BITS, PILANI - DUBAI CAMPUS

INTERNATIONAL ACADEMIC CITY, DUBAI
FIRST YEAR – I SEMESTER (2011-12)

MATHEMATICS-I (MATH C191 / MATH F111)

COMPREHENSIVE EXAMINATION (CLOSED BOOK)

Date: 02.01.2012 | Max. Marks: 120
Time: 3 hours | Weightage: 40 %

Answer Part A, Part B and Part C in separate Answer Books.
Answer all the questions.

PART A

1. Sketch the ellipse $r = \dfrac{4}{2 - \sin\theta}$ by labeling the vertices, foci and the directrix that corresponds to the focus at the origin. (6)

2. Find the area of the region inside the outer loop and outside the inner loop of the limacon $r = 1 + 2\cos\theta$. (8)

3. (a) Graph the following function:

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$

Is $f(x)$ continuous at $x = -1, 2$? Give reasons. Also find $\lim\limits_{x \to 1} f(x)$ (6)

(b) Find $\lim\limits_{h \to 0^+} \dfrac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$ (4)

4. Find the velocity and acceleration at the given value of t and plot them on the path (cycloid) $\vec{r}(t) = (t - \sin t)\vec{i} + (1 - \cos t)\vec{j}, \quad t = \dfrac{3\pi}{2}$ (8)

5. Without finding $\hat{T}$ and $\hat{N}$, find $\vec{a} = a_T\hat{T} + a_N\hat{N}$ for the curve $\vec{r}(t) = t\cos t\,\hat{i} + t\sin t\,\hat{j} + t^2\hat{k}$ at $t = 0$ (8)

PART B

1. (a) Sketch the curve $x^2 - y = 1$ together with ∇f at $P_0\ (\sqrt{2}, 1)$ and the tangent line at the given point. Also find the equation of the tangent line. (5)

(b) Find the parametric equation for the line tangent to the curve of intersection of the surfaces $xyz = 1, \quad x^2 + 2y^2 + 3z^2 = 6$ at the point $(1, 1, 1)$. (5)

2. The plane $x+y+z=1$ cuts the cylinder $x^2+y^2=1$ in an ellipse. Find the point on the ellipse that lies closest to the origin. (8)

3. Sketch the region of integration, reverse the order and evaluate the integral (7)

$$\int_0^3 \int_{x^2}^9 x\cos(y^2)\,dy\,dx$$

4. Find the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane $\frac{x}{4}+\frac{y}{2}+z=1$. (7)

5. Using the transformation $x=\frac{u}{3}-\frac{v}{3}$, $y=\frac{2u}{3}+\frac{v}{3}$, evaluate the integral (8)

$$\int_0^1 \int_0^{1-x} \sqrt{x+y}\,(y-2x)^2\,dy\,dx$$

PART C

1. Find the flow along the given curve in the increasing direction of t, given the velocity field of a fluid $\vec{F}=-4xy\,\hat{i}+8y\,\hat{j}+2\,\hat{k}$ and $\vec{r}(t)=t\,\hat{i}+t^2\,\hat{j}+\hat{k},\quad 0\le t\le 2$. (8)

2. Using Green's theorem, find the counterclockwise circulation for the field $\vec{F}=2xy^3\,\hat{i}+4x^2y^2\,\hat{j}$ over the region enclosed by the x-axis, $x=1$ and the curve $y=x^3$. (8)

3. Using the divergence theorem, calculate the outward flux of $\vec{F}=x^2\,\hat{i}+xz\,\hat{j}+3z\,\hat{k}$ across the boundary of the region D: the cube bounded by $x=1,\ y=1,\ z=1,\ x=0,\ y=0,\ z=0$ (8)

4. Find the Taylor's polynomial up to degree 2 for $f(x)=x^2\sin x$ about $x=\frac{\pi}{4}$. (6)

5. Test the convergence for the following series:

   (a) $\sum_2^{\infty}(-1)^{n+1}\frac{1}{n\ln n}$ (by checking the absolute convergence) (5)

   (b) $\sum_1^{\infty}\frac{(3n)!}{n!(n+1)!(n+2)!}$ (5)

ALL THE BEST!

BITS PILANI, DUBAI CAMPUS
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First Year – Semester I (2011-12)

MATHEMATICS-I (MATH F111 & MATH C191)

TEST – 2 (Open Book)

Date: 20.11.2011
Time: 50 minutes

Max. Marks: 60
Weightage: 20%

Answer all the questions

1. If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ (10)

2. (a) Find $\frac{dy}{dx}$ if $y^x + x^y = 0$ using the chain rule. (5)

   (b) If $w = \ln(x^2 + y^2 + z^2)$, check the Laplace equation for w. (5)

3. An object starts from rest at the point (0, 3, 0) and moves with an acceleration $\vec{a}(t) = 4t\,\hat{i} + 3\cos t\,\hat{j} + 3\sin t\,\hat{k}$. Find the location of the object after t = 2 seconds. (10)

4. Determine the tangential and normal components of the acceleration if
   $\vec{r}(t) = \left(\frac{1}{2}t^2 + 1\right)\hat{i} + (t^2 + t - 2)\,\hat{j} + (t^3 - t + 3)\hat{k}$ (10)

5. Find the unit normal vector and the curvature of the curve
   $\vec{r}(t) = (12t)\,\hat{i} + (8t^{3/2})\,\hat{j} + (3t^2)\hat{k}$ (10)

6. If $w = 2e^{3x} + 4y^3 - 3xz, \quad x = \ln t, \quad y = s^2 + 2t \text{ and } z = \ln t + 2s$, find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$ at $(1, 1)$ using only the Chain rule. (10)

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First Year – Semester I (2011-12)

MATHEMATICS-I (MATH F111)

TEST – I (Closed Book)

Date: 02.10.2011 **Max. Marks: 75**
Time: 50 minutes **Weightage: 25%**

Answer all the questions

1. (a) Sketch the curve $r = 2\cos 3\theta$. (7)

   (b) Sketch: $-\frac{3\pi}{4} \le \theta \le -\frac{2\pi}{3}, \quad |r| \ge 2$ (6)

2. (a) Convert to Cartesian form: $r\cos\left(\theta - \frac{\pi}{3}\right) = 4$ (3)

   (b) Convert to polar form: $(x-3)^2 + (y+1)^2 = 4$ (3)

3. Find the area inside the inner loop of the curve $r = 2 + 4\cos\theta$. (12)

4. Find the area of the region shared by the curves $r = 1$ and $r = 2(1 - \sin\theta)$. (12)

5. Find the area inside the cardioid $r = 2 + 2\sin\theta$ and outside the circle $r = 2\sin\theta$. (12)

6. Find the length of the curve $r = \sqrt{1 + \cos 2\theta}$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ (10)

7. Find the length of the curve $r = 2\sin\theta + 2\cos\theta$, $0 \le \theta \le \frac{\pi}{2}$ (10)

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BITS, Pilani - Dubai Campus
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First year – I Semester 2011 – 2012
Mathematics I (MATH F111/MATH C191)

A

Quiz - 2

08.12.2011

Time: 20 Minutes | Max Marks: 21 | Weightage: 7%

Name: | ID: | Section:

Answer all the questions:

1. Sketch the region and reverse the order of integration: (5)

$$\int_0^{\frac{1}{\sqrt{2}}} \int_x^{\sqrt{1-x^2}} f(x,y)\, dy\, dx$$

2. Maximize the area of a room subject to the condition that the sum of the lengths of any two adjacent walls is 10 meter. (5)

3. Find all the local maxima and local minima and saddle point of the function:

$$f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$$ (5)

4. Find the directions in which $f(x,y)=\cos x\cos y$ increases and decreases most rapidly at $P_0\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ and find the derivative in each direction. Also find the directional derivative of $f(x,y)$ at P_0 in the direction of $\vec{v}=3\hat{i}+4\hat{j}$. (6)

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First year – I Semester 2011 – 2012
Mathematics I (MATH F111/MATH C191)

B

	Quiz - 2	08.12.2011
Time: 20 Minutes	Max Marks: 21	Weightage: 7%
Name:	ID:	Section:

Answer all the questions:

1. Sketch the region and reverse the order of integration: (5)

$$\int_0^{\frac{1}{\sqrt{2}}} \int_y^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$$

2. Minimize the perimeter of a rectangular room such that the area is 16 sq meter. (5)

3. Find all the local maxima and local minima and saddle point of the function:

$$f(x,y) = x^2 - 2xy + 2y^2 - 2x + 2y + 1$$

(5)

4. Find the directions in which $f(x, y) = \sin x \sin y$ increases and decreases most rapidly at $P_0\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ and find the derivative in each direction. Also find the directional derivative of $f(x, y)$ at P_0 in the direction of $\vec{v} = 3\hat{i} + 4\hat{j}$. (6)

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FIRST YEAR – I SEMESTER 2011 – 2012

MATHEMATICS 1 (MATH F111)

A

QUIZ 1

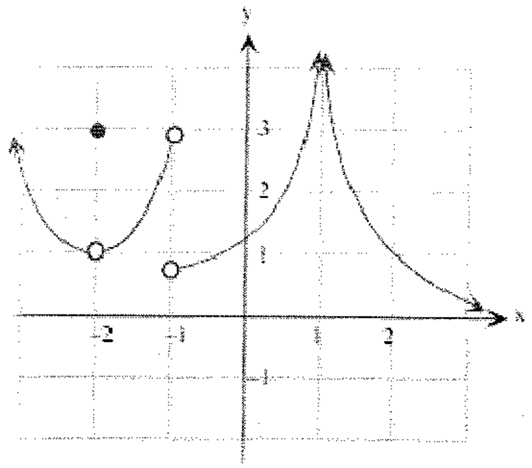
17.10.2011

DURATION: 20 MINUTES	MAXIMUM: 24 MARKS
NAME: ______________________________	ID: ______________________

Answer all the questions:

1. Find $\lim\limits_{x \to 2^+} \dfrac{|x-2|(x^2+2)}{x^2-4}$ (4)

2. Given the following graph,



a) Find $\lim\limits_{x \to -2} f(x)$

b) Find $\lim\limits_{x \to -1^+} f(x)$

c) Is f(x) continuous at $x = -1$ & $x = -2$? If not give reason? (5)

3. If $f(x)=\sqrt{1-5x}, \quad x_0=-3, \quad \varepsilon=0.5$ Find $L=\lim\limits_{x\to x_0} f(x)$. Then find a number $\delta>0$ such that for all x : $0<|x-x_0|<\delta \Rightarrow \quad |f(x)-L|<\varepsilon$ (5)

4. Sketch the velocity and acceleration on the path of the particle with $\vec{r}(t)=t^2\ \hat{i}+2t\ \hat{j}, \quad t=0,1$ (5)

5. Find the centre and foci of $r = \dfrac{6}{2+\cos\theta}$ (5)

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FIRST YEAR – I SEMESTER 2011 – 2012

MATHEMATICS 1 (MATH F111)

QUIZ 1

B

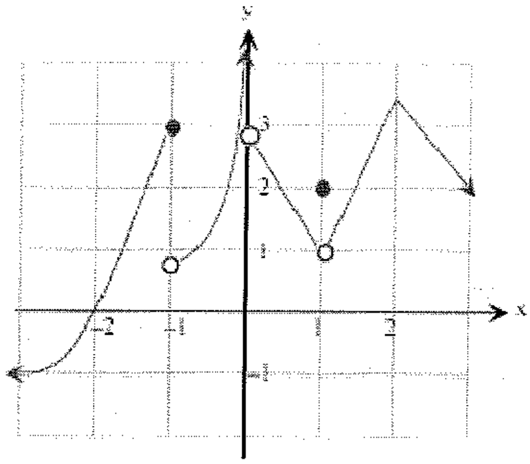
17.10.2011

DURATION: 20 MINUTES	MAXIMUM: 24 MARKS
NAME: ______________________________	ID: ____________________

Answer all the questions:

1. Find $\displaystyle \lim_{x \to 2^-} \frac{|x-2|(x^2+2)}{x^2-4}$ (4)

2. Given the following graph,



   a) Find $\displaystyle \lim_{x \to -1} f(x)$ =

   b) Find $\displaystyle \lim_{x \to 0^+} f(x)$

   c) Is f(x) continuous at $x = -1 \;\&\; x = 1$? If not give reason? (5)

3. If $f(x)=\sqrt{1-3x}, \quad x_0=-5, \quad \varepsilon=0.5$ Find $L=\lim_{x\to x_0} f(x)$ Then find a number $\delta>0$ such that for all x : $0<|x-x_0|<\delta \Rightarrow \quad |f(x)-L|<\varepsilon$ (5)

4. Sketch the velocity and acceleration on the path of the particle with

$$\vec{r}(t)=2t\,\hat{i}+t^2\,\hat{j}, \quad t=0,-1$$

(5)

5. Find the centre and foci of $r = \dfrac{6}{2 - \cos\theta}$ (5)