

BITS, PILANI – DUBAI CAMPUS
DUBAI INTERNATIONAL ACADEMIC CITY
I SEMESTER – I YEAR , 2011-12

COMPREHENSIVE EXAMINATION (CB)

Course: Probability and Statistics

Course No.: MATH F113/AAOC C111

Total Marks: 120

Weightage: 40%

Date: 11-01-2012

Time: 3 hours

Instruction:

1. Write answers of Part A and Part B in separate answer books.
2. Necessary table values are given, so statistical table is not required.
3. Non-programmable calculator is allowed.
4. Attempt all the questions.

Table Values (in standard notation):

$$Z_{0.05} = 1.645, Z_{0.01} = 2.328, Z_{0.025} = 1.96$$

PART – A

1. A continuous random variable X is uniformly distributed between 20 and 60.
 - a. Write down the expression for probability density function of X .
 - b. What are the mean and variance of the distribution?
 - c. Find $P(X > 50)$.
 - d. Find $P(30 \leq X < 70)$.
 - e. Find $P(X = 45)$. [12]
2. An engineer is studying early morning traffic patterns at a particular intersection. The observation period begins at 5.30a.m. Let X denote the time of arrival of the first vehicle from the north-south direction; Let Y denote the first arrival time from the east-west direction. Time is measured in fraction of an hour after 5.30 a.m. Assume that the joint density for (X, Y) is given by
$$f(x, y) = k/x, 0 < y < x < 1.$$
 - a. Find the value of k .
 - b. Find $P(X \leq 0.5 \text{ and } Y \leq 0.25)$.
 - c. Find the marginal densities for X and Y .
 - d. Find $P(X < 0.5)$.
 - e. Are X and Y independent? Explain. [12]
3. Calculate the correlation coefficient r between X and Y for the following data:

x	20	30	40	50	60	70	80	90
y	5.5	6.2	9.1	12.5	14.9	19.2	22.8	26.5

Write down the formula for r and then calculate its value. Are X and Y independent? Justify your answer. [12]

4. The following table shows the number of weeks worked by 8 persons at an automobile inspection station and the number of cars each one inspected between noon and 2 p.m. on a given day.

No. of weeks employed (x):	2	7	8	1	5	12	10	14
No. of cars inspected (y):	13	21	23	14	15	21	14	32

- Find the regression line of y on x .
 - Estimate how many cars someone who has been working at the inspection station for 9 weeks can be expected to inspect during the given 2-hour period. [12]
5. Consider the following data:
20, 25, 32, 30, 20, 24, 25, 26, 28, 20.
- Find a point estimate of the population mean μ .
 - Find a 90% confidence interval on μ . The population standard deviation is known to be $\sigma = 4$.
 - Find the minimum sample size required to get μ to within 0.5 units with 90% confidence. Given that $\sigma = 4$. [12]

PART – B

6. At a nuclear plant tests are performed to check corrosion inside the cooling pipes. The test has the probability 0.7 of detecting the corrosion when it is present but it has the probability 0.2 of falsely indicating internal corrosion. Suppose the probability that any section of pipe has internal corrosion is 0.1.
- Find the probability that a section of pipe has internal corrosion, given that test indicate presence.
 - Find the probability that a section of pipe has internal corrosion, given that test indicates absence of corrosion. [12]
7. A committee of 4 members is to be appointed from 3 officers of the production department, 4 officers from purchase department, 2 officers from sales department and 1 chartered accountant.
- Find the number of ways in which committee can be formed.
 - Find the number of ways in which committee can be formed with one member from each category.
 - Find the number of ways in which committee can be formed so that the chartered accountant is in the committee.
 - Find the probability that the chartered accountant is in the committee. [12]

8. The probability that a student passes a written test in an attempt is 0.7. Assume that all the attempts are independent. Let X denotes the number of attempts required by a student to pass the test.
- Write down the density function of X .
 - Find the probability that he will pass in the 3rd attempt.
 - Find the probability that he will pass in less than 4 attempts.
 - Find the mean of the number of attempts required to pass the test. [12]
9. The average number of accidents on a certain section of a highway is 1.5 per week. Assume that accidents take place in Poisson process.
- Find the probability that there will be no accident on this section during a 2-week period.
 - Find the probability that there will be at most three accidents on this section during a 2-week period.
 - Find the probability that there will be at least 4 accidents on this section during a 2-week period. [12]

10. Consider the null hypothesis

$$H_0: p = 0.05 \text{ against the alternative } H_1: p < 0.05.$$

Here p denotes the proportion of defective items produced by a machine. A sample of size 150 showed 6 defective items.

- Write down the test statistic for the above test.
- What is the critical region at $\alpha = 0.01$ level?
- Can we reject H_0 at 0.01 level of significance? Justify. [12]

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TEST– II(OB)

COURSE: Probability and Statistics
Max. Marks: 60 Weightage: 20%

COURSE NO.: AAOC C111/MATH F113
Date: 18-12-2011 Time: 50 minutes

Answer all the questions.

1. The lifetime of a particular type of inverter follows normal distribution with mean life 10.5 years and standard deviation 2 years.
 - a) What is the probability that a randomly selected inverter will fail before 8 years?
 - b) What percentage of inverter will fail between 9 years and 12 years?
 - c) The manufacturer wants to set an warranty period for the inverters. If any inverter fails during warranty period, manufacturer will repair it free of charge. If manufacturer wants to repair only 3% of the inverters during warranty period, what should be the length of the warranty period? [15]

2. Simulate two values of uniform distribution with parameters $\alpha = 4, \beta = 12$. Take the random numbers 0.844, 0.015 for simulating the values. [6]

3. To estimate the population mean μ , a random sample is drawn and the sample values are obtained as follows: 2.5, 4.6, 3.0, 5.4, 4.4, 1.8, 5.8, 3.2, 1.9, 5.5. The population variance $\sigma^2 = 4$.
 - a) Based on the above sample find a point estimate of μ .
 - b) Find 95% confidence interval on μ .
 - c) If somebody claims that the mean is 4.0, will it be surprising? [12]

4. The joint density of two random variables X and Y is given by $f(x, y) = kxy^2, 0 \leq x \leq 1, 0 \leq y \leq 1$.
 - a) Find the value of k .
 - b) Find marginal densities of X and Y .
 - c) Are X and Y independent?
 - d) Find $P(0 \leq x \leq 0.5)$. [15]

5. Each of the 6 tubes of a radio set has a life length (in years) which may be considered as random variable that follows a Weibull distribution with parameter $\alpha = 25$ and $\beta = 2$. If these tubes function independently of one another,
 - a) What is the probability that no tube will have to be replaced during the first 2 months of service?
 - b) Find the mean and variance of the distribution. [12]

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TEST– I(CB)

COURSE: Probability and Statistics
Max. Marks: 75 Weightage: 25%

COURSE NO.: AAOC C111/MATH F113
Date: 23-10-2011 Time: 50 minutes

Answer all the questions.

1. John takes two different routes A and B to come to his office. If he takes route A, then the probability that he will reach office no later than 7:30 a.m. is 0.8. If he takes route B, then the probability that he will reach office after 7:30 a.m. is 0.3. There is 40% chance that he will take route A.
 - a) Find the probability that on a particular day he will reach office no later than 7:30 a.m.
 - b) If he reaches office after 7:30 a.m. today, find the probability that he took route B.
 - c) Find the probability that he will take route A and reach office after 7:30 a.m. [15]

2. The daily amount of coffee in litres dispensed by a machine at a plaza is uniformly distributed with mean 6 litres and standard deviation $\frac{2}{\sqrt{3}}$ litre.
 - a) Write the density function of the random variable representing the daily amount dispensed.
 - b) What is the moment generating function of this random variable?
 - c) Calculate the probability that this amount will be at least 6 litres but less than 10 litres. [15]

3. In a small class, each of the 4 students must submit a written report on one of 8 field trips. In how many different ways can they submit the written reports,
 - a) if no two students can choose the same field trip?
 - b) if there is no restriction on their choice? [7]

4. In a large computer network user log-ons to the system can be modeled as a Poisson process with a mean of 25 log-ons per hour. What is the probability that there is no log-on in an interval of 6 minutes. Also find the probability that there are exactly 3 log-ons during this 6-minute period. [8]

5. 27 microprocessor chips are in stock. Three have etching errors that cannot be detected by the naked eye. Five chips are selected and installed in field equipment.
 - a) Find the density for X , the number of chips selected that have etching errors.
 - b) Find $E(X)$ and $Var(X)$.
 - c) Find the probability that at least one chip with an etching error will be chosen. [15]

6. If the probability density of a random variable X is given by
$$f(x) = \begin{cases} k(1-x)^2 & \text{when } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$
 - a) Find the value of k .
 - b) Find the probability that a random variable having this probability density will take on a value greater than 0.6.
 - c) Find the mean and variance of X . [15]

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QUIZ – II(CB)

COURSE: Probability and Statistics
 Max. Marks: 21 Weightage: 7%

COURSE NO.: MATH F113/AAOC C111
 Date: 05-12-2011 Time: 20 minutes

NAME:

ID. NO.:

SEC:

Attempt all the questions. No marks will be awarded for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks. No extra sheet will be given for rough works. Non-programmable Calculator is permitted.

Fill in the blanks with correct answers:

- The moment generating function of a binomial distribution with $n = 16$ and $p = 0.2$ is _____.
- The density table for a discrete random variable X is given below:

x	1	3	4	6
$f(x)$	0.1	0.2	0.4	--

 The missing value in the table is _____ and $E(X) =$ _____.
- If mean of a geometric distribution is 4, its variance is _____.
- The lower limit of the probability $P(|X - \mu| < 4)$ obtained from Chebyshev's theorem is _____ where μ is the mean of X . Given that the standard deviation of X is 1.

Tick(√) the correct answer:

- The mean of a Weibull distribution with parameters $\alpha = 2$ and $\beta = \frac{1}{3}$ is

a) 0.75 b) 1 c) 1.5 d) None of these
- The normal approximation to the binomial $(n, 0.6)$ distribution gives a very good result if n is greater than

a) 17 b) 13 c) 25 d) None of these
- A gamma distribution with parameters α and β will be reduced to the following distribution if the value of α is 1:

a) exponential b) Weibull c) Normal d) None of these

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Attempt all the questions. No marks will be awarded for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks. No extra sheet will be given for rough works. Non-programmable Calculator is permitted.

Fill in the blanks with correct answers:

- The moment generating function of a binomial distribution with $n = 20$ and $p = 0.4$ is _____.
- The density table for a discrete random variable X is given below:

x	1	3	4	6
$f(x)$	0.1	0.3	0.2	--

 The missing value in the table is _____ and $E(X) =$ _____.
- If mean of a geometric distribution is 3, its variance is _____.
- The lower limit of the probability $P(|X - \mu| < 2)$ obtained from Chebyshev's theorem is _____ where μ is the mean of X . Given that the standard deviation of X is 1.

Tick(√) the correct answer:

- The mean of a Weibull distribution with parameters $\alpha = 2$ and $\beta = \frac{1}{4}$ is

a) 2.5 b) 1 c) 1.5 d) None of these
- The normal approximation to the binomial $(n, 0.7)$ distribution gives a very good result if n is greater than

a) 17 b) 13 c) 25 d) None of these
- A gamma distribution with parameters α and β will be reduced to the following distribution if the value of α is 1:

a) Weibull b) exponential c) Normal d) None of these

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Fill in the blanks with correct answers:

- The moment generating function of a binomial distribution with $n = 15$ and $p = 0.2$ is _____.
- The density table for a discrete random variable X is given below:

x	1	3	4	6
$f(x)$	0.2	0.2	0.4	--

 The missing value in the table is _____ and $E(X) =$ _____.
- If mean of a geometric distribution is 2, its variance is _____.
- The lower limit of the probability $P(|X - \mu| < 3)$ obtained from Chebyshev's theorem is _____ where μ is the mean of X . Given that the standard deviation of X is 1.

Tick(✓) the correct answer:

- The mean of a Weibull distribution with parameters $\alpha = 1$ and $\beta = \frac{1}{3}$ is

a) 0.75 b) 6 c) 1.5 d) None of these
- The normal approximation to the binomial $(n, 0.8)$ distribution gives a very good result if n is greater than

a) 17 b) 13 c) 25 d) None of these
- A gamma distribution with parameters α and β will be reduced to the following distribution if the value of α is 1:

a) Normal b) Weibull c) exponential d) None of these

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Fill in the blanks with correct answers:

- The moment generating function of a binomial distribution with $n = 10$ and $p = 0.2$ is _____.
- The density table for a discrete random variable X is given below:

x	1	3	4	6
$f(x)$	0.2	0.2	0.1	--

 The missing value in the table is _____ and $E(X) =$ _____.
- If mean of a geometric distribution is 6, its variance is _____.
- The lower limit of the probability $P(|X - \mu| < 5)$ obtained from Chebyshev's theorem is _____ where μ is the mean of X . Given that the standard deviation of X is 1.

Tick(✓) the correct answer:

- The mean of a Weibull distribution with parameters $\alpha = 1$ and $\beta = \frac{1}{4}$ is

a) 0.75 b) 6 c) 1.5 d) None of these
- The normal approximation to the binomial $(n, 0.9)$ distribution gives a very good result if n is greater than

a) 17 b) 13 c) 25 d) None of these
- A gamma distribution with parameters α and β will be reduced to the following distribution if the value of α is 1:

a) Normal b) Weibull c) exponential d) None of these

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QUIZ – I(CB)

COURSE: Probability and Statistics
 Max. Marks: 24 Weightage: 8%

COURSE NO.: AAOC C111/MATH F113
 Date: 26-9-2011 Duration: 2:10 pm to 2:30 pm

NAME:

ID. NO.:

SEC:

Attempt all the questions. No marks will be awarded for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks. No extra sheet will be given for rough works. Non-programmable Calculator is permitted.

Fill in the blanks with correct answers:

1. Six blue marbles, two red marbles and five white marbles will be arranged in a row. If all the marbles of same colour are indistinguishable from each other, the number of different arrangements possible is _____.
2. There are 10 engineers, 8 programmers and 5 mathematicians. A team of 10 people will be formed taking 5 engineers, 3 programmers and 2 mathematicians. The number of ways the team can be formed is _____.
3. Let $P(A) = 0.2$, $P(B) = 0.12$. If A and B are mutually exclusive, then the probability of neither A nor B is _____.
4. If the probability that a communication system will have high fidelity is 0.71 and the probability that it will have high fidelity and selectivity is 0.17. The probability that a system will have high selectivity, given that it has high fidelity is _____.
5. Let $P(A) = 0.4$, $P(A \cap B) = 0.12$. If A and B are independent, then $P(A \cup B) =$ _____.

Tick(√) the correct option:

6. A keyboard has 10 switches. Each switch can be set at 2 positions, ON or OFF. In how many ways these keys can be set
 a) 1024 b) 20 c) 100 d) none of these.
7. The probability of getting a sum 11 in throwing two dice is
 a) $\frac{1}{9}$ b) $\frac{1}{36}$ c) $\frac{1}{18}$ d) none of these
8. Let $P(A \cap B) = 0.25$ and $P(A) = 0.4$. if A and B are independent, then $P(A' \cap B)$ is
 a) 0.625 b) 0.375 c) 0.125 d) none of these

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 Date: 26-9-2011 Duration: 2:10 pm to 2:30 pm

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SEC:

Attempt all the questions. No marks will be awarded for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks. No extra sheet will be given for rough works. Non-programmable Calculator is permitted.

Fill in the blanks with correct answers:

1. Six blue marbles, four red marbles and five white marbles will be arranged in a row. If all the marbles of same colour are indistinguishable from each other, the number of different arrangements possible is _____.
2. There are 8 engineers, 10 programmers and 5 mathematicians. A team of 10 people will be formed taking 5 engineers, 3 programmers and 2 mathematicians. The number of ways the team can be formed is _____.
3. Let $P(A) = 0.22$, $P(B) = 0.12$. If A and B are mutually exclusive, then the probability of neither A nor B is _____.
4. If the probability that a communication system will have high fidelity is 0.56 and the probability that it will have high fidelity and selectivity is 0.17. The probability that a system will have high selectivity, given that it has high fidelity is _____.
5. Let $P(A) = 0.6$, $P(A \cap B) = 0.12$. If A and B are independent, then $P(A \cup B) =$ _____.

Tick(✓) the correct option:

6. A keyboard has 8 switches. Each switch can be set at 2 positions, ON or OFF. In how many ways these keys can be set
 a) 1024 b) 256 c) 100 d) none of these.
7. The probability of getting a sum 11 in throwing two dice is
 a) $\frac{1}{18}$ b) $\frac{1}{36}$ c) $\frac{1}{9}$ d) none of these
8. Let $P(A \cap B) = 0.25$ and $P(A) = 0.4$. if A and B are independent, then $P(A' \cap B)$ is
 a) 0.625 b) 0.125 c) 0.375 d) none of these

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 Date: 26-9-2011 Duration: 2:10 pm to 2:30 pm

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Attempt all the questions. No marks will be awarded for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks. No extra sheet will be given for rough works. Non-programmable Calculator is permitted.

Fill in the blanks with correct answers:

1. Six blue marbles, four red marbles and four white marbles will be arranged in a row. If all the marbles of same colour are indistinguishable from each other, the number of different arrangements possible is _____.
2. There are 8 engineers, 10 programmers and 5 mathematicians. A team of 9 people will be formed taking 4 engineers, 3 programmers and 2 mathematicians. The number of ways the team can be formed is _____.
3. Let $P(A) = 0.22$, $P(B) = 0.32$. If A and B are mutually exclusive, then the probability of neither A nor B is _____.
4. If the probability that a communication system will have high fidelity is 0.56 and the probability that it will have high fidelity and selectivity is 0.25. The probability that a system will have high selectivity, given that it has high fidelity is _____.
5. Let $P(A) = 0.6$, $P(A \cap B) = 0.24$. If A and B are independent, then $P(A \cup B) =$ _____.

Tick(√) the correct option:

6. A keyboard has 6 switches. Each switch can be set at 2 positions, ON or OFF. In how many ways these keys can be set
 a) 1024 b) 256 c) 100 d) none of these.
7. The probability of getting a sum 11 in throwing two dice is
 a) $\frac{1}{36}$ b) $\frac{1}{18}$ c) $\frac{1}{9}$ d) none of these
8. Let $P(A \cap B) = 0.25$ and $P(A) = 0.4$. if A and B are independent, then $P(A' \cap B)$ is
 a) 0.375 b) 0.125 c) 0.675 d) none of these

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Attempt all the questions. No marks will be awarded for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks. No extra sheet will be given for rough works. Non-programmable Calculator is permitted.

Fill in the blanks with correct answers:

1. Six blue marbles, four red marbles and three white marbles will be arranged in a row. If all the marbles of same colour are indistinguishable from each other, the number of different arrangements possible is _____.
2. There are 8 engineers, 8 programmers and 5 mathematicians. A team of 9 people will be formed taking 4 engineers, 3 programmers and 2 mathematicians. The number of ways the team can be formed is _____.
3. Let $P(A) = 0.22$, $P(B) = 0.52$. If A and B are mutually exclusive, then the probability of neither A nor B is _____.
4. If the probability that a communication system will have high fidelity is 0.56 and the probability that it will have high fidelity and selectivity is 0.4. The probability that a system will have high selectivity, given that it has high fidelity is _____.
5. Let $P(A) = 0.8$, $P(A \cap B) = 0.24$. If A and B are independent, then $P(A \cup B) =$ _____.

Tick(✓) the correct option:

6. A keyboard has 9 switches. Each switch can be set at 2 positions, ON or OFF. In how many ways these keys can be set
 a) 18255 b) 27 c) 19683 d) none of these.
7. The probability of getting a sum 11 in throwing two dice is
 a) $\frac{1}{36}$ b) $\frac{1}{11}$ c) $\frac{1}{9}$ d) none of these
8. Let $P(A \cap B) = 0.2$ and $P(A) = 0.4$. if A and B are independent, then $P(A' \cap B)$ is
 a) 0.375 b) 0.125 c) 0.3 d) none of these