BITS, Pilani-Dubai Campus
Dubai International Academic City, Dubai

Comprehensive Examination – III year I Semester 2010-2011

Date: 21.12.10 Total Marks: 120 Weightage: 40%
Course: Optimization Course No. AAOC C222

Answer all questions

Use separate answer books for Part – A, Part – B and Part – C

Part – A

1. A factory manufactures tables and chairs. Both require a certain number of carpentry hours and a certain number of labor hours in the painting department. Each table takes 4 hours in carpentry and 2 hours in painting department. Each chair needs 3 hours in carpentry shop and just 1 hour in painting. During the current production period only 300 hours of carpentry time and 120 hours of painting time are available. Each table sold yields a profit of Rs. 70 and each chair produced may be sold for a profit of Rs 50. Formulate the above problem to maximize the profits and find the solution using Graphical method. **[10]**

2. Solve the following LPP by Big-M method.

Maximize $z = x_1 + 2x_2 + x_3$

Subject to

$$x_1 + x_2 + x_3 = 7$$
$$2x_1 - 5x_2 + x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

[10]

3. A cement factory manager is considering the best way to transport cement from his three manufacturing centers P, Q, R to depots A, B, C, D and E. The weekly production and demands along with transportation costs per ton are given below:

Factory	A	B	C	D	E	Supply
P	4	1	3	4	4	60
Q	2	3	2	2	3	35
R	3	5	2	4	4	40
Demand	22	45	20	18	30	

Find the initial basic feasible solution by Vogel's approximation method and the optimal solution using u-v method. **[10]**

4. Solve the following assignment problem.

Operators	Jobs				
	A	B	C	D	E
1	2	9	2	7	1
2	6	8	7	6	1
3	4	6	5	3	1
4	4	2	7	3	1
5	5	3	9	5	1

(a) Specify the minimum time required for the completion of the jobs.
(b) Also specify the allocation of jobs to the operators. **[10]**

5. For the following LPP, identify three alternative optimal basic solutions, and then write a general expression for all the non basic alternative optima using any two basic solutions

Maximize z = $x_1 + 2x_2 + 3x_3$

Subject to

$$x_1 + 2x_2 + 3x_3 \le 10$$
$$x_1 + x_2 \le 5$$
$$x_1 \le 1$$
$$x_1, x_2, x_3 \ge 0$$

[10]

6. Camyo Manufacturing produces four parts that require the use of a lathe and a drill press. The two machines operate 10 hours a day. The following table provides the time in minutes required by each part:

	Production time in min	
Part	Lathe	Drill press
1	5	3
2	6	2
3	4	6
4	7	4

It is desired to balance the two machines by limiting the difference between their total operation times to at most 30 minutes. The market demand for each part is at least 10 units. Additionally, the number of units of part 1 may not exceed that of part 2. Formulate the problem as a goal programming model. **[10]**

Part – B

7. Solve the following integer programming problem

Minimize $Z = 5x_1 + 4x_2$

Subject to

$$3x_1 + 2x_2 \ge 5$$
$$2x_1 + 3x_2 \ge 7$$
$x_1, x_2 \ge 0$ and are integers. **[6]**

8. Solve the following game graphically and find the value of game. The payoff for Player A is given below: [4]

	B_1	B_2	B_3
A_1	1	−3	7
A_2	2	4	−6

9. Solve the following nonlinear programming problem by using Khun-Tucker conditions

$$\text{Maximize } z = 7x_1^2 + 6x_1 + 5x_2^2$$
Subject to
$$x_1 + 2x_2 \le 10$$
$$x_1 - 3x_2 \le 9$$
$$x_1, x_2 \ge 0$$
[10]

10. Solve the following LP by using Revised Simplex method

$$\text{Minimize } Z = -2x_1 + 4x_2 - 4x_3$$
Subject to
$$2x_1 - 4x_2 + x_3 \le 2$$
$$x_1 + x_3 \le 5$$
$$x_1, x_2, x_3 \ge 0$$
[10]

Part – C

11. For the particular project the activities are specified along with the optimistic time 'a', pessimistic time 'b' and most likely time 'm' (time is in weeks). *(Values up to two decimal places)*

Activity	a	b	m
(1,2)	6	11	8
(1,3)	19	23	20
(1,4)	27	41	33
(2,5)	17	21	18
(2,6)	16	26	20
(3,6)	7	13	9
(4,7)	8	13	10
(5,7)	8	10	8
(6,7)	4	6	4

(a) Construct a project network.
(b) Find the critical path.
(c) Find the probability that the entire project is completed in 44 weeks. [12]

12. Consider the LPP

$$\text{Maximize } z = 35x_1 + 50x_2$$
Subject to
$$4x_1 + 6x_2 \le 120$$
$$x_1 + x_2 \le 20$$
$$2x_1 + 3x_2 \le 40$$
$$x_1, x_2 \ge 0$$

The associated optimal table for the primal is given as:

Basic	x_1	x_2	x_3	x_4	x_5	Solution
z	0	0	0	5	15	700
x_3	0	0	1	0	-2	40
x_1	1	0	0	3	-1	20
x_2	0	1	0	-2	1	0

a) Write the associated dual problem.

b) If the RHS of the constraints is changed from $(120, 20, 40)^T$ to $(150, 10, 60)^T$. Find the new optimum solution by applying sensitivity analysis. **[12]**

13. Solve the following using dynamic programming:

$$\text{Minimize } z = y_1^2 + y_2^2 + y_3^2$$
Subject to
$$y_1 y_2 y_3 = 64$$
$$y_1, y_2, y_3 \ge 0$$

[6]

TABLE AS PER STANDARD NOTATION:

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879

BITS, PILANI – DUBAI
DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI
III – Year – Semester – I (2010-11)
OPTIMIZATION (AAOC C222)
TEST – 2 (Open Book)

Time: 50 Minutes | Max. Marks 60
Date: November 07, 2010 | Weightage: 20%

Answer all questions

1. Find the initial basic feasible solution of the following Transportation problem by Vogel's approximation method and hence find the optimal solution. **[20]**

8	15	12	12	200
8	10	11	9	150
11	12	13	14	120
140	120	80	220	

2. A company has four salesmen who are to be assigned to four different sales territories. The monthly sales increases, estimated for each salesman in different territories (in lakhs of rupees) are shown in the following table. Determine the maximum sales increase. **[10]**

	Sales territory			
Salesman	I	II	III	IV
A	140	112	98	154
B	90	72	63	99
C	110	88	77	121
D	80	64	56	88

3. (i) The following game give A's payoff. Determine the values of p and q that will make the entry (2, 2) a saddle point. **[2]**

	B_1	B_2	B_3
A_1	12	q	4
A_2	p	6	q
A_3	17	3	8

(ii) By using graphical method find the value of the game, probability of selecting optimal strategies for players A and B **[8]**

Player B

$$\text{Player A} \begin{pmatrix} 1 & 3 \\ 3 & 0 \\ 5 & -1 \\ 6 & -6 \end{pmatrix}$$

4. Consider the LPP

$$\text{Max } z = 2x_1 - x_2 + x_3$$

Subject to

$$3x_1 + x_2 + x_3 \le 60$$
$$x_1 - x_2 + 2x_3 \le 10$$
$$x_1 + x_2 - x_3 \le 20$$
$$x_1, x_2, x_3 \ge 0$$

The associated optimal table for the primal is given as:

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	0	0	$3/2$	0	$3/2$	$1/2$	25
x_4	0	0	1	1	-1	-2	10
x_1	1	0	$1/2$	0	$1/2$	$1/2$	15
x_2	0	1	$-3/2$	0	$-1/2$	$1/2$	5

a) Write the associated dual problem.
b) Determine the optimal solution of the dual variables.
c) A new constraint $3x_1 - 2x_2 + x_3 \le 28$ is added. Find the new optimal solution.

[4+3+13]

BITS, PILANI – DUBAI

Dubai International Academic City, Dubai
III Year-Semester I (2010-2011)
OPTIMIZATION (AAOC C222)
Test I (Closed Book)

Time: 50 Minutes — Max. Marks: 75
Date: 26 Sept. 2010 — Weightage: 25%

ANSWER ALL QUESTIONS

1. Jack is an aspiring freshman at a University. He realizes that "all work and no play make Jack a dull boy". As a result, Jack wants to apportion his available time of about 10 hours a day between work and play. He estimates that play is twice as much fun as work. He also wants to study at least as much as he plays. However, Jack realizes that if he is going to get all his homework assignments done, he can not play more than 4 hours a day. How should Jack allocate his time to maximize his pleasure from both work and play? Formulate it as a LPP. **15 Marks**

2. Solve the following LP by using graphical method **10 Marks**

$$Maximize\ Z = 6x_1 + 4x_2$$

Subject to

$$x_1 - x_2 \le 2$$
$$-2x_1 + x_2 \le 2$$
$$3x_1 + 2x_2 \le 9$$
$$x_1, x_2 \ge 0$$

3. Solve the following LP by using Simplex method **25 Marks**

$$Minimize\ Z = -2x_1 + 4x_2 - 4x_3$$

Subject to

$$x_1 - 2x_2 + \frac{1}{2}x_3 \le 1$$
$$x_1 + x_3 \le 5$$
$$x_1, x_2, x_3 \ge 0$$

4. Solve the following LP using Big M method **25 Marks**

$$Maximize\ Z = 5x_1 + 2x_2 + 3x_3$$

Subject to

$$x_1 + 5x_2 + 2x_3 = 30$$
$$x_1 - 5x_2 - 6x_3 \le 40$$
$$x_1, x_2, x_3 \ge 0$$

BITS, PILANI – DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

III – Year – Semester – I (2010-11)

OPTIMIZATION (AAOC C222)

Quiz – 2 (Closed Book)

Time: 20 Minutes
Date: December 01, 2010

Max. Marks 21
Weightage: 07%

Answer all questions

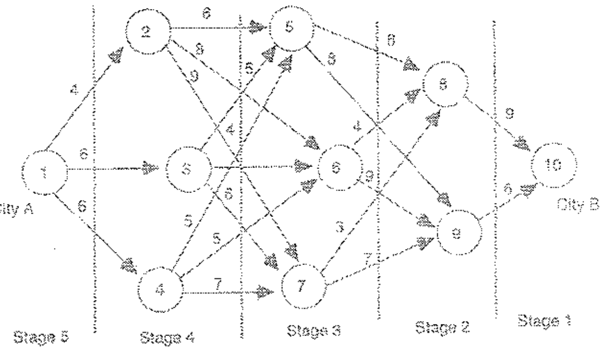
1. A manufacturing firm produces two types of products A and B. According to past experiences, production of either product A or B requires an average of one hour in the plant. The plant has a normal production capacity of 400 hours a month. The marketing department of the firm reports that because of limited market the maximum number of product A and B that can be sold in a month are 240 and 300 respectively. The net profit from the sale of product A and product B are Rs.800 and Rs.400 respectively. The manager of the firm has set the following goals arranged in the order of importance.

G1: He wants to avoid any under utilization of normal production capacity.

G2: He wants to sell maximum possible units of product A and product B.

Since the net profit from the sale of product A is twice the amount from that of product B, the manager has twice as much as desire to achieve sales for product A as for product B. Formulate it as a goal programming.

2. A salesman located in city A decided to travel to city B. He knew the distances (in km) of alternative route from city A to city B given in the following network. Find the shortest distance between the two cities using dynamic programming technique.



3. Consider the following LPP:

Maximize $z = 3x_1 + 2x_2 + 5x_3$

Subject to $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

$x_1, x_2, x_3 \ge 0$

The associated optimum table for the primal is given as

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	4	0	0	1	2	0	1350
x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230
x_6	2	0	0	-2	1	1	20

Investigate the optimality of the objective function $z = 2x_1 + x_2 + 4x_3$

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DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

III – Year – Semester – I (2010-11)

OPTIMIZATION (AAOC C222)

Quiz – 1 (Closed Book)

Time: 20 Minutes — Max. Marks 24

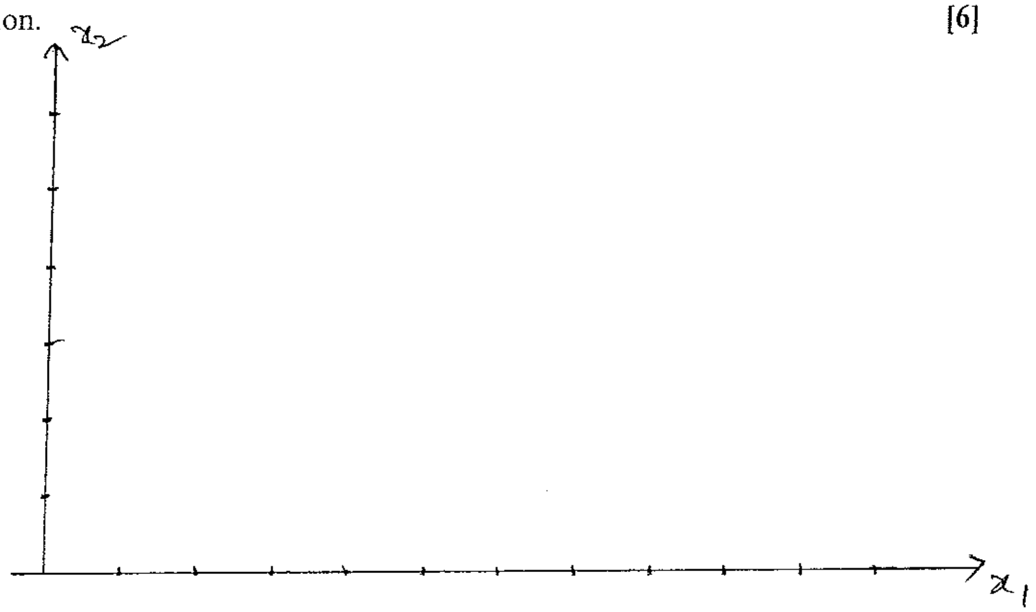
Date: October 20, 2010 — Weightage: 08%

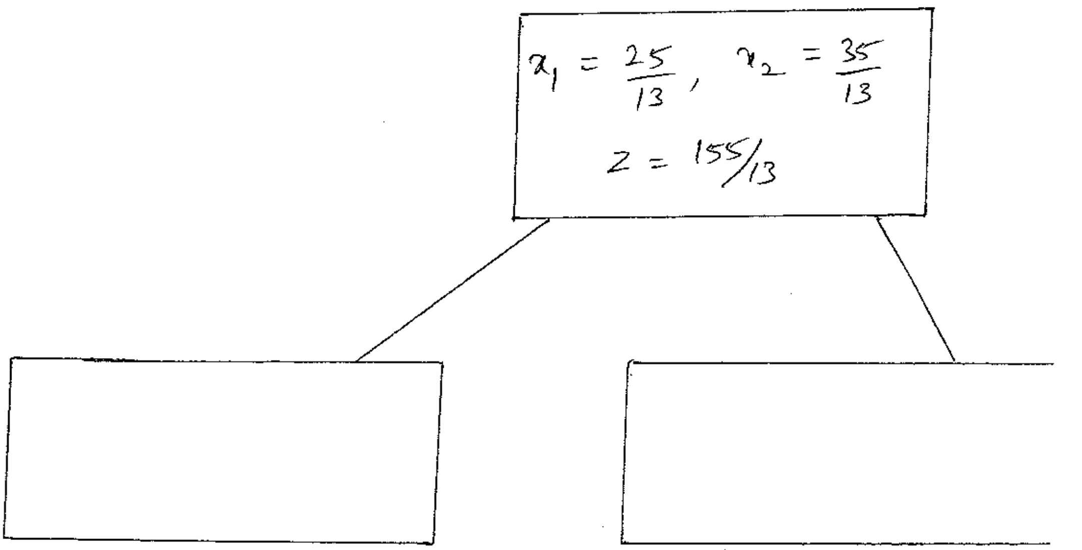
Answer all questions

1. Consider the following integer programming problem

$$\text{Maximize } z = 2x_1 + 3x_2$$
$$\text{Subject to } 6x_1 + 5x_2 \le 25$$
$$x_1 + 3x_2 \le 10$$
$$x_1, x_2 \text{ are non-negative integers}$$

The initial solution of the above integer programming problem is $x_1 = \frac{25}{13}, x_2 = \frac{35}{13}$ and $z_{\max} = \frac{155}{13}$. Write the sub-problems with respect to the variable x_2 and find the optimal integer solution. **[6]**





2. Give the initial basic feasible solution for the following transportation problem by North West corner rule. **[6]**

	I	II	III	Supply
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
Demand	75	20	50	

3. For a **Maximization** LPP the starting simplex table is given below:

Basic Variable	x_1	x_2	x_3	x_4	Solution
Z-Row	-6	-10	0	0	0
x_3	3	2	1	0	12
x_4	1	1	0	1	6

Find the optimal solution and classify it. **[6]**

4. (a) Write the following LPP in the standard form to get an initial basic feasible solution by introducing minimum number of artificial variables. Also write the objective function of Phase I and indicate the initial basic variables. [4]

Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to $x_1 + 2x_2 + 3x_3 = 15$

$2x_1 + x_2 + 5x_3 \geq 20$

$x_1 + 2x_2 + x_3 + x_4 \geq 10$

$x_1, x_2, x_3, x_4 \geq 0$

(b) If the given LPP is of the maximization type, explain why we do not maximize the sum of the artificial variables in Phase I when we solve by Two Phase method. [2]