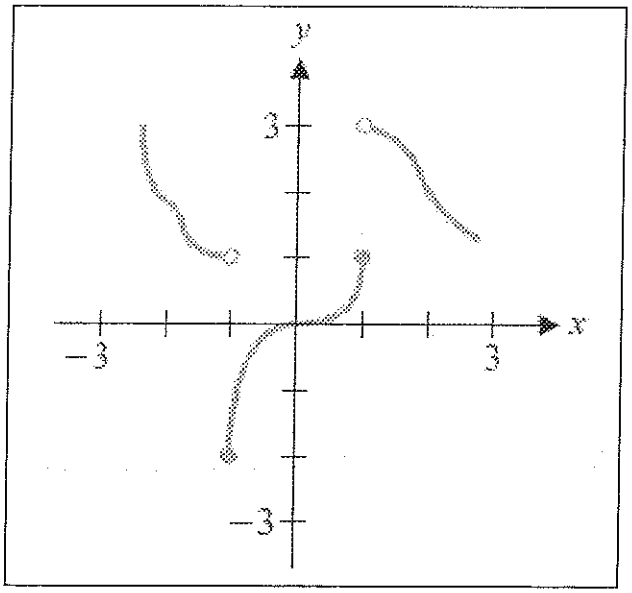
Time: 20 Minutes Max Marks: 24 Weightage: 8%

Name: ID: FACULTY'S NAME:

Answer all the questions

1) Find an open interval about $x_0 = 2$ for which the inequality $|(2x^2 - 1) - 7| < 0.1$ holds. Also find a δ such that above inequality is true for all x in $0 < |x - 2| < \delta$. [4]

2) Given the following graph, [4]



a) Find $\lim_{x \to 0} f(x)$

b) Find $\lim_{x \to -1^-} f(x)$

c) Is $f(x)$ continuous at $x = 1$? If not give reason?

3) Find $\lim_{x\to 1^-} \frac{x^2-4x-3}{x+1}$ [4]

4) Find the arc length of $\vec{r}(t) = \cos t\,\hat{i} + t\,\hat{j} + \sin t\,\hat{k}$ in $\frac{\pi}{4} \le \theta \le \pi$. [4]

5) For the position vector of the particle $\vec{r}(t) = 3\cos t\,\hat{i} + 2\sin t\,\hat{j}$, find the velocity at $t = \frac{\pi}{2}$ and sketch it on path curve of the particle. [4]

6) Solve the initial value problem $\frac{d\vec{r}}{dt} = (t^3 + 4t)\,\hat{i} + t\,\hat{j} + 2t^2\,\hat{k}$, given that $\vec{r}(0) = \hat{i} - \hat{j}$. [4]

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B

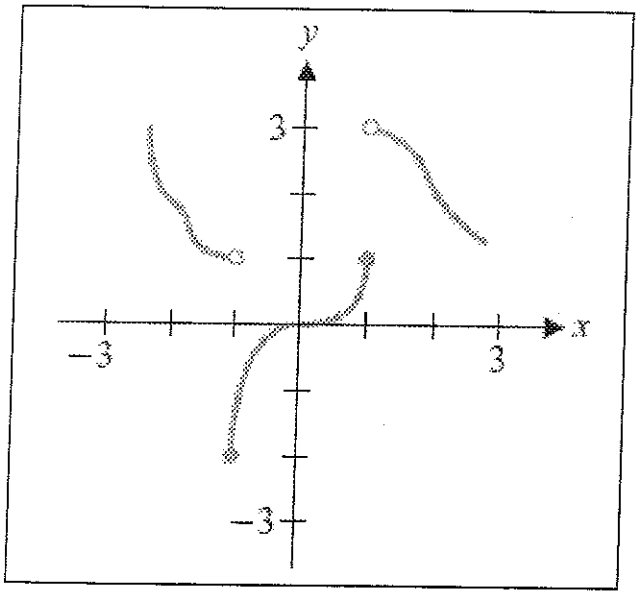
Quiz - 1 **22.10.2013**

Time: 20 Minutes Max Marks: 24 Weightage: 8%

Name: ID No.- Faculty's Name:

1) Find an open interval about $x_0 = 2$ for which the inequality $|(2x^2 - 1) - 7| < 0.2$ holds . Also find a δ such that above inequality is true for all x in $0 < |x - 2| < \delta$. [4]

2) Given the following graph, [4]



a) Find $\lim_{x \to 1^+} f(x)$

b) Find $\lim_{x \to 2} f(x)$

c) Is $f(x)$ continuous at $x = -1$? If not give reason?

3) Find $\lim_{x \to 1^+} \frac{x^2 - 4x - 3}{x + 2}$ [4]

4) Find the arc length of $\vec{r}(t) = 2\cos t\,\hat{i} + 2t\,\hat{j} + 2\sin t\,\hat{k}$ in $\frac{\pi}{4} \le \theta \le \pi$. [4]

5) For the position vector of the particle $\vec{r}(t) = 3\cos t\,\hat{i} + 2\sin t\,\hat{j}$, find the velocity at $t = \pi$ and sketch it on path curve of the particle. [4]

6) Solve the initial value problem $\frac{d\vec{r}}{dt} = (t^3 + 4t)\,\hat{i} + t\,\hat{j} + 2t^2\,\hat{k}$, given that $\vec{r}(0) = \hat{i} + \hat{j}$. [4]