BITS, PILANI – DUBAI

Dubai International Academic City, Dubai

II Year- Semester I (2010-2011)

MATHEMATICS - III (MATH C241)

Comprehensive Examination

Time: 3 Hours | Max. Marks: 120

Date: 28-12- 2010 | Weightage: 40%

NOTE: 1) There are two sections (A and B) in the question paper and they should be answered in two separate answer books.

2) All questions are compulsory and should be answered sequentially.

PART-A

1) a) Solve the equation by finding an Integrating factor

$(y\log(y)-2xy)dx+(x+y)dy=0$ (5 Marks)

b) Solve the equation $(1+x^2)dy+2xydx=\cot(x)dy$ (5 Marks)

2) a) Solve the equation by reducing the order $x^2\frac{d^2y}{dx^2}=2x\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^2$ (5 Marks)

b) Verify that one solution of $x\frac{d^2y}{dx^2}-(2x+1)\frac{dy}{dx}+(x+1)y=0$ is given by $y=e^x$, and find the general solution. (5 Marks)

3) a) Solve the equation $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+3y=0$. (5 Marks)

b) Find the particular solution of $\frac{d^3y}{dx^2}-2\frac{dy}{dx}+y=2x^3-3x^2+4x+5$ (5 Marks)

4) Find the solution using method of variation of parameters

$\frac{d^2y}{dx^2}+\frac{dy}{dx}=\cot^2(x)$ (10 Marks)

5) Find the Fourier Series for the periodic function defined by

$$f(x)=\begin{cases}-\pi & ;-\pi\le x\le 0\\ x & ;\ 0\le x\le\pi\end{cases}$$

and $f(x+2\pi)=f(x)$. Sketch the graph of the sum of this series on the interval $-3\pi\le x\le 3\pi$. Hence find the sum of the series $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\infty$. (10 Marks)

6) A tightly stretched string with fixed end points $x=0$ and $x=\pi$, and with the initial shape as given by $f(x)=kx\left(1-\frac{x}{\pi}\right)$; $0\le x\le\pi$ where *k* is a real constant, is released from this deformed shape at *t* = 0 and made to vibrate in the XOY plane. Find the displacement y for any given *x* and *t*. (10 Marks)

PART-B

1) Find power series solution of the equation $(1+x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-2y=0$. (10 Marks)

2) a) Find series solution of the equation $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4xy+4y=0$ by using Frobenius series method. (15 Marks)

b) Solve the following Hyper geometric series about $x=0$. (5 Marks)

$2(x-x^2)\frac{d^2y}{dx^2}+(3-4x)\frac{dy}{dx}+4y=0$

3) a) For Bessel's function $J_p(x)$ of p^{th} order, prove that (10 Marks)

$px^{p-1}J_p(x)+x^pJ_p'(x)=x^pJ_{p-1}(x)$

b) For Bessel's function $J_p(x)$ of p^{th} order, find $J_{\frac{1}{2}}(x)$ in terms of $Sin(x)$ or $Cos(x)$.

(5 Marks)

4) a) Find Laplace transform of greatest integer function *i.e.* $L\{[x]\}$ where [x] denotes the greatest integer ≤ x (5 Marks)

b) Solve following differential equation by using Laplace transform. (5 Marks)

$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0$, with the condition $y(0)=0$, $y'(0)=3$.

c) Find the eigenvalues λ_n and eigen function $y_n(x)$ for the boundary value problem $y''+\lambda y=0$, $y(0)=0$ and $y(L)=0$, where $L>0$. (5 Marks)

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Dubai International Academic City, Dubai
II Year- Semester I (2010-2011)
MATHEMATICS - III (MATH C 241)
Test II (Open Book)

Time: 50 Minutes
Date: 28-11- 2010

Max. Marks: 60
Weightage: 20%

All questions are compulsory and should be answered sequentially.

1. Find series solution of the equation $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-xy=0$ in terms of the constants a_0, a_1 of power series $\sum_{n=0}^{n=\infty} a_n x^n$. **15 Marks**

2. For Hyper geometric series, Find the value of

$$\left[\frac{d^2}{dx^2}F(-n,1,1,-x)\right] \text{ at } x=0$$ **5 Marks**

3. For Chebyshev polynomial $T_n(x)$, prove that

$$(T_n(x))^2 - T_{n+1}(x).T_{n-1}(x) = 1-x^2$$ **10 Marks**

4. Show that $\frac{1-t^2}{(1-2xt+t^2)^{\frac{3}{2}}} = \sum_{n=0}^{\infty}(2n+1)P_n(x)t^n$ **15 Marks**

5. Find $J_{\frac{7}{2}}(x)$ where $J_n(x)$ denotes the Bessel's function. **5 Marks**

6. $J_2'(x) = \left(1-\frac{4}{x^2}\right)J_1(x)+\frac{2}{x}J_0(x)$ where $J_n(x)$ is the Bessel's function **10 Marks**

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Dubai International Academic City, Dubai

II Year- Semester I (2010-2011)

MATHEMATICS - III (MATH C 241)

Test I (Closed Book)

Time: 50 Minutes
Date: 17-10- 2010

Max. Marks: 75
Weightage: 25%

All questions are compulsory and should be answered sequentially.

1. Solve $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$ **15 Marks**

2. Solve the following equation by finding an Integrating factor.
$e^x dx + (e^x \cot y + 2y \csc y)dy = 0$ **10 Marks**

3. Solve $xy' + y = x^4 y^3$ **10 Marks**

4. If $y_1 = x$ is one solution of $x^2y'' - x(x+2)y' + x(x+2)y = 0$ then find other solution y_2 and also find general solution. **10 Marks**

5. By using Method of Variation of Parameters find particular solution of
$y'' + y = \sec x \tan x$. **15 Marks**

6. By using Operator Method find particular solution of
$y''' - 2y' + y = 2x^3 - 3x^2 + 4x + 5$ **15 Marks**

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Dubai International Academic City, Dubai
II Year - Semester I (2010-2011)
MATH III (MATH C 241)
Quiz-2 (Closed Book)

Time: 20 Minutes
Date: 06-12- 2010

Max. Marks: 21
Weightage: 7%

Name: ***ID No:***

Write Final answer only

1. Laplace Transform $L(Sin^2 x + Cosh2x)$ is _____

6 Marks

2. Inverse Laplace Transform $L^{-1}\left(\frac{p-3}{p^2-6p+13}\right)$ is _____

5 Marks

3. Laplace Transform $L(xCos2x)$ is _____

5 Marks

4. Convolution of two function $e^x * e^{2x}$ is _____

5 Marks

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II Year- Semester I (2010-2011) **MATHEMATICS - III** (MATH C 241)

Quiz I (Closed Book)

NAME: **ID. No:** **SEC:**

Time: 20 Minutes **SET-A** Max. Marks: 24

Date: 09-11- 2010 Weightage: 8%

1. The Wronskian $W(e^{-2x}, \cot x)$ is

(a) $2e^{-2x}(Cotx - Co\sec^2 x)$ (b) $2e^{-2x}(Cotx + Co\sec^2 x)$

(c) $-2e^{-2x}(Cotx + Co\sec^2 x)$ (d) $2e^{-2x}(Cotx - Co\sec x)$ (e) None of these (4 Marks)

2. If auxiliary roots for any given differential equation are $m = -2, 2, 2, -2 \pm i$ then general solution will be

(a) $e^{-2x}(C_1 + C_4 Cos2x + C_5 Sin2x) + C_2 e^{-2x} + xC_3 e^{-2x}$ (b) $e^{-2x}(C_1 + C_4 Cos2x + C_5 Sin2x) + C_2 e^{-2x} + xC_3 e^{-2x}$

(c) $e^{2x}(C_4 Cosx + C_5 Sinx) + C_2 e^{-2x} + x^2 C_3 e^{-2x}$ (d) $e^{-2x}(C_1 + C_4 Cosx + C_5 Sinx) + C_2 e^{2x} + xC_3 e^{2x}$

(e) None of these (4 Marks)

3. If $y = C_1 e^{-x} + C_2 e^{-2x}$ is the general solution of $y'' + 3y' + 2y = 0$ then find C_1 and C_2 for which $y(0) = 1$ $y'(0) = -1$.

(a) $C_1 = 3$ and $C_2 = 2$ (b) $C_1 = 3$ and $C_2 = -2$ (c) $C_1 = 3$ and $C_2 = 3$ (d) $C_1 = 3$ and $C_2 = -3$

(e) None of these (4 Marks)

4. Find the first three coefficients in the Power series solution of the differential equation $\frac{dy}{dx} - 2y = 0$; $y(0) = 1$ are

(a) 1, 2, -2 (b) 1, 2, 2 (c) 1, 0, 2 (d) -2, 2, -2 (e) None of these (4 Marks)

5. Find the Indicial roots of the differential equation $3x\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$ while solving it by Frobenius Power series method near the regular singular point x = 0,

(a) 0, 2 (b) 0, -2 (c) 0, 2/3 (d) 0, -2/3 (e) None of these (4 Marks)

6. For the differential equation $\left(x^2 - 1\right)^3 (x - 2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y = 0$, select the correct combination of singular points and their nature.

(a) x=0,1 and 2 are Ordinary point, regular singular point and irregular singular point respectively.

(b) x=0,-1 and 2 are Ordinary point, regular singular point and irregular singular point respectively.

(c) x=0,-1 and 2 are, regular singular point, irregular singular point and Ordinary point respectively.

(d) x=0,1 and 2 are Ordinary point, irregular singular point and regular singular point respectively.

(e) None of these. (4 Marks)

BITS, PILANI – DUBAI
Dubai International Academic City, Dubai
II Year- Semester I (2010-2011) **MATHEMATICS - III** (MATH C 241)
Quiz I (Closed Book)

NAME: **ID. No:** **SEC:**

Time: 20 Minutes **SET- B** Max. Marks: 24
Date: 09-11- 2010 Weightage: 8%

1. Wronskian $W(e^x \sin x, \sec x)$ is
(a) $\tan^2 x - \tan x + 1$ (b) $e^x(\tan^2 x - \tan x + 1)$ (c) $e^x(\tan^2 x - \tan x - 1)$ (d) $e^x(Sin^2 x - Co\sec x + 2)$
(e) None of these (4 Marks)

2. If auxiliary roots for any given differential equation are $m = -3, -3, 3, 3 \pm 3i$ then general solution will be
(a) $e^{-3x}(C_1 + C_2 x) + e^{3x}(C_3 + C_4 Cos3x + C_5 Sin3x)$ (b) $e^{-3x}(C_3 + C_4 Cos3x + C_5 Sin3x) + e^{-3x}(C_1 e^{-2x} + C_2 x)$
(c) $e^{-3x}(C_3 + C_4 Cos3x + C_5 Sin3x) + e^{-3x}(C_1 e^{-2x} + C_2 x)$ (d) $e^{-3x}(C_1 e^{-2x} + C_2 x) + e^{3x}(C_3 + C_4 Cos3x + C_5 Sin3x)$
(e) None of these (4 Marks)

3. If $y = C_1 e^x + C_2 e^{2x}$ is the general solution of $y'' - 3y' + 2y = 0$, then find C_1 and C_2 for which $y(0) = -1$ $y'(0) = 1$
(a) $C_1 = 3$ and $C_2 = 2$ (b) $C_1 = 3$ and $C_2 = -2$ (c) $C_1 = 3$ and $C_2 = 3$ (d) $C_1 = 3$ and $C_2 = -3$
(e) None of these (4 Marks)

4. Find the first three coefficients in the Power series solution of the differential equation $\frac{dy}{dx} + 2y = 0$; $y(0) = 1$ are
(a) - 1,2,-2 (b) 1,-2,2 (c) 1,0,2 (d) -2,2,-2 (e) 1,2,2 (4 Marks)

5. Find the Indicial roots of the differential equation $3x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 6y = 0$ while solving it by Frobenius Power series method near the regular singular point x = 0,
(a) 0, 2 (b) 0, -2 (c) 0, 4/3 (d) 0, -4/3 (e) None of these (4 Marks)

6) For the differential equation $(x^2 - 9)^3 (x+2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y = 0$, select the correct combination of singular points and their nature.

(a) x=0,3 and -2 are Ordinary point, regular singular point and irregular singular point respectively.

(b) x=0,-3 and -2 are Ordinary point, regular singular point and irregular singular point respectively.

(c) x=0,-3 and -2 are, regular singular point, irregular singular point and Ordinary point respectively.

(d) x=0,3 and -2 are Ordinary point, irregular singular point and regular singular point respectively.

(e) none of these (4 Marks)