BITS PILANI, DUBAI CAMPUS

INTERNATIONAL ACADEMIC CITY, DUBAI

FIRST YEAR – I SEMESTER (2012-13)

MATHEMATICS-I (MATH C191 / MATH F111)

COMPREHENSIVE EXAMINATION (CLOSED BOOK)

Date: 10.01.2013	**Max. Marks: 120**
Time: 3 hours	**Weightage: 40 %**

<u>Answer Part A and Part B in separate Answer Books.</u>
<u>Answer all the questions.</u>

PART A

1. Find the area of the region inside the circle $r = 6$ above the line $r = 3\cos ec\theta$. (7)

2. If $e = \dfrac{1}{3}$ and $r\sin\theta = -6$, find the polar equation of the conic and sketch it by labeling the vertices, centre and foci with the appropriate polar coordinates (7)

3. If $f(x) = x^2 - 5, \quad x_0 = 4, \quad \varepsilon = 0.05, \quad L = 11$ find a number $\delta > 0$ such that for all x: $0 < |x - x_0| < \delta \Rightarrow \quad |f(x) - L| < \varepsilon$ by finding an open interval about x_0 such that the inequality holds? (6)

4. Find the point on the curve $\vec{r}(t) = 2t\,\hat{i} + 3\sin(2t)\,\hat{j} + 3\cos(2t)\,\hat{k}$ at a distance $\dfrac{\pi\sqrt{10}}{3}$ units along the curve from the origin in the direction of increasing arc length. (7)

5. Find $\hat{T}$, $\hat{N}$ and curvature for the curve $\vec{r}(t) = \cos^3 t\,\hat{i} + \sin^3 t\,\hat{j}, \quad 0 < t < \dfrac{\pi}{2}$ (8)

6. Without finding $\hat{T}$ and $\hat{N}$, find $\vec{a} = a_T\hat{T} + a_N\hat{N}$ for the curve
$\vec{r}(t) = e^t\cos t\,\hat{i} + e^t\sin t\,\hat{j} + \sqrt{2}\,e^t\,\hat{k}$ at $t = 0$ (7)

7. Using the chain rule, find the first order derivatives of *W* at *(u, v) = (-2, 0)* if
$W = \ln(x^2 + y^2 + z^2), \ x = u\,e^v\sin v, \quad y = u\,e^v\cos v, \quad z = u\,e^v$ (8)

8. Find the directions in which the function increases and decreases most rapidly at P_0:
$f(x, y, z) = \ln(xy) + \ln(yz) + \ln(xz)$ at P_0 (1, 1, 1) and find the derivative in those directions. (5)

PART B

1. Find all the local maxima, local minima and saddle points of the function:
$f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$ (9)

2. Evaluate the integral by changing to polar coordinates: $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2\,dy\,dx}{(1+x^2+y^2)^2}$ (8)

3. Find the volume of the region in the first octant bounded by the coordinate planes and the plane $x+y+z=4$ (8)

4. Using the transformation $u = x - y,\ \ v = 2x + y$, evaluate $\iint_R (2x^2 - xy - y^2)dx\ dy$ where R is the region in the xy- plane bounded by the lines $y=-2x+4$, $y=-2x+7$, $y = x-2$ and $y = x+1$ in the first quadrant. (8)

5. Evaluate $\int_C f(x,y)ds$ along the curve $C: y=\frac{1}{2}x^2$ from (1, 1/2) to (0, 0), where

$$f(x,y)=\frac{x+y^2}{\sqrt{1+x^2}}\ .$$ (8)

6. Apply Green's theorem to find $\int_C (2x^2-y^2)dx+(x^2+y^2)dy$ where C is the boundary of the region enclosed by the x-axis and the upper half of circle $x^2+y^2=a^2$. (8)

7. Evaluate $\int_C \vec{F}\,.\,d\vec{r}$ by Stoke's theorem, where $\vec{F}=y^2i+x^2j-(x+z)k$ and C is the boundary of triangle with vertices at (0, 0, 0), (1, 0, 0) and (1, 1, 0). (8)

8. Test whether the following series convergence or divergence: (8)

a) $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^3}$

b) $\sum_{n=1}^{\infty}\left(\frac{\tan^{-1} n}{e^{\frac{1}{n}}}\right)^n$

ALL THE BEST!

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First Year – Semester (2012-13)

MATHEMATICS-I (MATH F111/MATH C191)

TEST – 2 (Open Book)

Date: 20.12. 2012 **Max. Marks: 60**
Time: 50 minutes **Weightage: 20%**

Answer all the questions
Only the handwritten class notes and the text book are permitted

1. Find the extreme values of $a^3x^2 + b^3y^2 + c^3z^2$ subject to the constraint $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ using the Lagrange's multiplier method. (8)

2. Evaluate: $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy\,dx}{1+x^2+y^2}$ (8)

3. Evaluate the integral by changing to polar coordinates: $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$ (9)

4. Using the transformation $x + y = u$ and $y = v$, evaluate $\int_0^{\pi} \int_0^{\pi} |\cos(x+y)|\,dx\,dy$ (9)

5. If $\vec{F}(x,y,z) = (2x\cos y - 2z^3)\hat{i} + (3 + 2ye^z - x^2 \sin y)\hat{j} + (y^2e^z - 6xz^2)\hat{k}$, check whether the vector field $\vec{F}$ is conservative. If so, find the potential function. (9)

6. A particle starts from the point *(-2, 0)* moves along the x-axis to *(2, 0)* and then along the semicircle $y = \sqrt{4-x^2}$ to the starting point. Verify Green's theorem on this particle by the force $\vec{F}(x,y) = x\,\hat{i} + (x^3 + 3xy^2)\hat{j}$. (9)

7. Use divergence theorem to evaluate $\iint_S \vec{F}.\,\hat{n}\,ds$ where S is a cube bounded by the planes $x = \pm a,\ y = \pm a,\ z = \pm a$ and $\vec{F}(x,y,z) = (x^3 - 3y^2z)\hat{i} + (y^3 - 5z^2x)\hat{j} + (z^3 - 9x^2y)\hat{k}$ (8)

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First Year – First Semester (2012-13)

MATHEMATICS-I (MATH F111/MATH C191)

TEST – 1 (Closed Book)

Date: 04.11.2012 **Max. Marks: 75**
Time: 50 minutes **Weightage: 25%**

Answer all the questions

1. Given $\frac{d^2\vec{r}}{dr^2}(t) = t\,\hat{j} + t\,\hat{k}$, find the velocity and position vectors when $\left(\frac{d\vec{r}}{dt}\right)(1) = 5\hat{j}$ and $\vec{r}(1) = 0$. Evaluate the position vector when $t = 2$. (11)

2. The position vector of a particle in the xy-plane at time t is given by
$\vec{r}(t) = (6\cos t)\hat{i} + (3\sin t)\hat{j},\ t = \frac{\pi}{4}.$ Graph the path of the curve and sketch their velocity and acceleration vectors at the given value of *t*. (11)

3. (a) If $f(x,y) = \frac{1}{\sqrt{16 - x^2 - y^2}}$, find the function's domain and its range. Also find the level curve. Check whether the domain is open/closed and bounded/unbounded. (6)

   (b) Show that the function $f(x,y) = \frac{x^2 + y^3}{x^2}$ does not have a limit at the origin. (5)

4. Check whether the given function $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$ satisfies the Laplace equation. (9)

5. By using the chain rule, find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ of $w = xy + yz + zx$ with $x = u + v,\ y = u - v,\ z = uv$ at $(u,v) = \left(\frac{1}{2}, 1\right)$. (11)

6. Find $\hat{T}$, $\hat{N}$ and curvature for the curve $\vec{r}(t) = \cos^3 t\,\hat{i} + \sin^3 t\,\hat{j}$, $0 < t < \frac{\pi}{2}$ (11)

7. Write $\vec{a}$ in the form $\vec{a}(t) = a_T\,\hat{T} + a_N\,\hat{N}$ at the given value of *t* without finding $\hat{T}$ and $\hat{N}$:

$$\vec{r}(t) = t^2\hat{i} + \left(t + \frac{t^3}{3}\right)\hat{j} + \left(t - \frac{t^3}{3}\right)\hat{k}, \quad t = \frac{1}{\sqrt{2}}$$ (11)

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First year – I Semester 2012 – 2013
Mathematics I (MATH F111/MATH C191)

A

Quiz - 2

21.10.2012

Time: 20 Minutes | Max Marks: 21 | Weightage: 7%

Name: | ID: | Section:

Answer all the questions:

1) Find the linear approximation of $f(x,y,z)=z^2+2y\cos(x^2+5)$ at the point $(0,1,4)$. (4)

2) Find f_x when $f(x,y,z)=x^2y+\frac{1}{\sqrt{x^2+y^2+z^2}}+\tan^{-1}(xyz)$. (4)

3) Find the directional derivative of the function $f(x,y,z)=2z^3-3x^2z+\sin^2 y$ at $(1,\frac{\pi}{4},1)$ in the direction of $\vec{A}=-3i+j-6k$. (4)

4) Find the equation of tangent plane to the surface $x^2+y^2-2xy-x+3y-z=-4$ at (2,-3, 18). (4)

5) Find the extreme point(s) (max or min) and the corresponding value(s) of $f(x,y)=\frac{1}{x}+xy+\frac{1}{y}$. (5)

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A

Quiz - 1 17.10.2012

Time: 20 Minutes | Max Marks: 24 | Weightage: 8%

Name: | ID: | Section:

Answer all the questions:

1. Find the area of the region shared by $r = 1$ and $r = 1 + \sin\theta$. (5)

2. Find the polar coordinates of the foci, centre and vertices of the ellipse $r = \dfrac{12}{2 - \cos\theta}$ (5)

3. Plot the graph of $|r| > 2, \quad 0 \le \theta \le \frac{\pi}{2}$ (4)

4. For $f(x) = \sqrt{19-x}, \; x_0 = 10, L = 3, \varepsilon = 1$, find a value for δ such that, for all x satisfying $0 < |x - x_0| < \delta,$ the inequality $|f(x) - L| < \varepsilon$ is valid. (5)

5. Find $\lim\limits_{x \to 1^-} \left(\frac{1}{x}\right)\left(\frac{x+6}{x}\right)\left(\frac{3-x}{7}\right)$ (5)