BITS, PILANI-DUBAI CAMPUS
Knowledge village, DUBAI

First Year - Semester-I (2006-07)

Mathematics-I (MATH UC191)

Comprehensive Examination (Closed Book)

Date: 27.12.2006 **Max. Marks: 120**
Time: 3 hours **Weightage: 40 %**

Answer all the questions.

Answer Part A , Part B and Part C in three separate Answer Books.

PART-A

1 (a) Find the area shared by the cardioids $r = 2(1+\cos\theta)$ and $r = 2(1-\cos\theta)$.

(b) Find the arc length of $r = \sin^3\left(\frac{\theta}{3}\right)$ from $\theta = 0$ to $\theta = \frac{3\pi}{2}$. (7+8)

2 (a) Find the curve's unit tangent vector. Also find the length of the indicated portion of the curve $\vec{r} = (\cos^3 t)\hat{j} + (\sin^3 t)\hat{k}, \quad 0 \le t \le \frac{\pi}{2}$.

(b) Find the unit normal vector and the curvature for the curve $\vec{r} = (e^t \cos t)\hat{i} + (e^t \sin t)\hat{j} + 2\hat{k}$

(8+10)

(c) Find an equation for the level curve for the function $f(x,y) = \int_x^y \frac{dt}{1+t^2}$ passing through the point $(-\sqrt{2}, \sqrt{2})$.

(7)

PART-B

3 (a) Does the $\lim_{(x,y)\to(0,0)} \tan^{-1}\left(\frac{|x|+|y|}{x^2+y^2}\right)$ exist ? If yes, find it.

(b) Is the curve $\vec{r}(t) = \sqrt{t}\,\hat{i} + \sqrt{t}\,\hat{j} + (2t-1)\hat{k}$ is tangent to the surface $x^2 + y^2 - z = 1$ when $t = 1$? Justify. (7+8)

4 (a) Sketch the curve $x^2 - xy + y^2 = 7$ together with its gradient and tangent line at the point $(-1, 2)$.

(b) If $u = u(x, y)$ and $x = r\cos\theta, y = r\sin\theta$, transform the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into a polar equation.

(7+10)

5. Use the transformation $u = 2x - 3y,\ v = -x + y$ to evaluate the integral $\iint_R 2(y - x)\,dxdy$ over the region R bounded by the lines $x = -3,\ x = 0,\ y = x$ and $y = x + 1$. (8)

PART-C

6. Discuss the convergence of the following series:

(a) $\sum_{n=1}^{\infty} \frac{1}{n(1 + \ln^2 n)}$

(b) $\sum_{n=3}^{\infty} \frac{5n^3 - 3n}{n^2(n-2)(n^2+5)}$

(c) $\sum_{n=1}^{\infty} \frac{n\,2^n (n+1)!}{n!\,3^n}$

(5+5+5)

7. Find the Taylor series of $f(x) = 3^x$ at $x = 2$. (5)

8. Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative field and hence find the scalar potential. Also find the work done by $\vec{F}$ along any smooth curve C joining the point *(0, 0, 10)* to *(1, -1, 4)*

(10)

9. Verify Green's theorem for field $\vec{F} = (y^2 - x^2)\hat{i} + (x^2 + y^2)\hat{j}$ over the triangle bounded by the lines $y = 0, x = 3, y = x$. (10)

BITS, PILANI-DUBAI CAMPUS
KNOWLEDGE VILLAGE, DUBAI.

First Year-First Semester (2006-07)
MATHEMATICS-I (MATH UC191)
TEST-II (OPEN BOOK)

Time: 50 minutes   Marks: 60   Weightage: 20%

Note: Only the Class Notes and the Text Book are permitted.

1. Find the linearization of $f(x,y,z) = e^{x^2+yz}\sin(\pi x + z^2) + y^2\cos(\pi x + z^2)$ at $(1,1,0)$. (9)

2. Find $\frac{dw}{dt}$ using partial differentiation if $w = x^2 e^{-2y} + y\sin z - \cos^2(2z)$ if $x = 2\sqrt{t}$, $y = t - 1 + \ln t$, $z = \pi t$ at $t = 1$. (9)

3. Find the directional derivative of $f(x,y) = 2x^2y^3 + 6xy$ at $(1,1)$ in the direction of a unit vector whose angle with the positive x-axis is $\frac{\pi}{4}$. (9)

4. Find the level curve of $f(x,y) = -x^2 + y^2$ passing through $(2,3)$. Graph the gradient at the point. (9)

5. In a plane triangle, find the maximum value of $\cos A \cos B \cos C$. (9)

6. Write $\vec{a}$ in the form $\vec{a} = a_T\hat{T} + a_N\hat{N}$ for $\vec{r}(t) = \frac{4}{9}(1+t)^{\frac{3}{2}}\hat{i} + \frac{4}{9}(1-t)^{\frac{3}{2}}\hat{j} + \frac{1}{3}t\hat{k}$ at $t = 0$ without finding $\hat{T}$ & $\hat{N}$. (7)

7. If $f(x,y) = \ln(4 - x^2 - y^2) + \ln(x^2 + y^2 - 1)$, then
   (a) find the function's domain,
   (b) find the function's range,
   (c) Determine whether the domain is an open region or a closed region,
   (d) Determine whether the domain is bounded or unbounded. (8)

BITS, PILANI-DUBAI CAMPUS, DUBAI.
I Year- Semester-I (2006-07)
MATHEMATICS-I (MATH UC191)

Quiz-II (Closed Book)

VERSION - B November 14, 2006

Time: 30 Minutes **Marks: 30** **Weightage: 10%**

ID No. : **Section No.:** **Name:**

Note: 1. *Write ID No. , Name, Section No. and Answers in the provided space.*
2. ***Overwriting will be treated as wrong answer.***
3. ***No marks for incorrect or partially correct answers.***

1. What is the level surface of $g(x,y,z)=\sum_{n=0}^{\infty}\frac{(x+y)^n}{n!\,z^n}$ at the point $(\ln 2, \ln 4, 3)$?

....

2. If $f(x-y, y-z, z-x)$ is differentiable, then

$\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}=$ ....

3. Given $u = x\ln(xy)$ and $x^3 + y^3 = -3xy$, then write the chain rule and find $\frac{du}{dx}$

....

....

4. Find $\frac{dz}{dt}$, if $z = e^{\tan^{-1}x}.\cos(y^2)$, $x = \tan t, y = t$ at $t = 0$.

....

5. The function $f(x,y,z)=\dfrac{|y|+2}{|x|+|yz|}$ is continuous at

....

6. The linearization $L(x,y,z)$ of the function $f(x,y,z)=e^{(x^2+y^2)}\sin(y^2-z^2)$ at $\left(-1, 0, \dfrac{\sqrt{\pi}}{\sqrt{2}}\right)$ is ....

7. If $z=\ln(q)$ and $q=\left(\sqrt{v+3}\right)\tan^{-1}u$, then draw a tree diagram and write the chain rule formula for each derivative; and also find $\dfrac{\partial z}{\partial u}$ at $u=1$ and $v=-2$.

8.. The curvature κ for the plane curve $\vec{r}(t)=(2t+3)\hat{i}+(5-t^2)\hat{j}$ is

....

9. Does $\displaystyle\lim_{(x,y)\to(0,0)}\frac{kx+ky}{|x+y|}$ exist?

....

10. The principal unit normal vector $\hat{N}$ for the plane curve $\vec{r}(t)=(\cos t+t\sin t)\hat{i}+(\sin t-t\cos t)\hat{j},\ t>0$ is

....

BITS-PILANI, DUBAI CAMPUS

MATHEMATICS-I (MATHUC 191)
I- Year-Semester-I (2006-07)

B

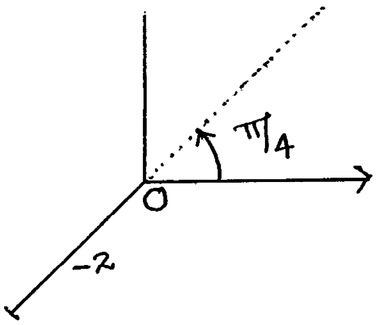
QUIZ – I (CLOSED BOOK)

TIME: 30 Minutes   September 26, 2006   **Max. Marks:10**

ID No.   **Section No.:**   **Name:**

Note: 1.Write ID No., Name, Sec.No. and Answers in the provided space.
2. Overwriting will be treated as wrong answer.

1. The equation of the given straight line is--------------------------



2. The Cartesian form of $r = 1 + \cos\theta$ is --

3. What are the vertices and centre of $r = \dfrac{6}{2-\cos\theta}$?

4. Is the curve $r = 1 - \sin\theta$ symmetric about the y-axis?

5. Sketch $\dfrac{-\pi}{2} \le \theta \le \dfrac{\pi}{2}$, $-1 \le r \le 3$

6. The polar equation for $x = -4$ is ----------------------------

7. The area inside the cardioid $r = a(1+\cos\theta)$ is ---------------------------.

8. Sketch $0 \le \theta \le \frac{\pi}{2}$, $r = -2$.

9. The area inside the circle $r = 6$ and above the line $r = 3\,\mathrm{cosec}\,\theta$ is ---------

10. The length of the curve $r = \cos^3\left(\frac{\theta}{3}\right)$, $0 \le \theta \le \frac{\pi}{4}$ is ---------------------------.

BITS, PILANI-DUBAI CAMPUS
Knowledge village, DUBAI

First Year - Semester-I (2006-07)

MATHEMATICS-I (MATH UC191)

TEST – I (Closed Book)

Date: 08.10.2006 **Max. Marks: 60**
Time: 50 minutes **Weightage: 20 %**
Answer all the questions.

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1. Solve the Boundary Value Problem for $\vec{r}$ as a vector function of t:

   Differential equation: $\dfrac{d^2\vec{r}}{dt^2} = -\hat{i} - \hat{j} - \hat{k}$,

   Boundary Conditions: $\vec{r}(0) = 3\,\hat{i} + 4\,\hat{j} + 5\,\hat{k}$ and $\left(\dfrac{d\vec{r}}{dt}\right)_{t=2} = \vec{0}$. (8)

2. Find parametric equations for the line that is tangent to the given curve at the given parameter value $t = t_0$: $\vec{r}(t) = (2\sin t)\,\hat{i} + (2\cos t)\hat{j} + 5t\,\hat{k}$, $t_0 = 4\pi$. (8)

3. Find the length of the curve $\vec{r}(t) = (\sqrt{2}t)\hat{i} + (\sqrt{2}\,t)\hat{j} + (1 - t^2)\hat{k}$ from (0, 0, 1) to $(\sqrt{2}, \sqrt{2}, 0)$. (8)

4. Find the point on the curve $\vec{r}(t) = (5\sin t)\,\hat{i} + (5\cos t)\,\hat{j} + 12t\,\hat{k}$ at a distance 26π units along the curve from the origin in the direction of increasing arc length. (8)

5. Find the length of the Cardioid $r = a\,(1 + \cos\theta)$, and show that a line $\theta = \dfrac{\pi}{3}$ divides upper half of the Cardioid. (8)

6. For the limit $\lim\limits_{x \to 0} \sqrt{x+1} = 1$, find a $\delta > 0$ algebraically that works for $\varepsilon = 0.1$. (6)

7. Find $\lim\limits_{x \to (-2)^-} (x+3)\dfrac{|x+2|}{(x+2)}$. (6)

8. Find the area shared by the circle $r = 2$ and the cardioid $r = 2(1 + \cos\theta)$. (8)