BITS, PILANI-DUBAI
International Academic City, Dubai

First Year - Semester-I (2008-09)

MATHEMATICS-I (MATH C191)

TEST – I (Closed Book)

Date: 19.10.2008
Time: 50 minutes

Max. Marks: 75
Weightage: 25 %

Answer all the Questions

1. Solve the Boundary Value Problem for $\vec{r}$ as a vector function of t:

   Differential equation: $\dfrac{d^2\vec{r}}{dt^2} = 10\hat{i} - 5\hat{j} + \hat{k}$, 9 marks

   Boundary Conditions: $\vec{r}(0) = \hat{i} - 4\hat{j} + 6\hat{k}$ and $\left(\dfrac{d\vec{r}}{dt}\right)_{t=3} = \vec{0}$.

2. Find unit tangent vector and the arc length parameter of the curve:
   $\vec{r}(t) = (e^t \sin t)\hat{i} + (e^t \cos t)\hat{j} + 5e^t \hat{k}$, $0 \le t \le \dfrac{\pi}{2}$ 8 marks

3. Find the length of the curve $\vec{r}(t) = (t\cos t)\hat{i} + (t\sin t)\hat{j} + \dfrac{2\sqrt{2}}{3} t^{\frac{3}{2}}\hat{k}$, $0 \le t \le \pi$.
   8 marks

4. Find the velocity and acceleration vectors at the stated times and mark them on the curve as vectors:
   Motion on the cycloid: $x = t - \sin t, y = 1 - \cos t$ and
   $\vec{r}(t) = (t - \sin t)\hat{i} + (1 - \cos t)\hat{j}$, $t = \dfrac{3\pi}{2}$, $t = \pi$. 9 marks

5. Find the length of the curve $r = \sqrt{1 + \sin 2\theta}$, $0 \le \theta \le \pi\sqrt{2}$. 8 marks

6. Find a $\delta > 0$ algebraically that works for $\varepsilon = 0.5$ if $f(x) = x^2 - 15, L = 1, x_0 = 4$.
   8 marks

7. Find $\displaystyle\lim_{x \to 2^+} \frac{\sqrt{x^2+5} - 3}{(x^2 - 2x)}$. 8 marks

8. Plot the graph, label the centre, vertices and foci of $r = \dfrac{25}{10 - 5\cos\theta}$ with appropriate polar coordinates. 9 marks

9. Find the area of the region inside the loop of the lemniscates $r^2 = 4\sin 2\theta$. 8 marks

BITS, PILANI-DUBAI
International Academic City, Dubai

First Year - Semester-I (2008-09)

MATHEMATICS-I (MATH C191)

TEST – II (Open Book)

Date: 10.12.2008
Time: 50 minutes

Max. Marks: 60
Weightage: 20 %

Answer all the Questions

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1. Evaluate by reversing the order of integration: $\int_0^{\frac{\pi}{2}} \int_0^{\cos x} x^2 \, dy\, dx$ — 7 marks

2. Using double integral, find the area of the region R bounded by $xy = 1$ and $2x + y = 3$. — 7 marks

3. Check whether $f(x, y) = e^{2x}(x\cos 2y - y\sin 2y)$ satisfies the Laplace equation $f_{xx} + f_{yy} = 0$ — 8 marks

4. Linearise the function $f(x, y, z) = e^x + \cos(y + z) + 2\sin(x + z)$ at the point $\left(0, \frac{\pi}{4}, \frac{\pi}{4}\right)$. — 7 marks

5. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ using the chain rule if $w = y^2 + \left(\frac{x}{y}\right)$, $x = u - 2v + 1,\ y = 2u + v - 2$ at the point $u = 1, v = 2$. — 8 marks

6. Maximize the function $x^2 + y^2 + z^2$ subject to the constraint $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$. — 7 marks

7. Find the directional derivative of $f(x, y, z) = x^2 - yz + 2z^3 e^{-3x}$ at the point $P_0(4, -1, 2)$ in the direction parallel to the line joining the points (2, 1, 3) and (3, -2, 5). — 8 marks

8. Find the extreme values of the function $f(x, y) = x^3 y^2 (12 - x - y)$ — 8 marks

BITS, PILANI-DUBAI

International academic City, Dubai

First Year - Semester-I (2008- 09)

Mathematics-I (MATH UC191)

Comprehensive Examination (Closed Book)

Date: 05.01.2009	Max. Marks: 120
Time: 3 hours	Weightage: 40 %

Answer all the questions.

Answer Part A, Part B and Part C in three separate Answer Books.

PART-A

1. Find δ algebraically $f(x)=\frac{1}{x},\ x_0=4,\ \varepsilon=0.05\ and\ L=\frac{1}{4}$. (7)

2. Sketch and label the vertices, foci and center of the curve $r=\frac{25}{10-5\cos\theta}$. (8)

3. Find the area of the region which is shared by the circles $r=2\cos\theta$ and $r=2\sin\theta$ (8)

4. Find the length of the curve $r=\cos^3\left(\frac{\theta}{3}\right),\quad 0\le\theta\le\frac{\pi}{4}$ (8)

5. With out finding $\hat{T}\ and\ \hat{N}$, write $\vec{a}=a_T\vec{T}+a_N\vec{N}$ for the curve $\vec{r}=t\cos t\ \hat{i}+t\sin t\ \hat{j}+t^2\ \hat{k}$ at $t=0$. (8)

PART-B

6. Find the length of the curve $\vec{r}=t\sqrt{2}\ \hat{i}+t\sqrt{2}\ \hat{j}+\left(1-t^2\right)\hat{k}$ from $(0,0,1)$ to $(\sqrt{2},\sqrt{2},0)$. (7)

7. Find $\frac{\partial w}{\partial r}and\frac{\partial w}{\partial s}$ at $r=1, s=-1$ using the chain rule if $w=(x+y+z)^2,\ x=r-s,\ y=\cos(r+s),\ z=\sin(r+s)$. (8)

(Please Turn Over)

8. Find the parametric equations to the tangent line to the curve of intersection of the surfaces $x^3+3x^2y^2+y^3+4xy-z^2=0$ and $x^2+y^2+z^2=11$ at the point $P_0(1,1,3)$ (7)

9. Find the maximum values of $49-x^2-y^2$ subject to the constraint $x+3y=10$. (8)

10. Discuss the convergence of the following series:

(a) $\sum_{n=1}^{\infty}\frac{(n+3)!}{3!\,n!\,3^n}$ (b) $\sum_{n=1}^{\infty}\frac{(10n+1)}{n(n+2)(n+3)}$ (7+7)

PART-C

11. Evaluate $\int_0^2\int_0^{4-y^2} y\,dx\,dy$ by reversing the order of integration. (7)

12. Convert into polar integral and evaluate $\int_0^2\int_0^{\sqrt{4-y^2}}\left(x^2+y^2\right)dx\,dy$. (8)

13. Find the work done by $\vec{F}=(y+z)\,\vec{i}+(z+x)\vec{j}+(x+y)\,\vec{k}$ along the curved path $\vec{r}(t)=t\,\vec{i}+t^2\,\vec{j}+t^4\,\vec{k}$, $0\le t\le 1$. (7)

14. Verify Green's theorem for the field $\vec{F}=(y^2-x^2)\,\vec{i}+(x^2+y^2)\vec{j}$ over the curve C: the triangle bounded by the lines $y=0, x=3$ and $y=x$. (8)

15. For what values of b and c will $\vec{F}=(y^2+2czx)\,\vec{i}+y(cz+bx)\vec{j}+(cx^2+y)\,\vec{k}$ be a gradient? If so, find the flow integral of $\vec{F}$ along any curve joining $(1,\ 0,\ -1)$ and $(2,3,5)$. (7)

ALL THE BEST

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