

BITS, PILANI – DUBAI
I YEAR – I SEMESTER
2009-2010

COMPREHENSIVE EXAMINATION (CB)

COURSE: Probability and Statistics

COURSE NO.: AAOC C111

Max. Marks: 120

Weightage: 40%

Date: 30-12-2009

Time: 3 hours

Instruction:

- Answer SECTION-A and SECTION-B in separate answer books.
 - Statistical tables are not allowed. Necessary table values are provided.
 - Non-programmable scientific calculator is allowed.
 - All questions are compulsory.
 - Answer all parts a), b), ... etc of each question together and in the given sequence.
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SECTION – A

1. A bag contains 5 balls and it is not known that how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. Calculate the probability that all the balls in the bag are white. [12]

2. a) A computer system uses passwords that consists of five letters followed by a single digit. How many passwords are possible? [4]
b) A box contains four bad and six good tubes. Two are drawn out from the box at a time. One of them is tested and found to be good. Find the probability that the other one is also good. [4]
c) A lot contains 10 good articles, 4 with minor defects and 2 with major defects. Two articles are chosen from the lot at random without replacement. Find the probability that at most one is good. Find also the probability that at least one is good. [4]

3. For 10 randomly selected observations, the following data were recorded:

<i>Overtime hours (x)</i>	:	1	1	2	2	3	3	4	5	6	7
<i>Additional Units (y)</i>	:	2	7	7	10	8	12	10	14	11	14

Fit a 2nd degree polynomial in the form $y = b_0 + b_1x + b_2x^2$. Write the normal equations and then use these equations to find b_0 , b_1 and b_2 . [12]

4. Let p denotes the true proportion of cracked ceramic pistons made for an experimental diesel engine. A random sample of 150 ceramic pistons contains 27 cracked pistons. Given $z_{0.025} = 1.96$.
- Find a point estimate of p .
 - Construct a 95% confidence interval on p .
 - Will it be surprising if someone reports that the true proportion is 0.20?
- [12]

5. Calculate the correlation coefficient for the following heights of fathers(x) and their sons (y):

x :	65	66	67	67	68	69	70	72
y :	67	68	65	68	72	72	69	71

Write down the computational formula and then calculate. [12]

SECTION - B

6. The following function defines a probability density function of a continuous random variable X :

$$f(x) = \begin{cases} k(1-x^2) & \text{for } 0 \leq x \leq 1 \\ 0 & \text{Otherwise} \end{cases}$$

- Find the value of k .
 - Derive the complete expression of the cumulative distribution function $F(x)$.
 - Find $P(\frac{1}{4} < X < \frac{3}{4})$. [12]
7. The following table defines the joint distribution of two discrete random variables X and Y :

$Y \rightarrow$	0	1	2	3
$X \downarrow$	0	1/42	1/21	1/14
1	1/21	1/14	2/21	5/42
2	2/21	5/42	1/7	p

- Find the value of p .
 - Find $Cov(X, Y)$.
 - Are X and Y independent? Justify. [12]
8. Let μ denotes the average time in minutes that workers at a certain large plant take to get to work. Assume the normality of the population and consider the null

hypothesis $\mu = 32.6$ against two sided alternative. To test this hypothesis a random sample of 20 workers is taken from that plant which showed an average 33.4 minutes to get to work and a sample standard deviation 4.2 minutes. *Given* $t_{19,0.05} = 2.861$.

- a) Write H_0 and H_1 in standard format.
- b) What is the suitable test statistic for testing this hypothesis?
- c) What is the critical region at 0.1 level of significance?
- d) Find the observed value of the test statistic based on the drawn sample.
- e) Can we reject H_0 at $\alpha = 0.1$ level? Justify.
- f) If an error is committed, what type of error will it be? [12]

9. X and Y are two independent uniform variates. X is defined on $[2,6]$ and Y is defined on $[8,10]$.

- a) Find the expectation $E[2X - 3Y]$.
- b) Find the variance $Var[2X - 3Y]$.
- c) Find $E(Y^2)$. [12]

10. Simulate a value of

- a) Weibull variate with $\alpha = 0.1$ and $\beta = 0.5$. Take the random number 0.424.
- b) Simulate a value of the discrete random variable X whose distribution is defined by

x	1	2	3	4	5
$f(x)$	0.125	0.225	0.425	0.152	0.073

Use the random number 812 to simulate it. [12]

BITS, PILANI – DUBAI
DUBAI INTERNATIONAL ACADEMIC CITY
I Year – I Semester 2009-2010

TEST – II (OB)

COURSE: Probability and Statistics
Max. Marks: 60 Weightage: 20%

COURSE NO.: AAOC C111
Date: 20-12-2009 Time: 50 min.

Instruction:

Only prescribed Text Book and handwritten class notes are allowed.

Attempt all the questions.

1. A study shows that 16 of 200 tractors produced on one assembly line required extensive adjustments before they could be shipped, while the same was true for 14 of 400 tractors produced on another assembly line. At 0.01 level of significance does this support the claim that second assembly line does superior work? Define appropriate null and alternative hypotheses and then test it. (Given $z_{0.01} = 2.33$).
[12]

2. The following are the numbers of minutes it took 10 mechanics to assemble a piece of machinery in the morning and in the late afternoon:

Morning (x)	11.1	10.3	12.0	15.1	13.7	18.5	17.3	14.2	14.8	15.3
Late Afternoon (y)	10.9	14.2	13.8	21.5	13.2	21.1	16.4	19.3	17.4	19.0

Find the correlation coefficient r . [12]

3. Consider the null hypothesis $H_0 : \mu \geq 28000$ against the alternative $H_1 : \mu < 28000$. A sample of size 40 taken from that normal population shows a sample mean 27463. If population standard deviation is 1348, find the P-value of the test. [6]

4. The joint density function of two random variables X and Y is given by

$$f(x, y) = \begin{cases} kxy & \text{when } x > 0, y > 0, x + 2y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- a) Find the value of k .
 b) Find $P(X \leq 1)$.
 c) Find $E[Y]$. [15]

5. Two statisticians Tom and John want to find the 95% confidence interval of the population mean μ of a normal population. Given that population standard deviation is 2. Tom has drawn a random sample of size 25 from the population which gives a sample mean 12. Based on this sample he has found the confidence interval.
- a) If John wants a confidence interval whose length will be not more than $1/4^{\text{th}}$ of that of Tom, find the minimum sample size that John needs to draw.
 b) If John draws a random sample of size 36 and the length of his confidence interval remains same as Tom, approximately at what confidence level he got the confidence interval? [15]

BITS, PILANI – DUBAI
Dubai International Academic City
(I year – I semester 2009-10)
TEST – I (CB)

Course Title : Probability & Statistics

Course No. : AAOC C111

Max. Marks : 75 Weightage : 25% Date : 08-11-2009

Time : 50 min.

Table values for standard normal probability:

$F(1) = 0.8413$; $F(-1) = 0.1587$; $F(-1.28) = 0.1$

Attempt all the questions.

1. The probability of a man hitting a target is $\frac{1}{4}$. He fires seven times. Using Binomial distribution,
 - a) Find the density function of X , number of times he will hit the target. [2]
 - b) Find the mean and standard deviation of X . [4]
 - c) What is the probability of his hitting the target at least twice? [4]

2. Suppose that X is hypergeometric with $N = 20$, $r = 3$, and $n = 5$.
 - a) What are the possible values for X ? [3]
 - b) What is $E(X)$? [3]
 - c) What is $\text{Var}(X)$? [4]

3. The local authorities in a certain city install 10,000 electric lamps in the streets of the city. If these lamps have an average life of 1,000 burning hours with a standard deviation of 200 hours, assuming normality,
 - a) What number of lamps might be expected to fail
 - i) in the first 800 burning hours? [4]
 - ii) between 800 and 1200 burning hours? [4]
 - b) After what period of burning hours would you expect that 10% of the lamps would fail? [7]

4. A consulting firm rents cars from three agencies A, B and C. The probabilities that the firm will rent a car from agencies A, B, C are 0.2, 0.3, 0.5 respectively. 10% cars from A, 15% from B and 5% from C have bad tires.
 - a) What is the probability that the firm will get a car with bad tires? [7]
 - b) If the firm gets a car with bad tires, what is the probability that the car belongs to agency C? [8]

BITS, PILANI – DUBAI
I YEAR – I SEMESTER
2009-2010

QUIZ – II (CB)

COURSE: Probability and Statistics

COURSE NO.: AAOC C111

Max. Marks: 21

Weightage: 7%

Date: 19-11-2009

Time: 20 mins.

NAME:

ID. NO.:

SEC:

Attempt all the questions. No marks for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks.

Total number of questions=7

Tick the correct answer:

1. If the function $f(x) = \lambda + 2^x$, $x = 1, 2, 3, 4$ defines the density function of the distribution of a discrete random variable X , then the value of λ is
 a) -7.50 b) -7.75 c) -7.25 d) none of these
2. If the mean and variance of a binomial distribution with density function $f(x)$ are 2 and 1.8 respectively, then $f(0)$ is
 a) 0.9^{20} b) 0.1^{20} c) $(0.1)^2 \cdot (0.9)^{15}$ d) none of these
3. The joint density function of continuous random variables X and Y is defined as $f(x, y) = 4xy$, $0 \leq x \leq 1, 0 \leq y \leq 1$. The marginal density of X is
 a) $f_x(x) = 2x, 0 \leq x \leq 1$ b) $f_x(x) = 4x, 0 \leq x \leq 1$ c) $f_x(x) = x, 0 \leq x \leq 1$
 d) none of these
4. If X is an exponential variate with parameter $\beta = 2$, then $P(X < 2)$ is
 a) e^{-1} b) $1 - e^{-1}$ c) $1 - e^{-2}$ d) none of these

Fill in the blanks with correct answers:

5. A gamma distribution with parameters α and β will be reduced to exponential distribution with parameter β if the value of α is _____.

6. The following table defines the joint density of discrete random variables X and Y :

$Y \rightarrow$	1	2	3
$X \downarrow$			
1	0.15	0.25	0.12
2	0.05	0.08	0.14
3	0.10	0.01	0.10

$P(X=1 \text{ and } Y \geq 2)$ is _____.

7. If arrivals of customers in a queue follows Poisson process with mean arrival rate 4, then the distribution of time of first arrival follows exponential distribution with parameter _____.

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I YEAR – I SEMESTER
2009-2010

QUIZ – II (CB)

COURSE: Probability and Statistics

COURSE NO.: AAOC C111

Max. Marks: 21

Weightage: 7%

Date: 19-11-2009

Time: 20 mins.

NAME:

ID. NO.:

SEC:

Attempt all the questions. No marks for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks.

Total number of questions=7

Tick the correct answer:

1. If the function $f(x) = \lambda - 2^x$, $x = 1, 2, 3$ defines the density function of the distribution of a discrete random variable X , then the value of λ is
 a) 4 b) 3 c) 6 d) none of these
2. If the mean and variance of a binomial distribution with density function $f(x)$ are 3 and 2.4 respectively, then $f(0)$ is
 a) 0.8^{15} b) 0.2^{15} c) 0.1^{15} d) none of these
3. The joint density function of continuous random variables X and Y is defined as $f(x, y) = 8xy$, $0 \leq x \leq 1, 0 \leq y \leq 1$. The marginal density of X is
 a) $f_x(x) = 2x, 0 \leq x \leq 1$ b) $f_x(x) = x, 0 \leq x \leq 1$ c) $f_x(x) = 4x, 0 \leq x \leq 1$
 d) none of these
4. If X is an exponential variate with parameter $\beta = 2$, then $P(X < 4)$ is
 a) e^{-1} b) $1 - e^{-2}$ c) $1 - 2e^{-2}$ d) none of these

Fill in the blanks with correct answers:

5. A gamma distribution with parameters α and β will be reduced to _____ distribution if the value of α is 1.

6. The following table defines the joint density of discrete random variables X and Y :

$Y \rightarrow$	1	2	3
$X \downarrow$			
1	0.15	0.25	0.12
2	0.05	0.08	0.14
3	0.10	0.01	0.10

$P(X = 1 \text{ and } Y \leq 2)$ is _____.

7. If arrivals of customers in a queue follows Poisson process with mean arrival rate 8, then the distribution of time of first arrival follows exponential distribution with parameter _____.

BITS, PILANI – DUBAI
I YEAR – I SEMESTER
2009-2010

QUIZ – I (CB)

COURSE: Probability and Statistics

COURSE NO.: AAOC C111

Max. Marks: 24

Weightage: 8%

Date: 01-10-2009

Time: 30 mins.

NAME:

ID. NO.:

SEC:

Attempt all the questions. No marks for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks.

1. A project consists of three independent tasks and the probabilities of these tasks being completed on time are: 0.90, 0.80 and 0.75, respectively. Then the probability that all three tasks will be completed on time is _____.

2. A certain product was found to have two types of minor defects. The probability that an item of the product has only a type 1 defect is 0.2, and the probability that it has only a type 2 defect is 0.4. Also, the probability that it has both defects is 0.1. Then the probability that an item has at least one defect is _____.

3. If $P(A \cap B) = 0.2$, $P(B) = 0.6$, then $P(A' \cap B) =$ _____.

4. A door key has 4 toggle switches, each switch can be independently set at 3 positions 'up', 'middle' and 'down'. The total number of different ways in which the switches can be set is _____.

5. If $P(A \cap B) = 0.6$, then $P(A' \cup B') =$ _____ .
6. Let two events A and B are independent. Are A and B' independent?
_____ . (yes/no)
7. Assume that an engine component of a spacecraft consists of two engines (main and backup) connected in parallel which function independently. If the probability that the engine component will function is 0.90 and the probability that the main engine will function is 0.80, then the probability that backup engine will function is _____ .
8. A box contains 5 white, 10 black and 15 red balls. If a ball is drawn from the box at random, then the probability that the ball is not black is _____