COURSE NO: AAOC UC 111
COURSE TITLE: PROBABILITY & STATISTICS
DATE: 15.01.04
TIME: 3 HRS.
MAX. MARKS: 40

1. Answer the following : (5x1 = 5 marks)

(i) If X_1 has mean 5 and variance 7 while X_2 has mean -4 and variance 7, and the two are independent, find
   a) $E(2X_1 + 3X_2 - 4)$
   b) $V(2X_1 + 3X_2 - 4)$

(ii) If the probability density of a random variable is given by
$$f(x) = \begin{cases} kx^3 & 0<x<1 \\ 0 & \text{elsewhere} \end{cases}$$
find the value of k.

(iii) Explain why there must be a mistake in the following statement: The probability that a drilling operation will be a success is 0.34 and the probability that it will not be a success is -0.66.

(iv) Explain why the following will not lead to random sample from the desired populations:
To determine what an average person spends on a vacation, a researcher interviews passengers on a luxury cruise.

2. In a bolt factory, machines M_1, M_2, M_3 manufacture respectively 25, 35 and 40 percent of the total output. Of their output 5, 4 and 2 percent respectively are defective bolts. One bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured in the machine M_2? (3 marks)

3. A manufacturer of ball-point pens claims that a certain pen he manufactures has a mean writing life of 400 pages with a standard deviation of 20 pages. A purchasing agent selects a sample of 100 pens and puts them for test. The mean writing life for the sample was 390 pages. Should the purchasing agent reject the manufacturer's claim at 5% level? (3 marks)

4. X_1, X_2 and X_3 is a random sample of size 3 from a population with mean μ and variance σ^2. T_1, T_2 and T_3 are the estimators used to estimate mean value μ, where $T_1 = X_1 + X_2 - X_3$,
$T_2 = 2X_1 + 3X_3 - 4X_2$ and $T_3 = \dfrac{(\lambda X_1 + X_2 + X_3)}{3}$

   (i) Find the value of λ such that T_3 is unbiased estimator for μ.
   (ii) Which is the best estimator?   (1+3 = 4 marks)

5. In a large corporate computer network, user log-ons to the system can be modeled as a Poisson process with a mean of 25 log-ons per hour. What is the probability that there are no log-ons in an interval of 6 minutes?   (3 marks)

6. The overall percentage of failures in a certain examination is 40. What is the probability that out of a group of 6 candidates at least 4 passed the examination?   (3 marks)

7. If two random variables have the joint density
   $$f(x,y) = \begin{cases} x_1 x_2 & 0 < x_1 < 1 \ , \ 0 < x_2 < 2 \\ 0 & \text{elsewhere} \end{cases}$$

   (i) Find the probability that the sum of the values taken on by the two random variables will be less than 1.
   (ii) Find the joint distribution function of the two random variables.   (2+2= 4 marks)

8. A shipment of 20 tape recorders contains 5 that are defective. If 10 of them are randomly chosen for inspection, what is the probability that 2 of the 10 will be defective?   (2 marks)

9. If a random variable has the gamma distribution with $\alpha = 2$ and $\beta = 2$, find the mean and standard deviation of the distribution.   (2 marks)

10. A sample of 100 measurements of breaking strength of cotton threads gave a mean of 7.4 ounces and a standard deviation of 1.2 ounces. Find 95% confidence limits for the mean breaking strength.   (3 marks)

11. The following measurements show the respective heights in inches of ten fathers and their eldest sons.
Father(X): 67, 63, 66, 71, 69, 65, 62, 70, 61, 72.
Son(Y)   : 68, 66, 65, 70, 69, 67, 64, 71, 60, 63.
(i)   Find the regression line of son's height on father's height.
(ii) Estimate the height of son for the given height of father as 70 inches.
(iii) Test the significance of the regression coefficient β at 95% confidence level.

(2+1+2 = 5 marks)

12. Two random samples gave the following results:

Sample	Size	Sample mean	Sum of squares of Deviations from the mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population at 5% level of significance.   (3 marks)

Tabulated Values:
$F_{0.05}(10,12) = 2.75$ $F_{0.05}(9,11) = 2.9$
$\chi^2_{0.05}(9) = 16.919$ $\chi^2_{0.05}(11) = 19.675$
$t_{0.05,\,8} = 2.306$ $Z_{0.05} = \pm1.645$
$Z_{0.025} = \pm1.96$ $Z_{0.01} = \pm 2.33$
$Z_{0.005} = \pm2.575$

TEST –I   DATE: 2.11.03

COURSE NO: AAOC UC 111

COURSE TITLE: PROBABILITY & STATISTICS

TIME: 50 minutes   MAX. MARKS: 20

1. Based on past experience , the main printer in a university computer center is operating properly 90% of the time. Suppose inspections are made at 10 randomly selected times.
   a) What is the probability that the main printer is operating properly for exactly 9 of the inspections?
   b) What is the probability that the main printer is not operating properly in no more than 1 inspection? (2+ 2 = 4 marks)

2. If the probability density of a random variable is given by
$$f(x) = \begin{cases} k(1-x^2) & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$
   a) Find the value of k .
   b) Find the probability that a random variable having this probability density will take on a value greater than 0.6.
   $$(2 + 2 = 4 \text{ marks})$$

3. In a certain manufacturing firm, the number of defective items produced per month is a random variable having a distribution with $\mu = 8.3$ and $\sigma = 2.5$. If this distribution can be approximated closely with a normal distribution , what is the probability that there will be more than 6 defectives in any one month?
   $$(4 \text{ marks})$$

4. In the inspection of tin plate produced by electrolytic process, 0.2 imperfection is spotted on the average per hour. What is the probability that the time between the spotting of successive imperfections will be less than 2 hours?   (3 marks)

5. The number of patients who visit a hospital on a week day is a random variable with $\mu = 25$ and $\sigma = 3$. With what probability can we assert that there will be between 16 and 34 patients?
   $$(3 \text{ marks})$$

PTO

6. A man rolls a balanced die. What is the probability that the man will obtain 6 on the face of the die for the first time on the 3rd throw? (2 marks)

$F(-1.12) = 0.1314 ,$ $F(-0.72) = 0.2358,$ $F(-0.92) = 0.1788$

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F(-1.12) = 0.1314 , F(-0.72) = 0.2358, F(-0.92) = 0.1788

TEST –II(Open Book)

DATE: 21.12.03

COURSE NO: AAOC UC 111

COURSE TITLE: PROBABILITY & STATISTICS

TIME: 50 minutes

MAX. MARKS: 20

1. We would like to know the average time that a child spends watching television over the weekend. We want our estimate to be within $\pm$ 1 hour of the true population average. Previous studies have shown the population standard deviation to be 3 hours. What sample size should be taken for this purpose, if we want to be 95% confident that the error in our estimate will not exceed the maximum allowable error?

(3 marks)

2. An insurance company claims that it takes 2 weeks on an average, to process an auto accident claim. The standard deviation is 6 days. To test the validity of this claim, an investor randomly selected 36 people who recently filed claims. The sample revealed that it took the company an average of 16 days to process these claims. At 99% level of confidence, check if it takes the company more than 14 days on an average to process a claim.

(4 marks)

3. It is desired to estimate the average age of students who graduate with an MBA degree in the university system. A random sample of 64 graduating students showed that the average age was 27 years with a standard deviation of 4 years. Estimate a 95% confidence interval estimate of the true average.

(3 marks)

4. The IQ scores of college students are normally distributed with a mean μ of 120 and standard deviation σ of 10. If a random sample of 35 is taken,what is the probability that the mean of this sample will be between 120 and 125?

(4 marks)

5. Find the value of $F_{0.99}$ (corresponding to a left – hand tail probability of 0.01) for the samples of size $n_1=10$ and $n_2=20$.

(3 marks)

6. The claim that the variance of a normal population is $\sigma^2 = 16.5$ is rejected if the variance of a random sample of size 20 exceeds 30. What is the probability that the claim will be rejected even though $\sigma^2 = 16.5$?

(3 marks)

TEST –II(Open Book)
COURSE NO: AAOC UC 111
COURSE TITLE: PROBABILITY & STATISTICS
TIME: 50 minutes

DATE: 21.12.03

MAX. MARKS: 20

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1

BITS , PILANI – DUBAI CAMPUS
KNOWLEDGE VILLAGE

TEST –II(Open Book)   -Make Up   DATE: 30.12.03
COURSE NO: AAOC UC 111
COURSE TITLE: PROBABILITY & STATISTICS
TIME: 50 minutes   MAX. MARKS: 20

1. Two random samples gave the following results:

Sample	Size	Sample mean	Sum of squares of deviations from the mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population at 5% level of significance.   (5 marks)

2. A gas station repair shop claims that it can do a lubrication job and oil change in at most 30 minutes. The Consumer Protection Department wants to test this claim. A sample of 6 cars were sent to the station for oil change and lubrication. The job took on an average of 34 minutes with a standard deviation of 4 minutes. Test the claim at the level of significance $\alpha = 0.05$.
(4 marks)

3. A random sample of 400 firms was taken to find out the average sale per customer. The sample mean was found to be Dhs. 900 and the standard deviation Dhs. 200. Construct an interval estimate of the population mean with the confidence level of 95.44%.   (3 marks)

4. Explain why the following may/may not lead to random samples from the desired populations:
   (i)   To determine the smoothness of shafts, a manufacturer measures the roughness of the first piece made each morning.
   (ii)   To measure the per capita income of people in UAE the sample is drawn from the salary of the government employees in UAE.
(4 marks)

5. A process for making certain bearings is under control if the diameters of the bearings have a mean of 0.5000 cm. What can we say about this process if a sample of 10 of these bearings has a mean diameter of 0.5060 am and a standard deviation of 0.0040cm.   (4 marks)