BITS, PILANI – DUBAI
INTERNATIONAL ACADEMIC CITY, DUBAI
(I YEAR – I SEMESTER 2008-2009)

TEST– I (CB)

PROBABILITY & STATISTICS
(AAOC C111)

Max. Marks: 75 Weightage: 25% Date: 02-11-2008 Time: 50 Minutes

Attempt all the questions.

1. If X has the probability density

$$f(x) = \begin{cases} k\,e^{-3x} & ; \text{for } x > 0 \\ 0 & ; \text{elsewhere} \end{cases}$$

   a) Find k. [5]
   b) $P(0.5 \le X \le 1)$ [5]

2. In a large corporate computer network, user log-ons to the system can be modeled as a Poisson process with a mean of 25 log-ons per hour. What is the probability that
   a) there are no log-ons in an interval of 6 minutes
   b) the time until next log-on is between 2 and 3 minutes [15]

3. Let X denote time to failure in years of a telephone modem used to connect a mainframe computer from a remote terminal. X follows Weibull distribution with parameters $\alpha = 2$ and $\beta = 0.5$.
   a) Write the density function of X; [2]
   b) Find the expected life of modem; [3]
   c) Find the probability that the modem will last for at least 2 years. [5]

4. Police response time to an emergency call is the time difference between the time the call is first received and the time that a patrol car has arrived at the scene. It has been observed that this response time follows normal distribution with mean 8.4 minutes and standard deviation 1.7 minutes.
   a) What percentage of response times are longer than 9 minutes? [5]
   b) What percentage of response times are between 6.7 minutes and 11.8 minutes? [5]
   c) How long is the response time if it is in the fastest 20% of all response times? [5]

5. Let X be Binomial with parameters n = 15 and p = 0.2.
   a) Find the expression for moment generating function for X. [5]
   b) Find $P(X \geq 3)$ [5]
   c) Find $P(2 \leq X < 5)$ [5]

6. Twenty microprocesssor chips are in stock. Three have etching errors that cannot be detected by the naked eye. Five chips are selected and installed in field equipment. Find the probability that
   a) no chips with an etching error will be chosen [5]
   b) at least one chip with an etching error will be chosen. [5]

Table Values:

As per the standard notation:

$F(0.3529) = 0.6379$
$Z_{0.8} = -0.8416$
$F(2) = 0.9772$
$F(-1) = 0.1587$

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TEST– II (OB)

PROBABILITY & STATISTICS
(AAOC C111)

Max. Marks: 60 Weightage: 20% Date: 21-12-2008 Time: 50 Minutes

NOTE:

- **Attempt all the questions.**
- **Attempt the questions in sequence.**

1. A drug research experimental unit is testing two drugs newly developed to reduce blood pressure levels. The drugs are administered to two different sets of animals. In group one, 350 of 600 animals tested respond to drug one and in group two, 260 of 500 animals tested respond to drug two. The research unit wants to test whether there is a difference between the efficacies of the said two drugs.

   (a) Set up the null and alternative hypotheses needed to test whether there is a difference between the efficacies of the said two drugs. [2]

   (b) Specify the critical points at 1% level of significance? [2]

   (c) Based on the above information, can H_0 be rejected at 1% level of significance? [6]

2. (a) Simulate a value of an exponential variable with $\beta = 0.352$. Use the random number 0.534. [5]

   (b) To test H_0: $\mu = 40$ versus H_1: $\mu > 40$, where μ is the mean of a normal population with variance 36, the critical region based on a random sample of size 25 is $C = \{\bar{x} \geq 43\}$. Find the probability of type I error. [5]

3. The joint density function of random variables X and Y is defined as follows:

$$f(x,y) = \begin{cases} 3y & \text{for } 0 \leq x \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   a) Find the marginal densities of X and Y. [5]
   b) Find $E(X)$ and $E(Y)$. [6]
   c) Find the covariance between X and Y. [4]

4. The joint distribution of X and Y is given below:

$X \rightarrow$ / $\downarrow Y$	1	2	3	4
1	0.01	0.04	0.05	0.02
2	0.02	0.06	0.01	0.04
3	0.12	0	0.25	0.12
4	0.08	0.05	0.03	----

a) Find the missing cell value. [3]

b) Calculate $P(X \geq 3 \; and \; Y \geq 3)$. [2]

5. A random sample of 500 apples was taken from a large consignment and 60 were found to be bad.

a) Obtain the 98% confidence limits for the percentage of bad apples in the consignment. [6]

b) How large a sample is required to estimate p to within 0.04 with 90% confidence? [4]

6. A random sample of 10 boys had the following I.Q's: 70, 120, 110, 101, 88, 83, 95, 98, 107, and 100. Find the reasonable range in which most of the mean I.Q values of the samples of 10 boys lie. [10]

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BITS, PILANI – DUBAI
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(I YEAR – I SEMESTER 2008-2009)

COMPREHENSIVE EXAMINATION

Course Title: Probability & Statistics
Max. Marks: 120
Date: 07-01-2009

Course No. : AAOC C111
Weightage: 40%
Time: 3 hours

Note:

- SECTION-A, SECTION-B and SECTION-C should be answered in separate answer books.
- Attempt all the questions in sequence.
- Non-programmable calculator is permitted
- Necessary ***statistical table values*** are given in the last page

SECTION-A

1. A factory produces a certain type of outputs by three types of machine. The respective daily production figures are:
Machine I: 3,000 Units; Machine II: 2,500 Units; Machine III: 4,500 units
Past experience shows that 1% of the output produced by Machine I is defective. The corresponding fraction of defectives for the other two machines is 1.2 % and 2% respectively. An item is drawn at random from the day's production run and is found to be defective.
a) What is the probability that the item drawn is defective? [5]
b) What is the probability that the item is produced by Machine I given that it is defective? [5]

2. a) It has been found that 80% of all printers used on home computers operate correctly at the time of installation. The rest require some adjustment. A particular dealer sells 10 units during a given month. Find the probability that at least nine of the printers operate correctly upon installation. [5]

b) A distributor of computer software wants to obtain some customer feedback concerning its latest package. 10 customers have purchased the package. Assume that 4 of these customers are dissatisfied with the product. 5 customers are randomly selected and questioned about the package. Let X denote the number of dissatisfied customers selected. Find the probability that at most 3 customers are dissatisfied with the package. [5]

3. a) A random variable X is said to be uniformly distributed over an interval (2, 4) if its density is given by

$$f(x) = \frac{1}{2}, \qquad 2 \le x \le 4$$

i) Show that this is a density for a continuous random variable. [3]
ii) Find $P(X \le 3)$ [4]

b) Find the general expression for the cumulative distribution of the uniform distribution

$$f(x) = \frac{1}{b-a}, \quad a \le x \le b$$

[5]

4. Consider the function $f(x) = kx, \quad 2 \le x \le 4$

i) Find the value of k that makes this a density for a continuous random variable.
ii) Find P $(2.5 \le X \le 3)$

[4 + 4]

SECTION –B

5. The time required to assemble a piece of machinery is a random variable having approximately a normal distribution with $\mu = 12.9$ minutes and $\sigma = 2.0$ minutes. What are the probabilities that the assembly of a piece of machinery of this kind will take

a) at least 11.5 minutes;
b) anywhere from 11.0 to 14.8 minutes?

[10]

6. a) Simulate a value of a Weibull variable with $\alpha = 0.25$ and $\beta = 0.5$. Use the random number 0.342.

[4]

b) The number of customers who visit a car dealer's showroom on a Saturday morning is a random variable with $\mu = 15$ and $\sigma = 2.5$. With what probability can we assert that there will be between 5 and 25 customers? Use Chebyshev's inequality.

[6]

7. If the joint probability density of two random variables is given by

$$f(x_1, x_2) = \begin{cases} 6e^{-2x_1-3x_2} & \text{for } x_1 > 0, x_2 > 0 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the probability that the first random variable will take on a value between 1 and 2 and the second random variable will take on a value between 2 and 3.

[4]

b) Find the marginal density of x_1 i.e. $f_1(x_1)$.

[3]

c) Find the conditional density of x_2 given $x_1 = 5$ i.e. $f(x_2 / x_1 = 5)$

[3]

8. a) A random sample of size n=100 is taken from a population with $\sigma = 5.1$.

Given that the sample mean is $\bar{x} = 21.6$, construct a 95% confidence interval for the population mean μ.

[8]

b) $m_x(t) = (0.4 + 0.6e^t)^5$ is the moment generating function of binomial distribution. Specify its parameters n and p.

[2]

SECTION –C

9. Leaders Private School (LPS) has 300 students. The principal of the school thinks that the average IQ of students at LPS is at least 110. To prove her point, she administers an IQ test to 20 randomly selected students. Among the sampled students, the average IQ is 108 with a standard deviation of 10.
   a) Set up appropriate null and alternative hypotheses. [2]
   b) Find the observed value of the test statistic. [2]
   c) Find the critical region of the test. [2]
   d) Calculate the P-value of the test. [2]
   e) Based on the above results, should the principal accept or reject her null hypothesis? Why? Assume a significance level of 0.01. [2]

10. The CEO of a large electric utility claims that 80 percent of his 1,000,000 customers are very satisfied with the service they receive. To test this claim, the local newspaper surveyed 100 customers, using simple random sampling. Among the sampled customers, 73 say they are very satisfied. Based on these findings, can we reject the CEO's hypothesis that 80% of the customers are very satisfied? Take 2-tailed alternative. Use a 0.05 level of significance. Based on the above sample find the 95% confidence interval on the true proportion p.
    [10]

11. Following are the marks of 8 students of BPD obtained in Test-1 and Test-2 components in the course Probability and Statistics:

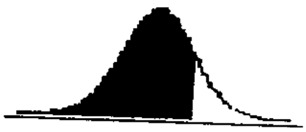
Test-1 (x)	25	28	15	22	18	30	20	28
Test-2 (y)	28	20	24	18	25	20	18	24

   a) Find the regression line of y on x in the form $y = a + bx$. [8]
   b) Estimate the Test-2 marks of a student who got 21 in Test-1. [2]

12. Calculate r, an estimate of the correlation coefficient between x and y based on the following observations:

x:	8	2	5	4	10	1	7	8
y:	5	3	0	1	2	4	2	6

[10]

The table shows the area to the left of a z-score: 

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1161	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	

As per the standard notation:

$P(t_{19} \leq -0.8944) = 0.19$

$P(t_{19} \leq 2.539) = 0.99$