BITS , PILANI – DUBAI CAMPUS
KNOWLEDGE VILLAGE

TEST – I (CB)
COURSE NO: AAOC UC 111
COURSE TITLE: PROBABILITY & STATISTICS
DATE: 17.10.04
TIME: 50 minutes
MAX. MARKS: 20
WEIGHTAGE : 20%

NOTE: 1. Each question should be attempted on a separate page.
2. Questions of each section must be attempted together.

SECTION - A

1. Check whether the following can define probability distribution and explain your answer:

$$f(x) = \frac{1}{2} \qquad \text{for } x = -1, 0.$$

(2 marks)

2. Customers enter a hyper market randomly at an average rate of 240 per hour. What is the probability that during:
   a) one- minute interval no one will enter
   b) two- minute interval two customers will enter. (4 marks)

3. The incidence of occupational disease in an industry is such that the workers have a 30% chance of suffering from it. What is the probability that out of six workers chosen at random, at most 2 will suffer from the disease ? (4 marks)

SECTION - B

4. The manufacturer of a new battery additive has to decide whether to sell her product of $ 1.50 a can or for $2.00 with a "money-back-if-not-satisfied" guarantee. How does she feel about the chances that a person will ask for double his or her money back if
(a) she decides to sell the product for $ 1.50
(b) she decides to sell the product for $ 2.00 with the guarantee
(c) she can not make up her mind ? (3 marks)

5. Engineers in charge of maintaining our nuclear fleet must continually check for corrosion inside the pipes that are part of the cooling systems. The inside condition of the pipes cannot be observed directly but a nondestructive test can give an indication of possible corrosion. The test is not infallible. The test has probability 0.8 of detecting corrosion when it present but it also has probability 0.2 of falsely indicating internal corrosion. Suppose the probability that any section of pipe has internal corrosion is 0.15

   (a)Determine the probability that a section of pipe has internal corrosion given that the test indicates the presence.

   (b)Determine the probability that that a section of pipe has corrosion given that the test is negative. (4 marks)

6. The probability that a new airport will get an award for its design is 0.06, the probability it will get an award for the efficient use of materials is 0.24 and the probability that it will get both awards is 0.11. What is the probability that it will get at least one of the two awards and what is the probability that it will get only one of the two awards ? (1.5 + 1.5= 3 marks)

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PSTAT03

BITS, PILANI – DUBAI CAMPUS
KNOWLEDGE VILLAGE, DUBAI, U.A.E

AAOC UC111

PROBABILITY AND STATISTICS

QUIZ 1 – CLOSED BOOK

27th October 2004

9.30 AM -10.30 AM

Note : (1) Mark the correct option in the answer sheet provided to you along with the question paper
(2) Check whether the question paper code is same as answer paper code
(3) More than one option will be treated as wrong answer

1. A box contains 16 marbles of 4 colours. There are equal number of marbles in each colour. 4 marbles are drawn one by one with replacement. The probability of getting one marble of each colour is

(1) 1/256 (2) 3/32 (3) 3/8 (4) 5/8

2. If the distribution function of a continuous random variable X is given by $F(x) = \frac{1}{\pi}\left[\frac{\pi}{2} + \tan^{-1} x\right], \forall x \in R$, then $P(1 < X < \sqrt{3})$ is

(1) 1/4 (2) 1/24 (3) 1/12 (4) 1/6

3. A fast bowler has the chance of getting a wicket in every 10 balls. The probability of the bowler getting his first wicket in the 10^{th} ball is

(1) $\frac{9^9}{10^{10}}$ (2) $\frac{9^{10}}{10^{10}}$ (3) $\frac{9^9}{10^9}$ (4) $\frac{8^9}{10^{10}}$

4. The mean of the random variable X with probability density function

$$f(x)=\begin{cases} e^{-x}, x>0 \\ 0, x\le 0 \end{cases}$$

(1) 2 (2) 0 (3) 1 (4) 3

5. The value of k for which the function

$$f(x)=\begin{cases} kx, 0<x<\frac{1}{2} \\ 1-x, \frac{1}{2}<x<1 \end{cases}$$

is (1) 9 (2) 5 (3) 3 (4) 7

6. If $f(x)=\begin{cases} \frac{1}{2}, 0<x<2 \\ 0, otherwise \end{cases}$

then P(X>3) is

(1) 1 (2) infinity (3) 0 (4) 1/2

7. The mean and standard deviation of a random variable X are 0 and 2 respectively. Then $P(|X|\geq 4)$ is
(1) at most ¼ (2) atleast ¼ (3) at most ¾ (4) atleast ¾

8. X has normal distribution with mean 2 and standard deviation 4. The value of standard normal variate corresponding to X=2 is

(1) 1/2 (2) 0 (3) 1 (4) 2

9. If X is a continuous random variable with mean 4 and variance 2 then the value of $\int_{-\infty}^{\infty} x^2 f(x)dx$ is

(1) 18 (2) 6 (3) 4 (4) 14

10. X has normal distribution with mean 45 and some standard deviation, If P(X>60) =0.25 then P(X<30) is
(1) 0.125 (2) 0.25 (3) 0.75 (4) -0.25

BITS, PILANI – DUBAI CAMPUS
KNOWLEDGE VILLAGE, DUBAI, U.A.E

AACC UC111

PROBABILITY AND STATISTICS

TEST-2 – OPEN BOOK

25th November 2004 **MAX MARKS :20**

4.00 PM – 4.50 PM

CARE MUST BE TAKEN TO WRITE ALL ANSWERS OF EACH SECTION TOGETHER IN THE ANSWER BOOK.

NO RECHECKING WILL BE ENTERTAINED IF ANSWERS ARE GIVEN IN A SCATTERED MANNER

SECTION 1

1. Let X be a discrete random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{5}e^{-\frac{x}{5}}, x > 0 \\ 0, otherwise \end{cases}$$

(a) Find the mean and variance of the above distribution (1+1)

(b) Using Chebysev's inequality obtain an upper bound for $P[|X-\mu| \geq 2\sigma]$ (1)

(c) Compute the actual value of $P[|X-\mu| \geq 2\sigma]$ $(1\frac{1}{2})$

(d) Find the difference between the actual value obtained in (c) and the bound obtained in (b) $(\frac{1}{2})$

2. Find the distribution function of the random variable X with probability density function

$$f(x) = \begin{cases} \frac{1}{2\pi}\frac{1}{1+x^2}, x < 0 \\ \frac{1}{2}e^{-x}, x > 0 \end{cases} \qquad (3)$$

Also compute P[-1<X<1] **using distribution function.** (2)

[should not use the denstity function to compute the probability]

SECTION 2

3. A sample of 100 items is taken at random from a batch known to contain 40% defectives. What is the probability that the sample contains
   (i) at least 44 defectives
   (ii) exactly 44 defectives. (4)

4. In the inspection of tin plate produced by a continuous electrolytic process, the time taken for spotting two successive imperfection is exponentially distributed with $\beta = 3$. Find the percentage of the time that the interval between spotting imperfections is
   a. less than 2 minutes
   b. at least 4 minutes. (3)

5. Metro trains on a certain line run every half hour between midnight and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least twenty minutes? (3)

BITS, PILANI – DUBAI CAMPUS
KNOWLEDGE VILLAGE, DUBAI, U.A.E

AAOC UC111

PROBABILITY AND STATISTICS

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BITS, PILANI – DUBAI CAMPUS
KNOWLEDGE VILLAGE, DUBAI, U.A.E
COMPREHENSIVE EXAMINATION, JANUARY,2005
AAOC UC111 PROBABILITY AND STATISTICS
CLOSED BOOK

11th January 5, 2005 **MAX:40**

10.00 AM – 1.00 PM **WEIGHTAGE:40**

ANSWERS FOR EACH SECTION SHOULD BE GIVEN IN SEPARATE ANSWER BOOKS. ANSWERS WRITTEN IN A MIXED FORM ARE LIABLE TO BE IGNORED. NO TABLES ARE PERMITTED.

Section A

ALL ANSWERS PERTAINING TO THIS SECTION SHOULD BE ANSWERED IN A SEPARATE ANSWER BOOK. WRITE ON THE COVER PAGE IN BOLD LETTER THE ALPHABET A

1. If two random variables have the joint density

$$f(x_1,x_2)=\begin{cases} x_1x_2, 0<x_1<1, 0<x_2<2 \\ 0, otherwise \end{cases}$$

a) Find the probability that the sum of the values taken on by the two random variables will be less than 1. (3 marks)
b) Find the joint distribution function of the two random variables for all points in the two dimensional real space .(2 marks)

2. If the annual proportion of erroneous income tax returns filed with the IRS can be looked upon as a random variable having a beta distribution with $\alpha = 1$ and $\beta = 7$, what is the probability that in any given year there will be fewer than 10% erroneous returns? (3 marks)

3. The arrival of ships at the receiving dock is a Poisson process with a mean arrival rate of 1 in 3 hours. Find the probability that exactly one ship arrives in 3 hour period and exactly 2 ships arrive in the next 3 hour period. (3 marks)

4. Show that for 1,000,000 flips of a balanced coin the probability is at least 0.99 that the proportion of heads will fall between 0.495 and 0.505. (3 marks)

5. A machine produces 10 percent defective items. Ten items are selected at random. Find the probability of not more than 2 items being defective. (2 marks)

6. (i) A man rolls a fair die until he obtains a 5 or 6 on the face of a die. What is the probability that he will require 5 throws. (2 marks)
(ii) If a random variable has the standard normal distribution, find the probability that it will take on a value greater than 0..85 (2 marks)

Section B

ALL ANSWERS PERTAINING TO THIS SECTION SHOULD BE ANSWERED IN A SEPARATE ANSWER BOOK. WRITE ON THE COVER PAGE IN BOLD LETTER THE ALPHABET B

7. A random sample from a company's very extensive files shows that orders for a certain piece of machinery were filled, respectively, in 10, 12,19,14,15,18, 11 and 13 days. Use the level of significance 0.01 to test the claim that on the average such orders are filled in 10.5 days. Choose the alternative hypothesis so that rejection of the null hypothesis $\mu = 10.5$ implies that it takes longer than indicated. Assume normality.
(4 1/2)

8. In a study designed to investigate whether certain detonators used with explosives in coal mining meet the requirement that at least 90% will ignite then explosive when charged, it is found that 174 of 200 detonators function properly. Test the null hypothesis P=0.90 against the alternative P<0.90 at the 0.05 level of significance
(4 1/2)

9. The following table shows how many weeks a sample of 6 persons have worked at an automobile inspection station and the number of cars each one inspected between noon and 2 P.M. on a given day.

Number of weeks employed x	Number of cars inspected y
2	13
7	21
8	23
1	14
5	15
12	21

(a) Find the equation of the least squares line which will enable us to predict y in terms of x (3)

(b) Estimate (using the result of part(a)) how many cars someone who has been working at the inspection station station for 8 weeks can be expected to inspect during the given 2-hour period. (1)

10,. A consulting firm rents cars from three agencies, 20% from agency D, 20% from agency E, and 60% from agency F. If 10% of the cars from D, 12% of the cars from E, and 4% of the cars from F have bad tyres, what is the probability that the firm will get a car with bad tyres ? What is the probability that the firm will get a car with bad tyres rented by the firm came from agency F ? (4)

11. Among 60 automobile repair parts loaded in a truck in San Francisco, 45 are destined for Seattle and 15 for Vancouver. If two of the parts are unloaded in Portland by mistake and the "selection" is random, what are the probabilities that

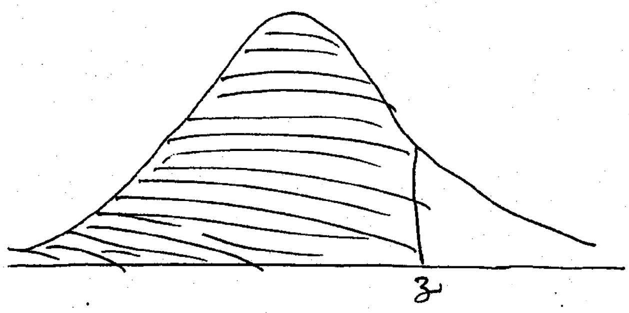
(a) both parts should have gone to Seattle (b) both parts should have gone to Vancouver (c) one should have gone to Seattle and one to Vancouver ?

(1+1+1)

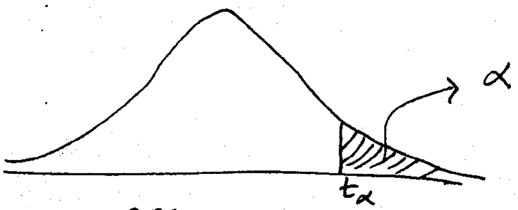
TABLE VALUES

Area of the shaded region corresponding to the standard normal curve

z	Area
0.15	0.5596
-0.15	0.4404
0.85	0.8023
-0.85	0.1977
-1.64	0.0505
1.64	0.9495
-1.96	0.0250
1.96	0.9750



Ordinate values for t-distribution for a given α



d.f	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.005$
6	1.440	1.943	3.143	3.707
7	1.415	1.895	2.998	3.499
8	1.397	1.860	2.896	3.355
9	1.383	1.833	2.821	3.250

BITS , PILANI – DUBAI CAMPUS
KNOWLEDGE VILLAGE

COMPREHENSIVE EXAMINATION-2003-2004
(MAKE UP)

COURSE NO: AAOC UC 111
COURSE TITLE: PROBABILITY & STATISTICS
DATE:
TIME:3 HRS.
MAX. MARKS: 40

SECTION - A

1. In a large corporate computer network, user log-ons to the system can be modeled as a Poisson process with a mean of 25 log-ons per hour. What is the probability that there are no log-ons in an interval of 6 minutes? (3 marks)

2. Two random variables are independent and each has a binomial distribution with success probability 0.3 and 2 trials.
   (i) Find the joint probability distribution.
   (ii) Find the probability that the second random variable is greater than the first.

   (4 marks)

3. The probability density f(x) of a continuous random variable X is given by

$$f(x) = \begin{cases} k(1+x) & ,-1<x\leq 0 \\ k(1-x) & ,0\leq x<1 \\ 0 & ,\text{elsewhere} \end{cases}$$

(i) Find k.
(ii) Find the cumulative distribution function F(x).
(iii) $P(X>0/ -0.5<X<0.5)$, using cdf as found in (ii).

(5 marks)

4. A continuous random variable is uniformly distributed between 20 and 60.
   (iii) What is the mean of the distribution?
   (iv) What is the probability a randomly selected value will be above 50?

(v) What is the probability a randomly selected value will be exactly 45?

(4 marks)

5. In the inspection of tin plate produced by electrolytic process, 0.2 imperfection is spotted on the average per hour. What is the probability that the time between the spotting of successive imperfections will be less than 2 hours? (4marks)