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BITS, PILANI – DUBAI CAMPUS
Knowledge Village, Dubai

(I year – I semester 2006-07)

QUIZ – II (CB)

Course Title: Probability & Statistics

Course No. : AAOC UC 111

Max. Marks : 30

Weightage : 10%

Time : 30 minutes

Date : 28-11-2006

Name:

Id. No.:

Sec.:

Instructions:

Write your Name, Id.no., Section in the space provided.

Calculator is permitted.

All questions are compulsory.

Each question carries 3 marks. No marks for incorrect or partly correct answers

1. A continuous random variable X is uniformly distributed on the interval $[35, 45]$. The probability that X is between 40 and 50 is _____.
2. If X has mean 2 and variance 5 and Y has mean -2 and variance 4 and the two are independent, find
 - a. $E\left[\frac{2X + 2Y - 5}{5}\right] =$ _____
 - b. $V\left[\frac{2X - 2Y - 5}{5}\right] =$ _____
3. The density function of exponential distribution is $\frac{e^{-x/\beta}}{\beta}; x > 0, \beta > 0$. The variance is 1/9 then $\beta =$ _____.
4. If the mean of a continuous uniform distribution defined in (α, β) is 3 and the variance is 1/3, then $\alpha =$ _____ and $\beta =$ _____.
5. In a gamma distribution, if the parameter α is made double and the parameter β is made half, then the mean of the distribution will
 - a) increase; b) decrease; c) not change. (Tick the correct answer)
6. If X is a beta variate with parameters $\alpha = 1$ and $\beta = 2$, then $P(X > 1.5) =$ _____.

7. Suppose the life time of a certain kind of an emergency back up battery (in hours) is a random variable X having a Weibull distribution with $\alpha = 0.12$ and $\beta = 0.6$. The probability that such a battery will last more than 400 hours is _____
8. If a random variable has the log-normal distribution with $\alpha = -1.9$ and $\beta = 3.5$, then its mean = _____ and standard deviation = _____
9. For the Weibull distribution with $\alpha = 0.04$ and $\beta = 2.1$, what is the formula for simulating the values of the variable when u is a uniform variable from 0 to 1 _____
10. The joint distribution of X and Y is defined as follows:

$X \rightarrow$	0	1	2
$Y \downarrow$			
0	0.21	0.10	0.32
1	0.01	0.02	α
2	0.10	β	0.04

If $P(X = 2, Y = 1) = 3P(X = 1, Y = 2)$, then $\alpha =$ _____ and $\beta =$ _____ .

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(I year – I semester 2006-07)

TEST – I (CB)

Course Title: Probability & Statistics

Course No. : AAOC UC 111

Max. Marks: 60

Weightage: 20%

Date: 29.10.2006

Time: 50 min.

NOTE

- (i) Attempt all the questions.
(ii) Attempt each question on a separate page.

1. In testing a certain kind of truck tire over a rugged terrain, it is found that 25% of the trucks fail to complete the test run without a blowout. Of the next 15 trucks tested, find the probability that
 - (a) fewer than 4 have blowouts
 - (b) from 3 to 6 have blowouts.[5+5]
2. Let X be the random variable that denotes the life in hours of a certain electronic device. The probability density function is
$$f(x) = \begin{cases} \frac{20,000}{x^3}, & x > 100 \\ 0, & \text{elsewhere.} \end{cases}$$
Find the expected life of this type of device. [10]
3. A consultant's office receives on the average 0.8 call per minute. Find the probabilities that
 - (a) there will be exactly 2 calls in a 5-minute period ;
 - (b) there will be exactly 2 calls in a 2-minute period and exactly 1 call in next 2-minute period .[12]
4. A box contains 2 white, 3 black and 3 blue balls. Draw a ball at random from the box. If it is white then stop, otherwise replace the ball in the box and again draw a ball at random. Continue this process until you get a white ball. What is the probability that the first white ball will be drawn at 5th draw ? [8]
5. Find the probability of scoring a total of 7 points (a) once & (b) at least once in two tosses of a pair of fair dice. [10]
6. Three candidates A, B and C compete for the post of President whose chances are $1/5$, $3/10$ and $1/2$ respectively. The chances of them introducing an insurance scheme to the employees are $2/7$, $2/7$ and $3/7$ respectively. What is the probability of having insurance scheme to the employees? [10]

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(I year – I semester 2006-07)

~~STAT~~^{SUTZ} – I (CB)

Course Title: Probability & Statistics

Course No. : AAOC UC 111

Max. Marks : 30

Weightage : 10%

Time : 30 minutes

Date : 07-11-2006

Name:

Id. No.:

Sec.:

Instructions:*Write your Name, Id.no., Section in the space provided.**Calculator is permitted.**All questions are compulsory.**Each question carries 3 marks. No marks for incorrect or partly correct answers.***Fill in the blanks:**

- If a binomial distribution is approximated by a normal distribution with $\mu = 20$ and $\sigma = 2\sqrt{3}$, then the parameters of that binomial distribution are $n = \underline{50}$ and $p = \underline{0.4}$.
- A box contains 6 red and 9 blue marbles. What is the probability of getting 4 red marbles in a draw of 7 marbles at random? $\frac{28}{153} = 0.1958$.
- The normal approximation to the binomial (n, p) distribution gives a very good result if both np and $n(1-p)$ are greater than $\underline{15}$.
- The variance of a negative binomial distribution with parameters $p = 0.5$ and $r = 4$ is $\underline{8}$.
- You are throwing a ball again and again to hit a spot on a nearby wall. It is known that 25% of the throws fail to hit the spot. The probability that you will hit the spot first time in 5th throw is $\underline{0.0029 \approx 0.003}$.
- If one out of ten bolts is defective, what is the expected number of defective bolts in a total of 400 $\underline{40}$.
- 10000 tickets are sold for a raffle draw which has two mega prizes each of Rs.10, 00,000 and one consolation prize of Rs.50, 000. The expected payoff of a person who is given one of the tickets is $\underline{Rs.25, 00, 205}$.

8. If $P(x=1) = 9P(x=2)$ for a Poisson distribution, then $\lambda = \underline{\frac{2}{9} = 0.222}$

9. The lower limit of the probability $P(|X - \mu| < 2)$ obtained from Chebyshev's theorem is 0.75, where μ is the mean of X .

Given that the standard deviation of X is 1.

10. The mean and variance of a standard normal random variable are 0
and 1

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(I year – I semester 2006-07)

TEST – II (OB)

Course Title: Probability & Statistics

Course No. : AAOC UC 111

Max. Marks: 60

Weightage: 20%

Date: 12.12.2006

Time: 50 min.

NOTE

- (i) *Attempt all the questions.*
(ii) *Attempt each question on a separate page.*
(iii) *Text Book, Class Notes and Calculator are allowed.*

1. A coin is tossed 900 times and head appears 490 times. Does it support the null hypothesis that the coin is unbiased at 5% level of significance. [7]
2. A random sample of 8 E-glass fiber specimens yielded an average interfacial shear yield stress of 30.2 with standard deviation of 3.1. Compute 90% confidence interval for the population average yield stress. Assume that the interfacial yield stress is normally distributed. [7]
3. If the simulated value of an exponential random variable is 1.45 corresponding to the value 0.30 of a uniform random variable. Find the mean of exponential distribution. [6]
4. The joint probability distribution of two discrete random variables X and Y is defined by $P(X = x, Y = y) = cxy$ where $X = 1, 2, 3$; $Y = 1, 2, 3$ and equals zero otherwise.
 - a) Find the value of c .
 - b) Find $P(X \leq 2, Y \geq 2)$.
 - c) Find $P(Y = 3)$.[4+4+2]
5. Certain electric bulbs manufactured by a company have a mean life of 800 hours and the standard deviation σ hours. If the probability that a random sample of 16 bulbs will have a mean life greater than 785 hours is 0.8413, find the value of σ . Assume normality. [10]
6. A random variable X has the density function $f(x) = \frac{1}{2}e^{-|x|}$, $-\infty < x < \infty$.
Find an upper bound of $P\{|X - \mu| > 2\}$ using the Chebychev's theorem and compare with the actual probability. [12]
7. A random variable X is Gamma variated with $\alpha = 3$ and $\beta = 2$. Find
 - (a) $P(X \leq 1)$
 - (b) $P(1 \leq X \leq 2)$[8]

BITS, PILANI – DUBAI CAMPUS,

Dubai Knowledge Village

(I year – I semester 2006-07)

Comprehensive Examination

Course Title: Probability & Statistics

Course No. : AAOC UC 111

Max. Marks: 120

Weightage: 40%

Date: 28.12.2006

Time: 3 hours.

NOTE

- (i) Answer Part A in the main answer sheet, Part B and Part C in two separate additional answer sheets.
- (ii) Attempt all the questions.
- (iii) Attempt each question on a separate page.
- (iv) Calculator is allowed.

TABLE VALUES: As per the standard notation:

$F_{6,10;0.05} = 3.22$, $F_{10,6;0.05} = 4.06$, $F_{6,10;0.01} = 5.39$, $F_{10,6;0.01} = 7.87$
 $t_{4;0.01} = 1.533$, $t_{5;0.01} = 3.365$, $t_{6;0.01} = 3.143$, $t_{6;0.005} = 3.707$, $t_{5;0.005} = 4.032$,
 $t_{4;0.005} = 4.604$, $t_{9;0.05} = 1.833$, $t_{10;0.05} = 3.169$, $t_{9;0.025} = 2.262$, $t_{10;0.025} = 2.228$
 $z_{0.05} = \pm 1.645$, $z_{0.025} = \pm 1.96$, $z_{0.01} = \pm 2.33$, $z_{0.005} = \pm 2.575$, $P(z \leq -1.5) = 0.0668$.

PART-A

1. Three groups of children consist of 2 boys and 3 girls, 3 boys and 4 girls, and 5 boys and 2 girls respectively. One child is selected at random from each group. Find the probability that the selected group consists of 2 boys and one girl. [10]
2. If the density function of a random variable is given by $f(x) = \begin{cases} cx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$
Find (a) the constant c ;
(b) the distribution function $F(x)$ and find $P(1 \leq x \leq 2)$ using distribution function. [10]
3. Out of 2000 families with 4 children each, how many you would expect to have at least one boy? (Assuming equal probabilities of male and female births) [10]
4. If the probability that an individual will suffer from a bad reaction from injection of a given serum is 0.002, determine the probability that out of 800 individuals,
(a) exactly 3 individuals will suffer from a bad reaction?
(b) more than 2 individuals will suffer from a bad reaction? [10]

PART-B

5. The number of mails that a department receives each day can be modeled by a distribution having mean 44 and standard deviation 8. For a random sample of 36 days, what can be said about the probability that the sample mean will be at most 42 or at least 46 using
(a) Chebyshev's theorem; (b) Central Limit Theorem? [10]

6. Taking the random numbers 0.524 and 0.255 in order, simulate a pair of values of
- a normal variate with mean 10 and the standard deviation 2;
 - a Weibull variate with parameters $\alpha = 2$ and $\beta = 4$.

[10]

7. If two random variables X_1 and X_2 have the joint density

$$f(x_1, x_2) = \begin{cases} x_1 x_2 & \text{for } 0 < x_1 < 1, 0 < x_2 < 2 \\ 0 & \text{elsewhere} \end{cases}$$

- Find
- the marginal density of X_2 ;
 - the probability that both the random variables will take on values less than 1;
 - the probability that the random variable X_2 will take on a value greater than 1.

[10]

8. The amount of time that a spy camera will run without having to be reset is looked upon as a random variable having exponential distribution with mean 60 days.

Find the probabilities that

- such a spy camera will require resetting in less than 30 days;
- such a spy camera will not require resetting in at least 50 days.

[10]

PART-C

9. Find the value of $F_{0.95}$ (corresponding to a left-hand tail probability of 0.05) for $v_1 = 6$ and $v_2 = 10$.

[5]

10. A sample of 10 camshafts intended for use in gasoline engines has an average eccentricity of 1.01 and a standard deviation of 0.039 inch. Assuming the data may be treated as a random sample from a normal population, determine a 95% confidence interval for the actual mean eccentricity of the camshaft.

[8]

11. In a study designed to investigate whether certain detonators used with explosives in coal mining meet the requirement that at least 90% will ignite the explosive when charged, it is found that 174 of 200 detonators function properly. Test the $H_0: p=0.90$ against the $H_1: p < 0.90$ at the 0.05 level of significance.

[7]

12. The following data pertains to the number of hours jet aircraft engines have been used and the number of hours required for repair.

No. of hours: (hundreds)	1	2	3	4	5	6
Repair time: (hours)	10	40	30	80	90	100

- Find the equation of the least squares line and use it to predict the mean repair time at $x = 4.5$.
- Construct 99% confidence interval for β , the repair time in hours of jet aircraft engines.

[14]

13. The following are the number of minutes it took 10 mechanics to assemble a piece of machinery in the morning, x , and in the late afternoon, y :

X:	11.1	10.3	12.0	15.1	13.7	18.5	17.3	14.2	14.8	15.3
Y:	10.9	14.2	13.8	21.5	13.2	21.1	16.4	19.3	17.4	19.0

Calculate r .

[6]