SECTION-II

11.12.2007

QUIZ-IV

① Change into polar integral:

Group A

1. $\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{0} f(x,y)\,dx\,dy$

$\int_{\pi/2}^{3\pi/2} \int_{r=0}^{1}$

Group B.

1. $\int_{-1}^{0} \int_{-\sqrt{1-x^2}}^{0} f(x,y)\,dy\,dx$ $\int_{0}^{3\pi/2} \int_{\pi}$

② ~~Evaluate~~ Find the Jacobian of

2. $x = u\cos v$, $y = u\sin v$

$J = u$

2. $x = u\sin v$, $y = u\cos v$

$|J| = u$ $J = -u$

③ Find the line integral of

3. $f(x,y,z) = -\sqrt{x^2+z^2}$ over the circle $\bar{r}(t) = a\cos t\,\bar{i} + a\sin t\,\bar{j}$, $0 \le t \le 2\pi$

$-2\pi a^2$

3. $f(x,y,z) = x+y+z$ over the line segment from $(1,2,3)$ to $(0,-1,1)$

$3\sqrt{14}$

BPD, Dubai

27.11.2007

Quiz - III - Mathematics - I

1.37 to 1.52 pm

SECTION - IV

Group A

1. Find the extreme pts of

$8x^3 + y^3 + 6xy$

2. Find the eqn. of the normal line to

$f(x,y,z) = 5x^2 + 4xy - 2y^2 + 3z = 4$

at $(1,1,-1)$.

3. Reverse the order:

$$\int_{-3}^{0} \int_{0}^{\sqrt{9-x^2}} f(x,y)\, dx\, dy$$

Group B

1. Find the extreme points of

$4x^2 - 6xy + 5y^2 - 20x + 24y$

2. Find the equation of the normal line for

$f(x,y,z) = x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0$

at $(1,1,3)$

3. Reverse the order:

$$\int_{-1}^{0} \int_{0}^{1-x^2} f(x,y)\, dy\, dx.$$

Answers

Group A

1. $(0,0)$ and $\left(-\frac{1}{2}, -1\right)$

2. $x = 1 + 14t$, $y = 1$,
$z = -1 + 3t$.

3. $\iint = \int_{y=0}^{3} \left(\int_{x=-\sqrt{9-y^2}}^{0} f(x,y)\, dx \right) dy$

Group B

1. $\left(\frac{14}{11}, \frac{-18}{11}\right)$

2. $x = 1 + 13t$,
$y = 1 + 13t$,
$z = 3 - 6t$

3. $\iint = \int_{y=0}^{1} \left(\int_{x=-\sqrt{1-y}}^{0} f(x,y)\, dx \right) dy$

BPD, Dubai — 29.11.2007

Quiz-III – Mathematics-I — 11.50 AM – 12.07 pm

SECTION-VII

Group A

1. Find the equation of the tgt. line to the curve of intersection of the surfaces

$xyz = 2$, $x^2+2y+3z^2=15$, $(1,1,2)$

2. Reverse the order of integration

$$\int_0^1 \int_{x^2}^{2-x} f(x,y)\,dy\,dx$$

3. Find the minimum of $2x+y$ subject to $xy=4$, $x>0, y>0$

Group B

1. $x^2+y^2=2$, $x^2+y^2-z=-1$, $(-1,-1,3)$

2. $$\int_0^1 \int_y^{10y} f(x,y)\,dx\,dy$$

3. Find the maximum of xy subject to $x+y=4$

BITS, PILANI-DUBAI,
INTERNATIONAL ACADEMIC CITY, DUBAI

MATHEMATICS-I (MATH UC 191)
QUIZ-III

SECTION-II

GROUP-A

1. Find the extreme points of $x^3 + y^3 - 3xy + 15$.

2. Find the tangent plane to
$f(x, y, z) = x^2 + y^2 + z = 4, \quad P_0(1, 1, 2)$.

3. Plot the region and change the order of integration:

$$\int_{-2}^{0} \int_{-\sqrt{4-x^2}}^{0} f(x, y)\,dA$$

GROUP-B

4. Find the extreme points of $x^3 + y^3 + 3axy$.

5. Find the tangent plane to
$f(x, y, z) = 2x^2 - 3y^2 + 3z^2 = 2, \quad P_0(1, 1, 1)$.

6. Plot the region and change the order of integration:

$$\int_{-1}^{0} \int_{0}^{\sqrt{1-x^2}} f(x, y)\,dA$$

BPD, Dubai, UAE

Mathematics-I (MATH UC191)

Quiz-II SECTION-VII

Group-A

① At what times in the interval $0 \le t \le \pi$, are the velocity and accn. of $\bar{r}(t) = \bar{i} + (5\cos t)\bar{j} + (3\sin t)\bar{k}$ orthogonal?

② Find the domain, its nature and range of

$$f(x, y) = \sqrt{1 - x^2 - y^2}$$

Group-B

① Find the first time when $\bar{r}$ is orthogonal to $\bar{i} - \bar{j}$ if

$$\bar{r}(t) = 2\bar{i} + 4\sin(t/2)\bar{j} + \left(3 - \frac{t}{\pi}\right)\bar{k}.$$

② Find the domain, its nature and range of

$$f(x, y) = \sqrt{x^2 + y^2 - 16}$$

~~ o ~~

BPD, Dubai, UAE

Mathematics-I (MATH UC 191)

Quiz-II SECTION-VI

Group-A

① plot $\bar{v}$ and $\bar{a}$ on the circle $x^2+y^2=1$ at $t=\pi/4$ given $\bar{r}(t) = \sin t\,\bar{i} + \cos t\,\bar{j}$

② Does $\lim_{(x,y)\to(0,0)} \frac{2x}{x^2+x+y^2}$ exist?

Group-B

① plot $\bar{v}$ and $\bar{a}$ on the circle ① at $t=\pi/2$.

② Does $\lim_{(x,y)\to(0,0)} \frac{x^3-xy^2}{x^2+y^2}$ exist?

—non—

BPD, Dubai, UAE

Mathematics-I (MATH UC 191)

Quiz-II SECTION-II

Group-A

① Plot $\bar{v}$ and $\bar{a}$ on the cycloid $x = t - \sin t$, $y = 1 - \cos t$, given $\bar{r}(t) = (t - \sin t)\bar{i} + (1 - \cos t)\bar{j}$ at $t = \pi$.

② Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ if

$w = u^2 - uv + 2v^3$, $u = x^2 + y$, $v = x^2 - y$

by (i) chain rule (ii) direct method.

Group-B

① For the curve ①, plot $\bar{v}$ and $\bar{a}$ at $t = 3\pi/2$

② Find $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial s}$, $\frac{\partial w}{\partial t}$ if

$w = u^2 + v^3$, $u = r^2 + s^2 + t$, $v = rs^3 - rt + 6t^3$.

BITS – PILANI

DUBAI

Form A

SURPRISE QUIZ 1 **MATHEMATICS – I** **DATE :02.10.07**

Name **ID No.** **Sec.** I

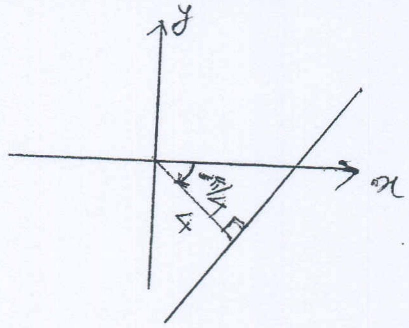
1.Draw the graph for the values of r and θ satisfy the following conditions.

$0 \le \theta \le \pi/3$ **and** $1 \le r \le 2$.

2.Convert the following Cartesian eqn. into its equivalent polar equation.

$(x-3)^2 + (y+1)^2 = 4$

3.Find the polar and Cartesian equation of the line



Form B

BITS – PILANI
DUBAI

SURPRISE QUIZ 1 | **MATHEMATICS – I** | **DATE : 02.10.07**

Name | **ID No.** | **Sec.** I

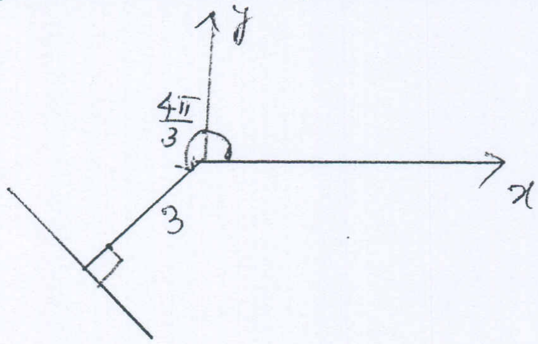
1.Draw the graph for the values of r and θ satisfy the following conditions.

$-\pi/2 \leq \theta \leq \pi/2$ and $-1 \leq r \leq 2$.

2.Convert the following polar eqn. into its equivalent cartesian equation.

$r = 1 - \cos(\theta)$.

3.Find the polar and Cartesian equation of the line



BITS – PILANI **Form A**

DUBAI

SURPRISE QUIZ 1 MATHEMATICS – I DATE :04.10.07

Name ID No. Sec.

1.Draw the graph for the values of r and θ satisfy the following conditions.

$0 \leq \theta \leq \pi/2$ and $-1 \leq r \leq 2$. **(5 marks)**

2.Convert the following polar eqn. into its equivalent cartesian equation and identify the graph.

$r^2 + 2r^2 \cos(\theta)\sin(\theta) = 1$ **(5 marks)**

3.Find the area shared by the cardioids $r = 2(1+\cos(\theta))$ and $r = 2(1-\cos(\theta))$.

(5 marks)

BITS – PILANI **Form B**

DUBAI

SURPRISE QUIZ 1 **MATHEMATICS – I** **DATE : 04.10.07**

Name **ID No.** **Sec.**

1.Draw the graph for the values of r and θ satisfy the following conditions.

$-\pi/2 \leq \theta \leq \pi/2$ **and** $-2 \leq r \leq 1$. **(5 marks)**

2.Convert the following polar eqn. into its equivalent cartesian equation and identify the graph. **(5 marks)**

$r = 2\cos(\theta) - \sin(\theta)$.

3.Find the area shared by the circle $r = 2$ and the cardioid $r = 2(1-\cos(\theta))$

(5 marks)

BITS – PILANI | **Form A**

DUBAI

SURPRISE QUIZ 1 | **MATHEMATICS – I** | **DATE :04.10.07**

Name | **ID No.** | **Sec.**

1.Draw the graph for the values of r and θ satisfy the following conditions. (5 marks)

$-\pi/3 \leq \theta \leq \pi/3$ and $-2 \leq r \leq 1$.

2.Convert the following Cartesian eqn. into its equivalent polar equation and identify the graph. (5 marks)

$(x-6)^2 + y^2 = 36$

3.Find the area inside one leaf of the four leaved rose $r = \cos(2\theta)$ (5 marks)

BITS – PILANI

DUBAI

Form B

SURPRISE QUIZ 1 MATHEMATICS – I DATE : 04.10.07

Name ID No. Sec.

1.Draw the graph for the values of r and θ satisfy the following conditions. (5 marks)

$\pi/4 \leq \theta \leq 3\pi/4$ and $1 \leq r \leq 2$.

2.Convert the following polar eqn. into its equivalent cartesian equation And identify the graph. (5 marks)

$r = 8\sin(\theta)$.

3.Find the area inside one loop of the lemniscate r = 4 sin(2θ). (5 marks)

BITS, PILANI-DUBAI

International academic City, Dubai

First Year - Semester-I (2007- 08)

Mathematics-I (MATH UC191)

Comprehensive Examination (Closed Book)

Date: 07.01.2008	**Max. Marks: 120**
Time: 3 hours	**Weightage: 40 %**

Answer all the questions.

Answer Part A and Part B in two separate Answer Books.

PART-A

1. (a) Find the area inside the circle $r=-2\cos\theta$ and outside the circle $r=1$ (8)

   (b) Find the length of the curve $r=\sqrt{1+\sin 2\theta}, \quad 0\le\theta\le\frac{\pi}{2}$ (8)

2. (a) Solve the initial value problem: $\frac{d^2\vec{r}}{dt^2}=-(\hat{i}+\hat{j}+\hat{k})$ with the initial conditions

   $\vec{r}(0)=10(\hat{i}+\hat{j}+\hat{k}), \ \frac{d\vec{r}}{dt}(0)=\vec{0}$ (8)

   (b) Find $\hat{T}$, $\hat{N}$ and κ for the curve $\vec{r}=6\sin 2t\,\hat{i}+6\cos 2t\,\hat{j}+5t\,\hat{k}$ (9)

3. (a) Find the directions in which the function $f(x,y,z)=\ln(xy)+\ln(yz)+\ln(xz)$ increase and decrease most rapidly at the point $P_0(1,1,1)$. Also find the derivatives of the function in those directions. (8)

   (b) Find $\frac{dw}{dt}$ at $t=3$ using the chain rule if $w=\frac{x}{z}+\frac{y}{z}, x=\cos^2 t, y=\sin^2 t, z=\frac{1}{t}$ (8)

4. (a) Find the tangent plane and normal line to the surface

   $f(x,y,z)=x^2+y^2-2xy-x+3y-z=-4$ at the point $P_0(2,-3,18)$ (8)

(Please Turn Over)

(b) Find the maximum and minimum values of $x^2 + y^2$ subject to the constraint

$x^2 - 2x + y^2 - 4y = 0$ (8)

PART-B

5. Discuss the convergence of the following series:

(a) $\sum_{n=3}^{\infty} \frac{\left(n^2 + 3n - 4\right)}{n^2 (n-2)(4+n^2)}$ (b) $\sum_{n=1}^{\infty} \frac{(3n)!}{n!\,(n+1)!\,(n+2)!}$ (7+7)

6. Use the transformation $u = 2x - 3y, v = -x + y$ to evaluate the integral

$$\iint_R 2(x-y)\,dxdy$$

for the region R in the *xy*-plane bounded by the lines

$y = x,\ \ y = x+1,\ \ x = -3\ \ and\ \ x = 0$ (8)

7. Reverse the order of integration and evaluate the integral $\int_0^{\frac{3}{2}} \int_0^{9-4x^2} 16x\,dydx$ (8)

8. Change into an equivalent polar integral and integrate $\int_0^2 \int_0^x y\,dydx$ (8)

9. Show that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is a conservative field and hence find the scalar potential. Also find the work done by $\vec{F}$ along any smooth curve C joining the point $(1, -1, 2)$ and $(3, 1, 5)$. (8)

10. Verify Green's theorem for $\int_C (x^2 - y^2)dx + 2xydy$ where C is the curve bounded by

$y = x^2$ and $x = y^2$. (9)

BITS, PILANI-DUBAI
INTERNATIONAL ACADEMIC CITY, DUBAI

First Year - Semester-I (2007-08)

MATHEMATICS-I (MATH UC191)

TEST – I (Closed Book)

Date: 28.10.2007 | **Max. Marks: 50**
Time: 50 minutes | **Weightage: 25 %**
Answer all the questions.

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1. Solve the Initial Value Problem for $\vec{r}$ as a vector function of t:

   Differential equation: $\dfrac{d\vec{r}}{dt}=\dfrac{3}{2}(1+t)^{\frac{1}{2}}\,\hat{i}+e^{-t}\,\hat{j}+\dfrac{1}{1+t}\,\hat{k}$,

   Initial Conditions : $\vec{r}(0)=\hat{i}-2\,\hat{k}$. **(6)**

2. Find the curvature and the tangential and normal components of acceleration without finding $\hat{T}$ and $\hat{N}$ for the curve at the indicated point $\vec{r}(t)=(t+1)\,\hat{i}+(2t)\hat{j}+t^2\,\hat{k}$, $t=1$. **(7)**

3. Find the unit tangent vector and also the length of the indicated portion of the curve $\vec{r}(t)=(t\cos t)\hat{i}+(t\sin t)\hat{j}+\left(\dfrac{2\sqrt{2}}{3}\right)t^{\frac{3}{2}}\,\hat{k}$;$0\le t\le\pi$. **(7)**

4. Find the length of the curve $r=\cos^3(\frac{\theta}{3})$, $0\le\theta\le\frac{\pi}{4}$. **(6)**

5. For the limit $\lim\limits_{x\to 10}\sqrt{19-x}=3$, find a $\delta>0$ algebraically that works for $\varepsilon=1$. **(4)**

6. Find $\lim\limits_{h\to 0^-}\dfrac{\sqrt{6}-\sqrt{5h^2+11h+6}}{h}$. **(4)**

7. (i) Plot the graph of $r=\frac{1}{2}+\sin\theta$. **(4)**

   (ii) Find the vertices and centre of $r=\dfrac{8}{4-\sin\theta}$. **(4)**

8. Find the area inside the lemniscates $r^2=6\cos(2\theta)$ and outside the circle $r=\sqrt{3}$. **(8)**