# BITS, PILANI-DUBAI

**International Academic City, Dubai**

**First Year - Semester-I (2009- 10)**

**Mathematics-I (MATH C191)**

**Comprehensive Examination (Closed Book)**

**Date: 27.12.2009** **Max. Marks: 120**
**Time: 3 hours** **Weightage: 40 %**

**Answer all the questions.**

**Answer Part A and Part B in TWO separate Answer Books.**

## PART-A

1. Find $\delta$ algebraically $f(x)=\sqrt{19-x},\ x_0=10,\ \varepsilon=1\ and\ L=3$. (7)

2. Sketch and label the vertices, foci and center of the curve $r=\dfrac{400}{16+8\sin\theta}$. (7)

3. Find the area of the region which is shared by the circle $r=2$ and the cardioid $r=2(1-\cos\theta)$. (8)

4. Find $\hat{T},\ \hat{N}\ and\ \kappa$ for the curve $\vec{r}=(\cos t+t\sin t)\,\hat{i}+(\sin t-t\cos t)\,\hat{j}+3\,\hat{k}$. (8)

5. Find the length of the curve $\vec{r}=\left(e^t\cos t\right)\hat{i}+\left(e^t\sin t\right)\hat{j}+\left(e^t\right)\hat{k}$, $-\ln 4\le t\le 0$ (7)

6. Find the linearization of $f(x,y,z)=e^{2xy}\sin z+4x^2y$ at $\left(1,-1,\dfrac{\pi}{2}\right)$ (7)

7. Find the extreme values of $f(x,y)=4xy-x^4-y^4$. (8)

8. Find the points lying on the plane $x+2y+z=5$, closer to the origin. (8)

## PART - B

9. Use Green's theorem to evaluate the line integral

$\oint_C (xy+e^{x^2})\,dx+(x^2-\ln(1+y))\,dy$, where C consists of the line segment from $(0,0)$ to $(\pi,0)$ and the curve $y=\sin x,\ \ 0\le x\le\pi$. (8)

**(P. T. O)**

10. Use Stokes' Theorem to evaluate $\int_C F \bullet dr$, where $\vec{F} = xz\,\hat{i} + 2xy\,\hat{j} + 3xy\,\hat{k}$, and C is the boundary of the part of the plane 3 x + y + z = 3 in the first octant. (8)

11. Show that the vector field

$\vec{F} = \left(3x^2 + y\cos(xy+z)\right)\hat{i} + \left(2y + x\cos(xy+z)\right)\hat{j} + \left(\cos(xy+z) - 2\right)\hat{k}$ is a conservative force field and hence find its scalar potential. (8)

12. Evaluate the triple integral $\iiint_E xy\,dV$, where *E* is the solid tetrahedron with the coordinate planes and the plane $6x + 3y + 2z = 6$. (7)

13. Evaluate $\iint_R xy\,dA$, where R is the region bounded by the lines 2 x - y = 1, 2 x - y = -3, 3 x + y = 1 and 3 x + y = -2, using the transformation u = 3x +y, v = 2x – y. (8)

14. Evaluate the integral by reversing the order of integration for $\int_0^1 \int_{3y}^3 e^{x^2}\,dx\,dy$. (7)

15. Find the Taylor Series for $f(x) = \ln(x)$ about $x = 2$. (6)

16. Determine if the following series is convergent or divergent.

(a) $\sum_{n=1}^{\infty} \left(\frac{1}{n^2 + 4n + 3}\right)$ (b) $\sum_{n=0}^{\infty} \frac{(64)^n}{5^{n-1}}$ (4+4)

**ALL THE BEST**

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**BITS, PILANI-DUBAI**
**International Academic City, Dubai**

First Year - Semester-I (2009-10)

**MATHEMATICS-I (MATH C191) TEST – 2 (Open Book)**

**Date: 10.12.2009** | **Max. Marks: 60**
**Time: 50 minutes** | **Weightage: 20 %**

**Instructions: Only the Handwritten Class notes and the Text book are permitted.**
**Answer all the questions in sequential order.**

1. Evaluate the double integral $\iint_R (x^2 - 2xy)\, dA$ where R is the region outside the curve $y = \sqrt{x}$ and below the line $x + y = 2$ in the first quadrant. (7 marks)

2. Use a double integral to find the area of one loop of the rose $r = \cos 3\theta$ (7 marks)

3. Using the transformation $u = xy\,;\, v = \frac{y}{x}$, evaluate the integral
$$\iint_R xy\, dA,$$
where R is the region in the first quadrant bounded by the lines $y = x$ *and* $y = 3x$ and the hyperbolas $xy = 1$ *and* $xy = 3$. (8 marks)

4. Evaluate $\displaystyle\int_0^1 \int_{\sin^{-1}x}^{\frac{\pi}{2}} \cos x \sqrt{1 + \cos^2 x}\, dx\, dy$ (8 marks)

5. Evaluate the line integral $\int_C xy^4\, ds$ where C is the right half of the circle $x^2 + y^2 = 16$. (7 marks)

6. Maximize the function $yz + xy$ subject to the constraints $xy = 1$ and $y^2 + z^2 = 1$. (8 marks)

7. Find the directional derivative of $f(x, y, z) = x^2 - y^2 z + 2z^2 e^{-2y}$ at the point $P_0(4, -1, 2)$ in the direction parallel to the line joining the points (-2, 1, 4) and (2, -2, 7). (8 marks)

8. Find the extreme values of the function $f(x, y) = 3x^2 y + y^3 - 3x^2 - 3y^2 + 2$. (7 marks)

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BITS , PILANI - DUBAI

INTERNATIONAL ACADEMIC CITY, DUBAI

First Year – Semester I (2009-10)

**MATHEMATICS-I (MATH C191)**

TEST – I (Closed Book)

**Date : 18.10.2009** **Max. Marks : 75**

**Time: 50 minutes** **Weightage : 25%**

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**Answer all the questions**

1. Find an equation in polar coordinates of the curve:

   $x = e^{2t} \cos t, \quad y = e^{2t} \sin t$ and find its length from

   $t = 0 \quad to \quad t = 2\pi$ (9)

2. Sketch the curve and determine the area inside the inner loop of the curve $r = 2 + 4\cos\theta$ (9)

3. The position vector of a particle in the plane at time t is given by

   $$\vec{r}(t) = (4\cos t)\hat{i} + (\sqrt{2}\sin t)\hat{j}, \quad t = 0, \frac{\pi}{4}$$

   Graph the path of the particle and sketch the velocity and acceleration vectors at the given values of t. (10)

4. Find the $\vec{T}$, $\vec{N}$ and the curvature of the curve

   $\vec{r}(t) = (\cos^3 t)\hat{i} + (\sin^3 t)\hat{j}$ (10)

5. Write $\vec{a}$ in the form $\vec{a} = a_T\vec{T} + a_N\vec{N}$ without finding $\vec{T}$ *and* $\vec{N}$

   for the curve $\vec{r}(t) = t^2\,\hat{i} + \left(t + \frac{t^3}{3}\right)\hat{j} + \left(t - \frac{t^3}{3}\right)\hat{k}, \ t = 0$ (9)

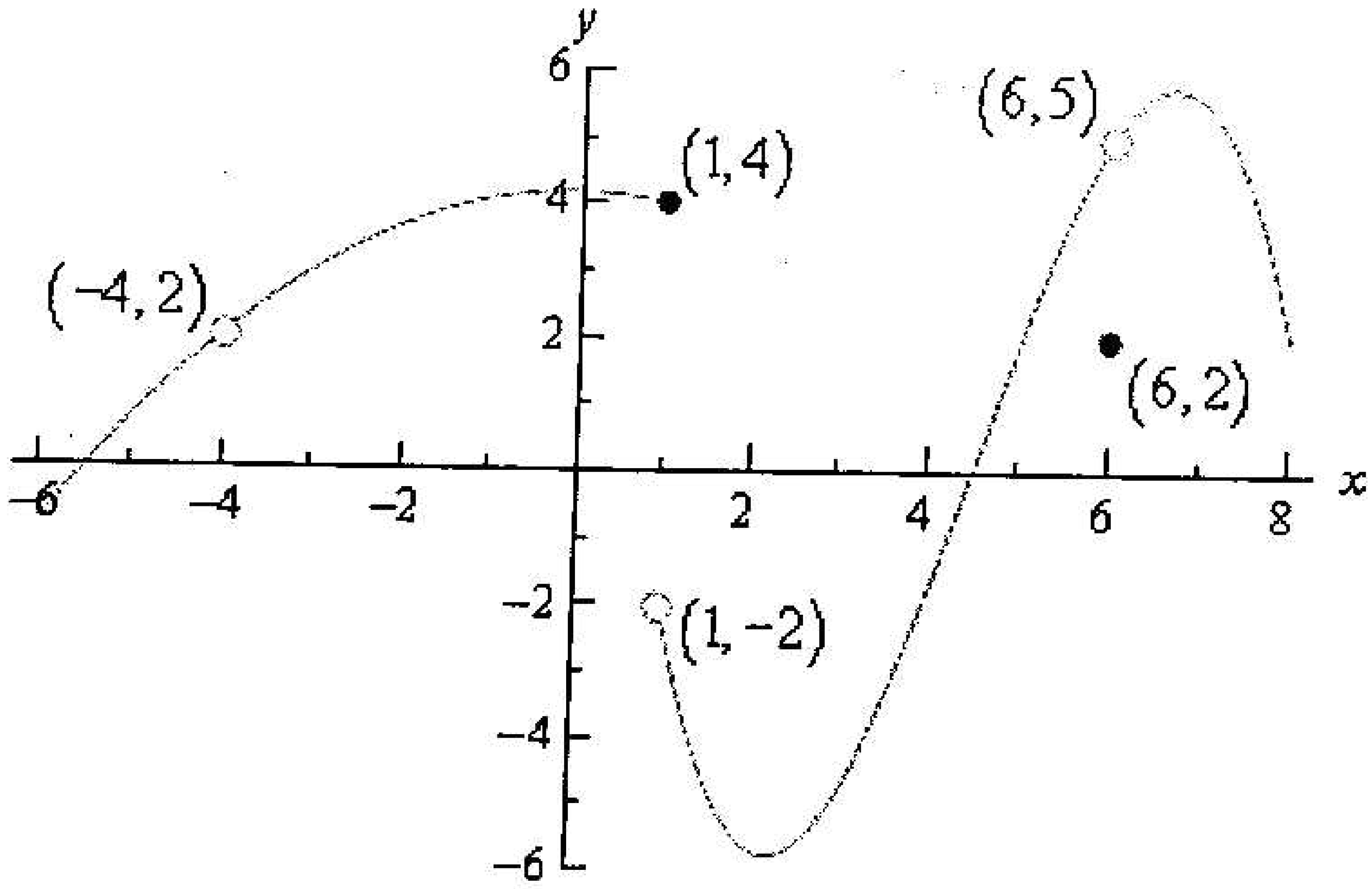
6. Solve the initial value problem for r as a vector function of t:

   $$\frac{d^2\vec{r}}{dt^2} = -32\,\hat{k}$$

   with the initial conditions: $\vec{r}(0) = 100\,\hat{k}$ and $\left(\frac{d\vec{r}}{dt}\right)_{t=0} = 8\,\hat{i} + 8\,\hat{j}$ (9)

**(P.T.O)**

7. Given the following graph,




a) Find $\lim_{x \to 1} f(x)$

b) Find $\lim_{x \to -4^-} f(x)$

c) Is f(x) continuous at x = 6 and – 4? If not give reason? (9)

8. (a) If $f(x) = \sqrt{1-5x}, \quad x_0 = -3, \quad \varepsilon = 0.5$ Find L= $\lim_{x \to x_0} f(x)$.

Then find a number $\delta > 0$ such that for all x:

$$0 < |x - x_0| < \delta \Rightarrow \quad |f(x) - L| < \varepsilon$$

(b) Evaluate the following:

(i) $\lim_{x \to -1^-} \sqrt{\frac{2x+1}{2x-1}}$

(ii) $\lim_{x \to 1^+} \frac{2x(x-1)}{|x-1|}$ (10)

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BITS, PILANI, DUBAI
MATHEMATICS-I (MATH C191)

A

QUIZ-2

**TIME: 25 Minutes** **Max. Marks: 21** **05.11.2009**

**ID No.** **Name:** **Section:**

Note: 1.Write ID No., Name, Sec.No. and **only the Answers** in the provided space.
2. Overwriting will be treated as wrong answer.

1. Find $\frac{\partial z}{\partial x}$ for the equation $z(x, y) = e^{x^2 - y + 2} \sin(x^2 + y^3)$ (4M)

2. Find $f_{xz}$ if $f(x, y, z) = x^{yz}$. (4M)

3. Use the Chain Rule to find $\frac{\partial z}{\partial t}$ for:
$z = y^2 \tan x, \quad x = t^2 uv, \quad y = u + tv^2, \quad when\, t = 2, u = 1, v = 0$ (4M)

**(P. T. O)**

4. Find $\frac{dy}{dx}$ for $2y^2 + \sqrt[3]{xy} = 3x^2 + 17$ at (1,1) (3M)

5. Find the tangent vector for $\vec{r}(t) = t\,\hat{i} + 3\sin t\,\hat{j} + 3\cos t\,\hat{k}$ (3M)

6. Find the domain and range of the function $f(x, y) = \frac{1}{|xy| + 4}$. (3M)

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MATHEMATICS-I (MATH C191)

A

QUIZ-1

**TIME: 25 Minutes** **Max. Marks: 24** **06.10.2009**

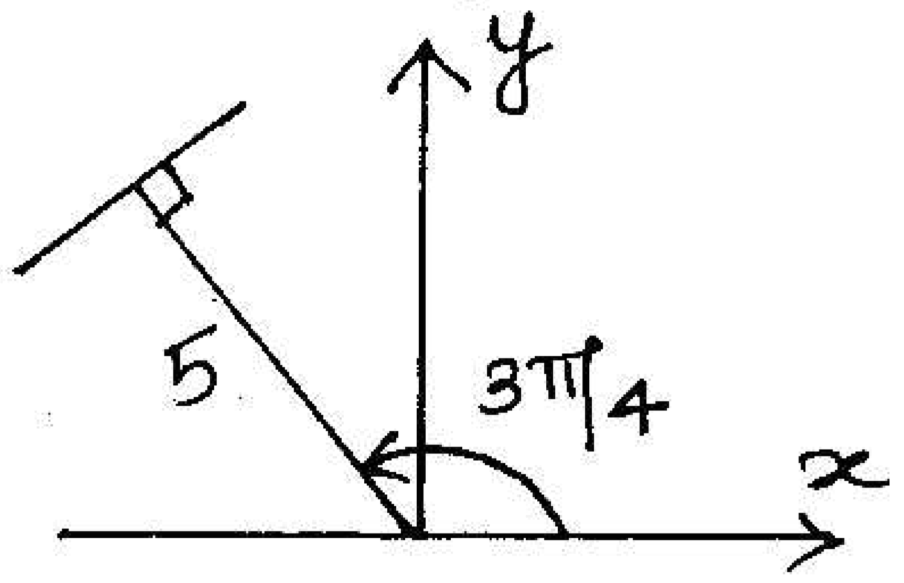
**ID No.** **Name:** **Section:**

Note: 1.Write ID No., Name, Sec.No. and Answers in the provided space.
2. Overwriting will be treated as wrong answer.

1. Plot the graph of $r = \frac{3}{2} + \cos\theta$ (4 marks)

2. Give the polar coordinates of the vertices, foci and centre given $r = \frac{12}{3 + \sin\theta}$. (4 marks)

3. Write the Polar and Cartesian equations of the straight line given below: (4 marks)



4. Find the area of the region that lies inside the circle r = 1 and outside the cardioid $r = 1 + \cos\theta$. (4 marks)

5. Find the length of the curve given by the polar equation: $r = \sqrt{1 - \cos 2\theta}$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ (4 marks)

6. Plot the graph of $r \ge 2$, $-\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$ (4 marks)