BITS, PILANI-DUBAI CAMPUS

Knowledge Village, DUBAI

First Year - Semester-I (2005-06)

MATHEMATICS-I (MATH UC191)

TEST – II (Open Book)

Date: 05.12.2005
Time: 50 minutes

Max. Marks: 20
Weightage: 20 %

Answer all the questions.

Text Book and Notes can be used.

1. Check whether $u(x,y) = e^x\left[(x^2 - y^2)\cos y - 2xy\sin y\right]$ satisfies the equation $u_{xx} + u_{yy} = 0$. (3)

2. Find the derivative of $f(x,y,z) = 2x^3 - \tan^{-1}\left(\frac{y}{x}\right) + e^{yz}$ in the direction of $\vec{A} = \vec{i} + 2\vec{j} + 3\vec{k}$ at $P_0(1,1,2)$. (4)

3. Find the point on the plane $x + y + z = 3$ nearest to the origin. (3)

4. Evaluate $\iint \sqrt{\frac{1 - x^2 - y^2}{1 + x^2 + y^2}}\,dxdy$ over the positive quadrant of the circle $x^2 + y^2 = 1$ (4)

5. Use the transformation $u = x + 2y,\ v = x - y$ to evaluate the integral $\int_0^{\frac{2}{3}} \int_y^{2-2y} (x + 2y)e^{(y-x)}\,dxdy$ (4)

6. Integrate $f(x,y,z) = \sqrt{1 + 30x^2 + 10y}$ over the path $\vec{r}(t) = t\vec{i} + t^2\vec{j} + 3t^2\vec{k},\quad 0 \le t \le 2$. (2)

BITS, PILANI-DUBAI CAMPUS
Knowledge Village, DUBAI
First Year - Semester-I (2005-06)
MATHEMATICS-I (MATH UC191)

TEST – II (OPEN BOOK) – MAKE MAKE UP

Date: 22.12.2005

Time: 50 minutes

Weightage: 20 %

Answer all the questions.

Text book and notes can be used.

1. Find the directional derivative of ϕ where $\vec{F} = \nabla\phi = 2xy^2z^3\,\hat{i} + 2x^2yz^3\,\hat{j} + 3x^2y^2z^2\,\hat{k}$ in the direction of the normal to the surface $xy + yz + zx = 3$ at $(1,1,1)$. (3).

2. Find the dimensions of a rectangular solid of maximum volume that can be inscribed in a sphere. (4).

3. Find the linearisation of $x^2+y^2+z^2$ at $(1,1,-1)$. (3).

4. Evaluate $\iint_R \sqrt{4-x^2-y^2}\,dx\,dy$ over the region bounded by the semi-circle $x^2+y^2-2x=0$ lying in the first quadrant. (4).

5. Evaluate $\iint_R \sqrt{4x^2-y^2}\,dx\,dy$ where R is the region bounded by the lines $y=0$, $y=x$ and $x=1$. (4).

6. Evaluate $\int_C (x^2\,dx + x\,dy)$ along the curve $y=x^2$ from $(-1,1)$ to $(2,4)$. (2).

BITS, PILANI-DUBAI CAMPUS
Knowledge village, DUBAI

First Year - Semester-I (2005-06)

Mathematics-I (MATH UC191)

Comprehensive Examination (Closed Book)

Date: 02.01.2006 | **Max. Marks: 40**
Time: 3 hours | **Weightage: 40 %**

Answer all the questions.

Answer Part A and Part B in separate Answer Books.

PART-A

1. Find the area of the region lying inside the lemniscates $r^2 = 6\cos 2\theta$ and outside the circle $r = \sqrt{3}$. (3)

2. Find an open interval about x_0 on which the inequality $|f(x) - L| < \varepsilon$ holds. Then find a $\delta > 0$ such that for all x satisfying $|x - x_0| < \delta$, the inequality $|f(x) - L| < \varepsilon$ holds: $x_0=10$, $\varepsilon=1$, $f(x) = \sqrt{19 - x}$, $L = 3$. (2)

3. Find the length of the curve $\vec{r}(t) = \sqrt{2}t\vec{i} + \sqrt{2}t\vec{j} + (1 - t^2)\vec{k}$, from the point $(0,0,1)$ to the point $(\sqrt{2}, \sqrt{2}, 0)$. (3)

4. Find $\vec{T}$, $\vec{N}$ *and* κ for the curve $\vec{r}(t) = (e^t \cos t)\vec{i} + (e^t \sin t)\vec{j} + 2\vec{k}$. (4)

5. Find the derivative of $f(x,y,z) = \tan^{-1}\left(\frac{y}{x}\right) + \sqrt{3}\sin^{-1}\left(\frac{xy}{2}\right) + z^3$ in the direction of $\vec{A} = 3\vec{i} - 2\vec{j} + \vec{k}$ at $P_0(1,1,1)$. (4)

6. Find the local extrema of $f(x,y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$. (4)

(P.T.O)

PART-B

1. Investigate the convergence of the following series:

(a) $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$ (b) $\sum_{n=1}^{\infty} \frac{n^p}{(n+1)^q}$ (2+2)

2. Evaluate the integral

$$\int_0^{2\pi} \int_0^{\theta/2\pi} \int_0^{3+24r^2} dz r dr d\theta.$$ (2)

3. Use the transformation $u = 3x + 2y,\ v = x + 4y$ to evaluate the integral

$\iint_R (3x^2 + 14xy + 8y^2) dxdy$ where R is the region bounded by the lines

$y = -\frac{3}{2}x + 1,\ y = -\frac{3}{2}x + 3,\ y = -\frac{1}{4}x$ and $y = -\frac{1}{4}x + 1.$ (4)

4. Evaluate $\int_C (xy + y + z) dS$ along the curve $\vec{r} = 2t\hat{i} + t\hat{j} + (2 - 2t)\hat{k};\ 0 \le t \le 1.$ (3)

5. Verify Green's theorem for $\oint_C (y^2 dx + x^2 dy)$ where C is the triangle bounded by $x = 0,\ x + y = 1,\ y = 0.$ (4)

6. Show that $\vec{F} = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$ is a conservative field. Also find the scalar potential and the work done by $\vec{F}$ along any smooth curve C joining the point (1,1,2) to (3,5,0) (3)

BITS-PILANI, DUBAI CAMPUS

MATHEMATICS-I (MATHUC 191)

A

I- Year-Semester-I (2005-06)

QUIZ – II (CLOSED BOOK)

TIME: 30 Minutes	November 15, 2005	**Max. Marks:10**
ID No.	**Section No.:**	**Name:**

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1. The domain and range of $f(x,y) = \sin^{-1}(y-x)$ are ------------------------ and --------- ----------------.

2. Does $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y}{x^2-y}$ exist? Ans. ------------------------

3. If $f(x,y) = -2x(x^2+y^2)^{-2}$, then f_x= ------------------------.

4. The linearization of $f(x,y,z) = x^3z + y^2x + e^{x-2y+3z}$ at (1, 1, 0) is $L(x,y,z) =$ ------------------------.

5. If $u = \dfrac{p-q}{q-r}$, $p = x+y+z, q = x-y+z, r = x+y-z$, then at $(\sqrt{3}, 2, 1)$, $\dfrac{\partial u}{\partial y}$ = ------------------------.

6. The point where $f(x,y) = 2x^2 + 3xy + 4y^2 - 5x + 2y$ has a local minimum is --------- ----------------.

7. The point on the plane $x + 2y + 3z = 13$ closest to the point (1, 1, 1) is ------------------ ------.

8. The equation for the tangent plane at the point (1, 1, 1) on the surface $x^2 + y^2 + z^2 = 3$ is --------------------------.

9. The derivative of the $xy + yz + zx$ at (1, -1, 2) in the direction of $\vec{A} = 3\vec{i} + 6\vec{j} - 2\vec{k}$ is --------------------------.

10. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then $\nabla\left(\frac{1}{r}\right) =$ --------------------------.

BITS-PILANI, DUBAI CAMPUS

MATHEMATICS-I (MATHUC 191)

B

I- Year-Semester-I (2005-06)

QUIZ – II (CLOSED BOOK)

TIME: 30 Minutes   November 15, 2005   **Max. Marks:10**

ID No.   **Section No.:**   **Name:**

1. The derivative of the $xy + yz + zx$ at (1, -1, 2) in the direction of $\vec{A} = 3\vec{i} + 6\vec{j} - 2\vec{k}$ is ------------------------.

2. If $u = \frac{p-q}{q-r}$, $p = x + y + z, q = x - y + z, r = x + y - z$, then at $(\sqrt{3}, 2, 1)$, $\frac{\partial u}{\partial y}$ = ------------------------.

3. The equation for the tangent plane at the point (1, 1, 1) on the surface $x^2 + y^2 + z^2 = 3$ is ------------------------.

4. If $f(x, y) = -2x(x^2 + y^2)^{-2}$, then f_x = ------------------------.

5. The point on the plane $x + 2y + 3z = 13$ closest to the point (1, 1, 1) is ------------------------.

6. Does $\lim_{(x,y)\to(0,0)} \frac{x^2 + y}{x^2 - y}$ exist? Ans. ------------------------

7. The linearization of $f(x,y,z) = x^3 z + y^2 x + e^{x-2y+3z}$ at $(1, 1, 0)$ is
$L(x,y,z) =$ --------------------------.

8. If $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$, then $\nabla\left(\frac{1}{r}\right) =$--------------------------.

9. The domain and range of $f(x,y) = \sin^{-1}(y-x)$ are ------------------------- and ---------
-----------------.

10. The point where $f(x,y) = 2x^2 + 3xy + 4y^2 - 5x + 2y$ has a local minimum is --------
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BITS, PILANI-DUBAI CAMPUS
Knowledge Village, DUBAI

First Year - Semester-I (2005-06)

MATHEMATICS-I (MATH UC191)

TEST – I (Closed Book)

Date: 09.10.2005
Time: 50 minutes

Max. Marks: 20
Weightage: 20 %

Answer all the questions.

1. Find $\delta > 0$ such that for all x satisfying $0 < |x - x_0| < \delta$, the inequality $|f(x) - L| < \varepsilon$ holds when $f(x) = e^{2x}$, $L = e$, $x_0 = 0.5$, $\varepsilon = 0.1$ (2)

2. Sketch the sets of points whose polar coordinates satisfy the inequality

(a) $-3\cos\theta \le r \le 2\cos\theta$ (b) $-\left(\frac{3\pi}{4}\right) \le \theta \le -\left(\frac{2\pi}{3}\right)$, $r \le -2$ (2)

3. Find the area lying inside the lemniscate $r^2 = 6\cos(2\theta)$ and outside the circle $r = \sqrt{3}$. (3)

4. Find the length of the parabolic segment $r = \dfrac{6}{1+\cos\theta}$, $0 \le \theta \le \dfrac{\pi}{2}$. (3)

5. Show that the curvature of a straight line is zero. (2)

6. Find the particle's velocity and acceleration vectors at the stated times and sketch them as vectors on the cycloid, given $\vec{r}(t) = (t - \sin t)\vec{i} + (1 - \cos t)\vec{j}$, $t = \pi$ *and* $\dfrac{3\pi}{2}$. (2)

7. Find $\vec{T}$, $\vec{N}$ *and* κ for the curve $\vec{r}(t) = (a\cos t)\vec{i} + (a\sin t)\vec{j} + bt\,\vec{k} : a, b \ge 0,\ a^2 + b^2 \ne 0$. (2)

8. Find $\vec{T}$ *and* $\vec{N}$ for the curve $\vec{r}(t) = (\cos t + t\sin t)\vec{i} + (\sin t - t\cos t)\vec{j} + 3\vec{k}$. (2)

9. Solve the equation $\dfrac{d\vec{r}}{dt} = (t^3 + 4t)\vec{i} + t\vec{j} + 2t^2\vec{k}$, $\vec{r}(0) = \vec{i} + \vec{j}$. (2)

BITS-PILANI, DUBAI CAMPUS,
DUBAI
FIRST YEAR - I SEMESTER
MATHEMATICS-I (MATH UC 191)
QUIZ-I (MAKE-UP)

TIME : 30 MINUTES

MAX. MARKS : 10

NAME :

ID NO :

Note : 1. Write your Name, ID No
2. Overwriting may be considered as wrong answer.

1. All the polar co-ordinates of the point $(-2, \pi/6)$ are

ⓐ $(-2,\ 2n\pi+\pi/6),\ (2,\ 2n\pi-5\pi/6)$

ⓑ $(-2,\ 2n\pi+\pi/6),\ (2,\ 2n\pi+5\pi/6)$

ⓒ $(-2,\ n\pi+\pi/6),\ (2,\ 2n\pi-5\pi/6)$

ⓓ None of the above.

(where n is an integer.

2. Which of the following label the point $(2, -\pi/3)$.

ⓐ $(2, -2\pi/3)$ ⓑ $(-2, \pi/3)$ ⓒ $(-2, 2\pi/3)$

ⓓ $(2, 2\pi/3)$

3. The curve $r = 1 + 2\sin\theta$ is symmetric

ⓐ about the x-axis ⓑ about the y-axis

ⓒ about the origin ⓓ None of the above.

4. The cartesian equation of the line $r\cos(\theta - \pi/3) = 4$ is

ⓐ $x - \sqrt{3}y = 8$ ⓑ $-x + \sqrt{3}y = 8$ ⓒ $x + \sqrt{3}y = 4$

ⓓ $x + \sqrt{3}y = 8$

(5) The center and radius of the circle $r = -2\sin\theta$ are

(a) $(1, \pi/2)$, 1
(b) $(1, -\pi/2)$, 1
(c) $(-1, \pi/2)$, 1
(d) None of the above.

(6) The directrix and the eccentricity of $r = \dfrac{4}{3+2\sin\theta}$ are

(a) $y = 2$, $e = 2/3$
(b) $y = -2$, $e = 2/3$
(c) $y = -2$, $e = 1/3$
(d) $y = 2$, $e = 1/3$.

(7) For what values of x, the function

$$f(x) = \frac{x+1}{x^2-4x+3}$$ is continuous?

(a) All x except $x = 1, 3$
(b) All x except $x = 0, 1$
(c) All x except $x = 0, 3$
(d) All x.

(8) For what $\delta > 0$, the inequality

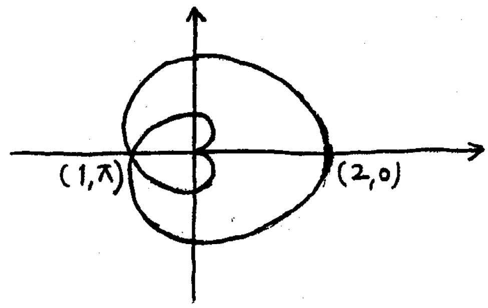
$$0 < |x - \tfrac{1}{2}| < \delta \Rightarrow \frac{4}{9} < x < \frac{4}{7} \quad \text{for all } x.$$

(a) $\delta = 0$
(b) $\delta = \frac{1}{9}$
(c) $\delta = \frac{1}{18}$
(d) $\delta = \infty$.

(9) The formula $S = \int_\alpha^\beta 2\pi r\cos\theta\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ is for

(a) area of surface of revolution about x-axis.
(b) area of surface of revolution about y-axis
(c) area of surface of revolution about the origin.
(d) None of the above.

(10). The polar equation of the curve



is

(a) $r = 1 + \cos\frac{\theta}{2}$,
(b) $r^2 = \sin 2\theta$,
(c) $r = 1 - \cos\theta$,
(d) $r = 2\cos\theta$.