BITS, PILANI – DUBAI CAMPUS
DUBAI KNOWLEDGE VILLAGE
I YEAR – I SEMESTER (2005-2006)

COMPREHENSIVE EXAMINATION
(CLOSED BOOK)

PROBABILITY AND STATISTICS

(AAOC UC 111)

Max. Marks : 40
Weightage : 40%

Date : January 03, 2006
Time : 03 hours

Instructions : i) Use separate Answer Scripts for Sections I and II.
ii) Answer all the questions.
iii) Calculators are permitted, but statistical tables or charts are not permitted in the Exam. Hall. Necessary statistical table values are given below.

Table values in usual notation :

$z_{0.0009} = 3.125$, $z_{0.05} = 1.645$, F(Z=1.5) = 0.9332, F(Z=0.43)=0.6664,
$t_{0.01} = 3.365$ for 5 degrees of freedom.

SECTION -I

1. A random sample of 100 mechanics in a city revealed a mean weekly salary of Rs.487 with a standard deviation of Rs.48. With what degree of confidence can we assert that the average weekly salary of all the mechanics in that city is between Rs.472 and Rs.502? (3)

2. An airline claims that only 6% of all lost luggage is never found. If, in a random sample, 17 of 200 pieces of lost luggage are not found, test the null hypothesis $p = 0.06$ against the alternative hypothesis $p > 0.06$ at 0.05 level of significance. Here p means the proportion of lost luggage not found. (3)

3. Suppose that India will play a 2-match one-day series against Pakistan in which Sachin and Saurav will open the innings. Assume that each of these two openers has 50% chance of scoring a century(100 runs or more) in a match. If X denotes the number of centuries to be scored by Sachin and Y denotes the number of centuries to be scored by Saurav in that series, find
   a) the joint distribution of X and Y assuming that X and Y are independent random variables;
   b) the probability that both the players will score at least one century;
   c) the probability that Sachin will fail to score a century. (2+1+1)

4. Use the following data to fit a straight line of the form $y = a + bx$ by the method of least square. Also find the correlation coefficient of x and y. (4)

x	3	5	2	8	6	9	3	4
y	86	118	49	193	164	232	73	109

5. Let X and Y are two independent variables such that $E[X] = 5$, $Var[X] = 15$, $E[Y] = 10$ and $Var[Y] = 25$. If $Var[aX + Y] = 85$, $E[aX + bY] = 60$, find the values of 'b'. (3).

6. A sample of size 6 is taken from a large population. The sample gives a mean 232.26 and a standard deviation 0.14. If this mean is used to estimate the population mean, what can one assert with 98% confidence about the maximum error? (3)

SECTION -II

7. Suppose that the four inspectors at a film factory are supposed to stamp the expiration date on each package of film at the end of the assembly line. John, who stamps 20% of the packages, fails to stamp the expiration date once in every 200 packages; Tom, who stamps 60% of the packages, fails to stamp the expiration date once in every 100 packages; Jeff, who stamps 15% of the packages, fails to stamp the expiration date once in every 90 packages; and Pat, who stamps 5% of the packages, fails to stamp the expiration date once in every 200 packages. If the customer complains that her package of film does not show the expiration date, what is the probability that it was inspected by John? (3)

8. The proportion of people who respond to a certain mail- order solicitation is a continuous random variable X that has the density function

$$f(x) = \frac{2(x+2)}{5}, 0 < x < 1$$
$$0 \quad , \text{elsewhere.}$$

(a) Show $P(0 < X < 1) = 1$
(b) Find the probability that more than 1/4 but less than 1/2 of the people contacted will respond to this type of solicitation. (2)

9. A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a

2

normal distribution and can be measured to any degree of accuracy, what is the probability that the resistors will have a resistance exceeding 43 ohms? (3)

10. As can easily be shown, the probabilities of getting 0, 1 or 2 heads with a pair of balanced coins are 1/4, 1/2 and 1/4. What is the probability of getting 2 tails twice, 1 head and 1 tail 3 times, and 2 heads once in 6 tosses of a pair of balanced coins? (3)

11. A random variable X has a mean 10 and variance 4. Using Chebychev's theorem, find
    a) $P(5 < X < 15)$
    b) The value of the constant c such that $P(|X - 10| \geq c) \leq 0.04$. (3)

12. Suppose that a system contains a certain type of component whose time in years to failure is given by T. The random variable T is modeled nicely by the exponential distribution with mean time to failure $\beta = 5$. If 5 of these components are installed in different systems, what is the probability that at least 2 are still functioning at the end of 8 years? (3)

13. In a certain city district the need for money to buy drugs is stated as the reason for 75% of all thefts. Find the probability that among the next 5 theft cases reported in this district,
    a) exactly 2 resulted from the need for money to buy drugs;
    b) at most 3 resulted from the need for money to buy drugs. (3)

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(I year – I semester 2005-06)

TEST – II (OB)

Course Title : Probability and Statistics **Course No. : AAOC UC111**

Max. Marks : 20   Weightage : 20%   Date : 11th December, 2005   Time : 50 min.

Text Book and Class Notes are allowed.
Answers of same section should be together.
Total no. of questions = 6.
Attempt all the questions.

Section –I

1. A box contains 10 cartons. Two of them contain check prizes, three of them have gift certificates, and the rest are empty. Two cartons are picked up from the box at random. If X denotes the number of cartons with check prizes drawn and Y denotes the number of cartons with gift certificates drawn, find
   a) the joint probability distribution of X and Y.
   b) the marginal distribution of X.

   2+2

2. A random variable X has Weibull distribution with parameters α and β (in standard notation). If the standard deviation of X is 10 and $\beta = \frac{1}{2}$, then find
   a) the value of α ;
   b) the expected value of X.

   2+1

3. An urn contains 4 red and 1 white balls. Two balls are drawn at random from the box. Assume that the first ball drawn will be replaced in the box before the second draw. Let X denotes the number of times a red ball is drawn. Distribute the 2-digit random numbers from 00 to 99 among the possible values of the random variable X so that they can be used to simulate the values of X.

   3

Section – II

4. If the cholesterol level of healthy men is normally distributed with a mean of 180 and a standard deviation of 20, at what level (in excess of 180) should men be diagnosed as not healthy, if you want the probability of a type one error to be 2%?

   2

5. The contents of 7 similar containers of sulphuric acid are 9.8, 10.2, 10.4, 9.8. 10.0, 10.2 and 9.6 liters.
   a) Find a 95% confidence interval for the mean of all such containers, assuming an approximate normal distribution.
   b) What can be asserted with 99% confidence about the maximum error?   4

6. In the past a production process used to produce items with a mean life of 1500 hours. It is claimed that a new process produces items with a mean life exceeding 1500 hours. 64 items were chosen at random from a lot produced by the new process and the average life of these items was found to be 1525 hours. Do the data support the claim that the new process is better at 5% level of significance when the maximum error permissible is 24.5 hours?   4

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(I year – I semester 2005-06)

QUIZ – II (CB)

Course Title : Probability & Statistics **Course No. : AAOC UC111**

Max. Marks : 10 Weightage : 10% Date :   **Time : 30 min.**

Name :   **Id. No. :**   **Sec :**

Instructions : i)Write your name , Id. No. and Section in the space provided.
       ii) All questions are compulsory.
       iv) No marks will be given for incorrect, overwritten, multiple answers.

Fill in the blanks with corrwect answers:

1. If X and Y are two identical independent binomial variates with n =10, p = 1/2, then Var[2X-5Y] is _____ . (1)

2. If the joint density function of X and Y is given by

$$f(x, y) = xy \text{ for } 0 < x < 1 \text{ and } 0 < y < 2$$
$$= 0 \text{ otherwise, then}$$

   a) $P(x < 1, y < 1)$ is _____ ;
   b) marginal density function of X is _____ . (1+1)

3. The Box-Muller-Marsaglia method is used to simulate two values of a normal variate with mean 50 and s.d. 5. If the random numbers 0.253 and 0.531 are used then the simulated values of the variate are _____ and _____ . (1)

4. If the covariance of two random variables X and Y is 2 and the variances of X and Y are 2 and 3 respectively, then the variance of 5X-3Y is _____ . (1)

5. The probability distribution function of Weibull distribution with parameters α and β is _____ . (1)

6. If the random number 0.45 is used, then the simulated value of an exponential variate with parameter β =2 is _____ . (1)

7. If two variables are independent, the covariance of the variables is _____ . (1)

Tick the correct answer :

8. The conditional density of Y given X for the joint p.d.f,
   $f(x,y) = 3 - x - y$ for $0 \leq x, y \leq 1$ is:

   a. $f(y / x) = \dfrac{3 - x - y}{\dfrac{5 - y}{2}}$

   b) $f(y / x) = \dfrac{3 - x - y}{\dfrac{5 - x}{2}}$

   c) $f(y / x) = \dfrac{\dfrac{5 - y}{2}}{\dfrac{5 - x}{2}}$

   d) none of the above   (1)

9. The probabilities that a computer software salesperson will make 0, 1, 2 or 3 sales on any one day are 0.2, 0.3, 0.4. 0.1.Distributing one digit random numbers from 0 to 9 among the four values of this random variable, so that the corresponding random numbers can be used to simulate the salesperson's sales. If the random numbers selected are 5, 7 & 2 it indicates that the sales were:

   a. 1, 2 & 0
   b. 2, 3, & 1
   c. 2, 2 & 1
   d. None of the above   (1)

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(I year – I semester 2005-06)

TEST – II (OB)

Course Title : Probability and Statistics   **Course No. : AAOC UC111**

Max. Marks : 20   Weightage : 20%   Date :   Time : 50 min.

Text Book and Class Notes are allowed.
Answers of same section should be together.
Total no. of questions = 6.
Attempt all the questions.

Section –I

1. Consider a random variable X having log-normal distribution with parameters α and β (in standard notation). It is known that mean of X is twice the standard deviation. If the standard deviation of X is 1, find the values of α and β.   (3)

2. Let X and Y be two independent binomial variates. If parameters of X are p=1/3, n=9, and the parameters of Y are p=1/5, n=25, find

    a) $E[6X - 2Y]$;
    b) $Var[6X - 2Y]$.   (3)

3. The joint probability distribution of two discrete random variables X and Y is given by $f(x,y) = c(2x + y)$ for $x = 0, 1, 2$; $y = 0, 1, 2, 3$ and $f(x,y) = 0$ otherwise.

    a) Find the value of c.
    b) Find $P(X \geq 1, Y \leq 2)$.   (4)

Section –II

4. A manufacturer of car batteries guarantees that his batteries will last, on the average, 3 years with a standard deviation of 1 year. If five of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5 and 4.2 years, is the manufacturer still convinced that his batteries have a standard deviation of 1 year? If 95% of the suitable test statistic values fall between 0.484 and 11.143 then what should the manufacturer infer? Assume that the battery lifetime follows a normal distribution.

5. Consider the following measurements of the heat producing capacity of the coal produced by two mines(in millions of calories per ton):

Mine 1: 8260	8130	8350	8070	8340	
Mine 2: 7950	7890	7900	8140	7920	7840

Can it be concluded that the two population variances are equal?

6. X_1, X_2 and X_3 is a random sample of size 3 from a population with mean value μ and standard deviation σ. T_1 and T_2 are the estimators used to estimate mean where,

$$T_1 = 4X_1 - X_2 - 2X_3 \qquad \& \qquad T_2 = \frac{X_1 + X_2 + X_3}{3}$$

   a) Are T_1 & T_2 unbiased estimators?
   b) Which is the best estimator?

A

QUIZ – II (CB)

PROBABILITY AND STATISTICS
AAOC UC111

Max. Marks : 10   Date : 29-11-2005
Weightage : 10%   Time : 30 Minutes

Name :   Id. No. :   Section :

Note : i) *Write your Name, Id. No.(full) and Section in the space provided.*
     ii) *Answer all the questions.*
     iii) *Overwritten and multiple answers will be treated as incorrect answer.*

Tick the correct answer:

Q1. The distribution of the random variable X is :

X	-2	3	1
f(x)	1/3	1/2	1/6

Then the mean and the variance of $2X+5$ are respectively
    a) 7, 20   b) 7, 15   c) 5, 20   d) None of these   [1]

Q2. The number of different samples of size 2 that can be drawn from a population of size 6 is
    a) 12   b) 20   c) 15   d) None of these   [1]

Q3. If (in usual notation) $f(1,1) = 0.2$ and $f_2(1) = 0.4$, then $f_1(1/1)$ is
    a) 0.5   b) 0.6   c) 0.2   d) None of these   [1]

Q4. Given the joint probability mass function of X and Y be
       $f(x,y) = \dfrac{x + y}{21}$; x = 1, 2, 3;   y = 1, 2
    The P (x = 3) is equal to:
    a) 3/ 7   b) 1/ 9   c) 4 /9   d) 4 /7   [1]

Q5. Let (X,Y) be jointly distributed with density function,

$$f(x,y) = e^{-x-y} \quad ; 0 < x < \infty, \ 0 < y < \infty$$
$$= 0 \ ; \text{ elsewhere}$$

Then

(i) X and Y are:independent

(ii) $f(x) = e^{-x}$

(iii) $f(x/y) = e^{-x}$

(iv) $f(y) = e^{-x}$

Which of the following options is true?

  a) (i), (ii) & (iii)   b) (i), (iii) & (iv)   c) (i) & (ii)   d) none of these   [1]

Q6. If X_1 has mean 2 and variance 4 while X_2 has mean -2 and variance 5 and the two are independent, then Var($2X_1 - X_2 + 5$) is

  a) 8   b) 16   c) 21   d) None of these   [1]

Q7. Simulating the observations 0.77 and 0.12 of a random variable having the Weibull distribution with $\alpha = 0.05$ and $\beta = 2$, the simulated values are:

  a) 2.454 & 1.599
  b) 5.422 & 1.599
  c) 5.422 & 2.459
  d) None of the above   [1]

Q8. Joint cumulative distribution function F(x, y) lies within the limits:

  a) -1 & 1   b) -1 & 0   c) $-\infty$ & ∞ d) 0 & 1   [1]

Fill in the blanks :

Q9.The joint distribution of two discrete random variables X_1 and X_2 is :

		X_1	
	0	1	2
X_2 0	0.2	0.2	k
1	0.1	0.4	0.1

i) The value of k is _____.

ii) The probability that $X_1 + X_2 \leq 1$ is _____.   [1+1]

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(I year – I semester 2005-06)

TEST – I (CB)

Course Title : Probability and Statistics   Course No. : AAOC UC111

Max. Marks : 20   Weightage : 20%   Date :   Time : 50 min.

NOTE

(i)   Attempt all the questions.
(ii)   Attempt each question on a separate page.

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1.   Suppose that a study of a certain computer system reveals that the response time in seconds has an exponential distribution with a mean of 3 seconds. What is the probability that response time exceeds 5 seconds?   [3]

2.   The complexity of arrivals and departures into an airport are such that certain computer simulation is often used to model the ideal conditions. For a certain airport containing three runways it is known that in the ideal setting the following are the probabilities that the individual runways are accessed by a randomly arriving commercial jet are 2/9, 1/6 and 11/18 respectively. What is the probability that 6 randomly arriving airplanes are distributed as 2 arriving on first runway, 1 arriving on second runway & 3 arriving on the third runway?   [3]

3.   The probability that a patient recovers from a rare blood disease is 0.4. If 100 people are known to have contracted this disease, what is the probability that less than 30 survive?   [4]
        TABLE VALUES:

        As per the standard notation of F(z),
        F(-2.04) = 0.0207   F(2.14) = 0.9838

4.   In a group of 160 graduate students, 92 are enrolled in Statistics, 63 are enrolled in Operations Research, and 40 are enrolled in both. How many of these students are not enrolled in either course?   [2]

5.   In a winner-take-all tournament among four professional tennis players, the prize money is Rs.500,000. Leander, one of the tennis players, figures his probability of winning is 0.20.
        a) What is Leander's mathematical expectation?   [1]
        b) Would he be better off if he made a secret agreement with the other tennis players to divide the prize money evenly regardless of who wins?   [2]

1

6. In a certain state, the proportion of highway sections requiring repairs in any given year is a random variable having the beta distribution with $\alpha = 3$ and $\beta = 2$.

        a) On the average what percentage of the highway sections require repairs in any given year? [1]

        b) Find the probability that at most half of the highway sections will require repairs in any given year. [2]

7. An expert shot hits a target 95% of the time. What is the probability that the expert will miss the target for the first time in the 15th shot? [2]

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(I year – I semester 2005-2006)

TEST – I (CB)

Course Title : Probability and Statistics   Course No. : AAOC UC111

Max. Marks : 20   Weightage : 20%   Date : 23 October, 2005   Time : 50 min.

NOTE

(i)   Attempt all the questions.
(ii)   Attempt each question on a separate page.

SECTION - A

1. A shipment of 120 burglar alarms contains 5 that are defective. If 3 of these alarms are randomly selected and shipped to a customer, find the probability that the customer will get one defective unit by using
    a) hypergeometric distribution;   [2]
    b) binomial approximation to hypergeometric distribution.   [2]

2. Find the probability of getting eight 'Heads' in a row with a balanced coin.   [2]

3. Two firms V and W consider bidding on a road building job, which may or may not be awarded depending on the amounts of bids. Firm V submits a bid and the probability is $\frac{3}{4}$ that it will get the job provided firm W does not bid.

    The probability is $\frac{3}{4}$ that W will bid, and if it does, the probability that V will get the job is $\frac{1}{3}$.

    a) What is the probability that V will get the job?   [2]
    b) If V gets the job, what is the probability that W did not bid?   [2]

SECTION – B

4. A bus arrives every 10 minutes at a bus stop between 6.00am to 9.00 pm. It is assumed that the waiting time for a particular individual is a random variable with a uniform distribution.
    a) What is the probability that the individual waits more than 7 minutes? [2]
    b) What is the average number of individuals arriving at the bus stop?   [2]

5. The arrival of customers at a certain automobile service facility follows Poisson process with a mean arrival rate of 3 per hour. Find the probability that exactly 3 customers arrive in one hour period and exactly 2 customers arrive in next one hour period. [3]

6. An electrical firm manufactures a 100-watt light bulb, which, according to specifications written on the package has a mean life of 900 hours with a standard deviation of 50 hours. At most what percentage of bulbs fail to last even 700 hours? Assume the distribution is symmetric about the mean. [3]

A

Course Title : Probability & Statistics   Course No. : AAOC UC111

Max. Marks : 10   Weightage : 10%   Date : 04th October, 2005   Time : 30 min.

Name :   Id. No. :   Sec :

Instructions : i)Write your name , Id. No. and Section in the space provided.
ii) All questions are compulsory.
iii) Each question carries 1 mark.
iv) No marks will be given for incorrect, overwritten, multiple answers.

1. Consider a random experiment of throwing a balanced die. If X denotes the number of throws up to and including that throw in which '4' comes up for the first time, then the name of the distribution of X is _____ and its mean is _____ .

2. If a bank receives on the average 0.2 incorrect drafts per day, the probability that 1 incorrect draft will be received by the bank in a 3-day period is _____ .

3. The variance of the hypergeometric distribution with parameters n, a and N is

_____ .

4. In a lottery there are 20 prizes of Rs.200 and 10 prizes of Rs.500. If 1000 ticket are sold, the mathematical expectation of a person who is given one of the tickets is _____ .

5. If the mean and variance of a binomial distribution with parameters n and p are 5 and 2.5 respectively, then n = _____ and p = _____ .

6. Sixty percent of the employees of the ABC Corporation are college graduates. Of these, 10% are in sales. Of the employees who did not graduate from the college, 80% are in sales. The probability that an employee selected at random is in sales is _____

1

7. The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable X that has the density function

$$f(x) = \begin{cases} x & ; 0 < x < 1 \\ 2 - x & ; 1 \leq x < 2 \\ 0 & ; \text{elsewhere} \end{cases}$$

The probability that over a period of one year, a family runs their vacuum cleaner between 50 and 150 hours is

(a) 0.375 (b) 0.25 (c) 0.75 (d) 0.625

8. A random sample of 15 people is taken from a population in which 40% favour a particular political stand. The probability that exactly 6 individuals in the sample do not favour this political stand is _____ .

9. The standard deviation of the probability distribution given by

$$f(x) = 1/3, \quad \text{for } x = 0, 1, 2$$

is _____

10. There will be three candidates for the position of principal- A, B & C- whose chances of getting the appointment are in the proportion 4: 2: 3 respectively. The probability that A if selected would introduce co-education in the college is 0.3. The probabilities that B & C doing the same are respectively 0.5 & 0.8. If there is co-education in the college after the appointment of the principal then the probability that C is the principal is

(a) 23/45 (b) 12/23 (c) 10/23 (d) 12/45

B

Name :   Id. No. :   Sec :

Instructions : i)Write your name , Id. No. and Section in the space provided.
            ii) All questions are compulsory.
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2. If the mean and variance of a binomial distribution with parameters n and p are 5 and 2.5 respectively, then n = _____ and p = _____ .

3. The standard deviation of the probability distribution given by
    $f(x) = 1/3$,   for x = 0, 1, 2
    is _____

4. There will be three candidates for the position of principal- A, B & C- whose chances of getting the appointment are in the proportion 4: 2: 3 respectively. The probability that A if selected would introduce co-education in the college is 0.3. The probabilities that B & C doing the same are respectively 0.5 & 0.8. If there is co-education in the college after the appointment of the principal then the probability that C is the principal is
    (a) 23/45   (b) 12/23   (c) 10/23   (d) 12/45

5. Consider a random experiment of throwing a balanced die. If X denotes the number of throws up to and including that throw in which '4' comes up for the first time, then the name of the distribution of X is _____ and its mean is _____ .

6. If a bank receives on the average 0.2 incorrect drafts per day, the probability that 1 incorrect draft will be received by the bank in a 3-day period is _____ .

7. Sixty percent of the employees of the ABC Corporation are college graduates. Of these, 10% are in sales. Of the employees who did not graduate from the college, 80% are in sales. The probability that an employee selected at random is in sales is _____

8. The variance of the hypergeometric distribution with parameters n, a and N is

_____ .

9. The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable X that has the density function

$$f(x) = \begin{cases} x & ; 0 < x < 1 \\ 2 - x & ; 1 \leq x < 2 \\ 0 & ; \text{elsewhere} \end{cases}$$

The probability that over a period of one year, a family runs their vacuum cleaner between 50 and 150 hours is

(a) 0.375   (b) 0.25   (c) 0.75   (d) 0.625

10. A random sample of 15 people is taken from a population in which 40% favour a particular political stand. The probability that exactly 6 individuals in the sample do not favour this political stand is _____ .