BITS, PILANI – DUBAI
DUBAI INTERNATIONAL ACADEMIC CITY
I SEMESTER – I YEAR , 2010-11

COMPREHENSIVE EXAMINATION (CB)

Course: Probability and Statistics   **Course No. AAOC C111**

Total Marks: 120   **Weightage: 40%**   **Date: 30-12-2010**   **Time: 3 hours**

Instruction:

1. *Write answers of Part A, Part B and Part C in separate answer books. Use blue, green and brown answer books for Part A, Part B and Part C respectively.*
2. *Necessary table values are given, so statistical table is not required.*
3. *Non-programmable calculator is allowed.*
4. *Attempt all the questions.*

Table Values (in standard notation):

$Z_{0.01} = 2.326$, $Z_{0.05} = 1.645$, $F(2.997) = 0.9987$, $F(0.37) = 0.6443$, $F(-2.62) = 0.0044$.

Here $F(z)$ stands for cdf of standard normal variate.

PART - A

1. It has been found that 90% of all the washing machines of a particular brand function correctly at the time of installation. In a given month 8 units of washing machines are sold.

   a) Find the probability that at least 7 of the washing machines function correctly on installation.

   b) Find the probability that atmost 7 function correctly on installation.

   c) Consider 5 months in which 8 units are sold per month. Calculate the probability that at least 7 units function correctly in each of the 5 months. [10]

2. A test has been developed to detect a particular type of arthritis in individuals over 50 years old. From a national survey it is known that approximately 10% of the individuals in this age group suffer from this form of arthritis. The proposed test was given to individuals with confirmed arthritic disease and a correct test result was obtained in 85% of the cases. When the test was administered to individuals of the same age group who were known to be free of the disease 4% were reported to have the disease. What is the probability that an individual has this disease given that the test indicates its presence. [10]

3. A garage-door opener has 5 toggle switches. Each switch can be set in 3 different positions- up, center and down.

   a) In how many different ways the switches can be set?

   b) How many settings are possible in which 2 are up, 2 are down and 1 is at center?

   c) What is the probability that a thief will be able to open the door in the first attempt? [10]

4. A random variable X has the following distribution.

x:	-2	-1	0	1	2	3
$f(x)$:	0.1	k	0.2	2k	0.3	3k

   a) Find k.

   b) Evaluate $P(X<2)$ and $P(-2 < X < 2)$

   c) Find the cumulative distribution of X

   d) Evaluate the mean of X. [10]

PART - B

5. Let X denote the number of syntax errors in a programme. The density function $f(x)$ is given below:

X	0	1	2	3	4	5	6	7
$f(x)$	0.005	0.105	0.225	0.005	0.235	0.164	0.145	0.116

   Simulate five values of X using the random numbers 650, 110, 235, 345 and 490. [10]

6. A random variable X has a distribution with density function

   $$f(x) = \frac{x}{6}, \ for \ 2 \leq x \leq 4$$

   Find $E(X)$, $E(X^2)$, $Var(X)$ and Standard Deviation (X). [10]

7. Although errors are likely when taking measurements from photographic images, these errors are often very small. For sharp images with negligible distortion, errors in measuring distances are often no larger than 0.0004 inch. Assume that the probability of a serious measurement error is 0.05. A series of 150 independent measurements are made. Let X denotes the number of serious errors made.

   a) Find the probability of making at least one serious error, using normal approximation and compare the result by using Binomial distribution.

b) Find the probability of making 6% to 10% serious errors. [10]

8. The joint density for (X, Y), where X and Y are inside and outside barometric pressures respectively on an air support roof is given by

$$f_{XY}(x, y) = \frac{1.72}{x}, \text{ for } 27 \leq y \leq x \leq 33$$

Find the $Cov(X, Y)$. [10]

PART - C

9. The weight of a calf taken at weekly intervals is given below:

Age in week (x)	1	2	3	4	5	6	7	8
Weight in kg. (y)	52.5	58.7	65.0	70.2	75.4	81.1	87.2	95.5

a) Find the regression line of y on x in the form $y = b_0 + b_1 x$. First write down the computational formulae for b_0, b_1 and then calculate their values.

b) What is the estimate of weight of the calf at 9th week? [10]

10. Calculate the correlation coefficient r between the age of husband (x) and the age of wife (y) for the following data:

Age of husband in years (x)	23	27	28	28	29	30	31	33	35	36
Age of wife in years (y)	18	20	22	27	21	29	27	29	28	29

[10]

11. Let μ be the mean of a normal population which is unknown. The population standard deviation σ is known to be 2. A random sample of size 30 is drawn from the population and the sample mean $\overline{x}$ is found to be 5.5. Test the following hypothesis at $\alpha = 0.01$ level of significance:

$H_0 : \mu = 5$ against the alternative $H_1 : \mu > 5$.

If you have committed some error, what type of error will it be? [10]

12. Two manufacturers A and B produce certain type of bolts. Let p_1 denotes the proportion of bolts defective if manufactured by A and p_2 be the same proportion for B. Random samples of bolts are taken from two manufacturers. Following is the test result:

Manufacturer	Sample size	No. of defective bolts
A	$n_1 = 220$	$x_1 = 15$
B	$n_2 = 180$	$x_2 = 14$

a) Find the point estimates of p_1, p_2 and $p_1 - p_2$. *Write results correct to 4 decimal places.*

b) Find a 90% confidence interval on $p_1 - p_2$. [10]

BITS, PILANI –DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY

I YEAR – I SEMESTER 2010-2011

TEST – II (OB)

COURSE: PROBABILITY AND STATISTICS   COURSE NO.: AAOC C111

MAX. MARKS: 60   WEIGHTAGE: 20%   DATE: 15-12-2010   TIME: 50 MINUTES

Instruction:

Only prescribed text book and handwritten class notes are allowed.

Non-programmable calculator is allowed.

Attempt all the questions:

1. A gas station repair shop claims that on average it can do a lubrication job and oil change less than 34 minutes . The consumer protection department wants to test this claim. A sample of 6 cars were sent to the station for oil change and lubrication. We observed that the sample mean is 30 minutes. The population standard deviation $\sigma = 4$ minutes.

   a) To test the claim, set up appropriate null and alternative hypotheses.   [2]

   b) What is the critical region of the test at the 0.05 level of significance.   [2]

   c) Find the observed value of the test statistic. Can we reject H_0?   [3]

   d) What is the p-value of the test?   [3]

   e) Use same sample to find a 98% confidence interval on μ (μ is the actual (population) mean time required for lubrication and oil change.   [5]

2. We would like to know the mean time that a child spends watching television over the weekend. We want our estimate to be within 1 hour with 95% confidence. Previous studies have shown the population standard deviation to be 3 hours. What is the minimum sample size required for this purpose ?   [5]

3. The joint probability distribution of X and Y is given by

X/Y	1	3	9
2	1/8	1/24	1/12
4	1/4	1/4	0
6	1/8	1/24	1/12

a) Find the marginal densities of X and Y.

b) Conditional density of Y given that $X = 6$. [10]

4. A random variable X has a distribution with density function

$f(x) = (\alpha + 1) x^{\alpha}$, for $0 \leq x \leq 1$, $\alpha > -1$

and a random sample of size 8 produces the data 0.2, 0.4, 0.8, 0.5, 0.7, 0.9, 0.8, 0.9. Find the maximum likelihood estimate of the unknown parameter α. [10]

5. A sample of size 20 is drawn to find the confidence interval of the population variance σ^2. The 95% confidence interval is obtained as [0.69, 2.5478]. What is the observed value of the sample variance? Find the 90% confidence interval on σ^2. [10]

6. A factory produces brake pads. Let p denotes the proportion of brake pads defective. To test the null hypothesis $H_0 : p = 0.05$ against the alternative $H_1 : p < 0.05$ a random ample of 400 brake pads are tested and 14 are found defective.

a) What is the test statistic of this test?

b) What is the observed value of the test statistic?

c) What is the critical region of this test at $\alpha = 0.02$ level?

d) Can we reject the null hypothesis H_0 at $\alpha = 0.02$ level?

e) What type of error might have committed? [10]

COURSE: PROBABILITY AND STATISTICS   COURSE NO.: AAOC C111

MAX. MARKS: 75   WEIGHTAGE: 25%   DATE: 28-10-2010   TIME: 50 MINUTES

Attempt all the questions.

1. The lifetime X (in days) of a certain type of battery follows Weibull distribution with parameters $\alpha = 0.2$ and $\beta = 0.5$.

   a) What is the probability density function of X?

   b) What is the expected lifetime of such batteries?

   c) What is the probability that such a battery will fail in less than 10 days?   [15]

2. Three identical boxes A, B, C contain 20, 40, and 50 switches respectively. Number of defective switches in A, B and C are 2, 5 and 8 respectively. A box is chosen at random and a switch is taken from that box at random. If this switch is defective, find the probability that box B is chosen. What is the probability of selecting a defective switch?   [10]

3. a) In a large corporate computer network, user log-ons to the system can be modelled as a Poisson process with a mean of 25 log-ons per hour. Find the probability that there is no log-ons in an interval of 6 minutes.   [7]

   b) A card is drawn at random from a well-shuffled pack of 52 playing cards. Find the probability that it is either a black jack or an ace. Also find the probability that the drawn card is not a jack.   [10]

   c) If for two events A and B, $P(A) = 0.4$ and $P(A \cap B) = 0.1$, find the values of $P(A \cap B')$ and $P(A' \cup B')$.   [8]

4. It is known that the probability of being able to log on to a computer from a remote terminal at any time is 0.6. Let X denote the number of attempts that must be made to gain access to the computer.

   a)   Find the first four terms of the density table.

a) Find the first four terms of the density table.

b) Write the expression for density function.

c) Find the probability that at most six attempts must be made to gain access
    to the computer.   [15]

5. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} \dfrac{x}{2}, & \text{if } 0 < x < 1 \\[2ex] \dfrac{3-x}{2}, & \text{if } 1 \leq x < 2 \end{cases}$$

Determine the cumulative distribution function $F(x)$. Use $F(x)$ to find
$P(0.5 \leq X \leq 1.5)$.   [10]

BITS, PILANI – DUBAI
I YEAR – I SEMESTER
2010-2011

QUIZ – II(CB)

COURSE: Probability and Statistics **COURSE NO.:** AAOC C111

Max. Marks: 21 Weightage: 7% Date: 16-12-2010 Time: 20 minutes

NAME: **ID. NO.:** **SEC:**

Attempt all the questions. No marks will be awarded for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks. No extra sheet will be given for rough works. Do rough works in the space provided.

Table values: $\chi^2_{9,0.05} = 16.9$, $\chi^2_{9,0.95} = 3.33$, $Z_{0.05} = 1.645$, $Z_{0.025} = 1.96$.

Fill in the blank with correct answer:

1. An unbiased estimate of population mean μ based on the sample 2, 3, 4, 3, 2 is

   _____ .

2. If two variables are independent, the covariance between them is _____.

3. The 90% confidence interval on σ^2 for a sample size 10 with variance 3.4, is _____.

4. Let H_0: $p \le 0.5$, H_1: $p > 0.5$. Critical point for $\alpha = 0.05$ level is _____.

5. If the random number 0.4 is used then the simulated value of an exponential variate with parameter $\beta = 2$ is _____

Tick the correct answer:

6. If the power of a test is 0.96, the probability of committing Type-II error is
   a) 0.03 b) 0.04 c) 0.05 d) none of these

7. A researcher wants to find a 90% confidence interval of the parameter p, the proportion of defective products in a production system. He wants it to within 0.05 units. The minimum sample size required is
   a) 269 b) 270 c) 271 d) none of these

↓ SPACE FOR ROUGH WORKS ↓

COURSE: Probability and Statistics **COURSE NO.:** AAOC C111

Max. Marks: 21   Weightage: 7%   Date: 16-12-2010   Time: 20 minutes

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NAME: **ID. NO.:**   **SEC:**

Attempt all the questions. No marks will be awarded for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks. No extra sheet will be given for rough works. Do rough works in the space provided.

Table values: $\chi^2_{9,0.05} = 16.9$, $\chi^2_{9,0.95} = 3.33$, $Z_{0.05} = 1.645$, $Z_{0.025} = 1.96$.

Fill in the blank with correct answer:

1. An unbiased estimate of population mean μ based on the sample 4, 3, 4, 3, 2 is _____ .

2. If two variables are independent, the covariance between them is _____ .

3. The 90% confidence interval on σ^2 for a sample size 10 with variance 2.4, is _____ .

4. Let H_0: $p \geq 0.5$, H_1: $p < 0.5$. Critical point for $\alpha = 0.05$ level is _____ .

5. If the random number 0.4 is used then the simulated value of an exponential variate with parameter $\beta = 1$ is _____ .

Tick the correct answer:

6. If the power of a test is 0.96, the probability of committing Type-II error is
   a) 0.04   b) 0.03   c) 0.05   d) none of these

7. A researcher wants to find a 90% confidence interval of the parameter p, the proportion of defective products in a production system. He wants it to within 0.05 units. The minimum sample size required is
   a) 269   b) 271   c) 270   d) none of these

↓ SPACE FOR ROUGH WORKS ↓

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QUIZ – II(CB)

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Table values: $\chi^2_{9,0.05} = 16.9$, $\chi^2_{9,0.95} = 3.33$, $Z_{0.05} = 1.645$, $Z_{0.025} = 1.96$.

Fill in the blank with correct answer:

1. An unbiased estimate of population mean μ based on the sample 4, 1, 4, 1, 2 is

    _____ .

2. If two variables are independent, the covariance between them is _____ .

3. The 90% confidence interval on σ^2 for a sample size 10 with variance 4.4, is _____ .

4. Let H_0: $p \geq 0.5$, H_1: $p < 0.5$. Critical point for $\alpha = 0.025$ level is _____ .

5. If the random number 0.8 is used then the simulated value of an exponential variate with parameter $\beta = 1$ is _____

Tick the correct answer:

6. If the power of a test is 0.92, the probability of committing Type-II error is
    a) 0.08   b) 0.03   c) 0.05   d) none of these

7. A researcher wants to find a 90% confidence interval of the parameter p, the proportion of defective products in a production system. He wants it to within 0.05 units. The minimum sample size required is
    a) 271   b) 269   c) 270   d) none of these

↓ SPACE FOR ROUGH WORKS ↓

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QUIZ – II(CB)

COURSE: Probability and Statistics   **COURSE NO.:** AAOC C111

Max. Marks: 21   Weightage: 7%   Date: 16-12-2010   Time: 20 minutes

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Attempt all the questions. No marks will be awarded for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks. No extra sheet will be given for rough works. Do rough works in the space provided.

Table values: $\chi^2_{9,0.05} = 16.9$, $\chi^2_{9,0.95} = 3.33$, $Z_{0.05} = 1.645$, $Z_{0.025} = 1.96$.

Fill in the blank with correct answer:

1. An unbiased estimate of population mean μ based on the sample 4, 1, 4, 1, 6 is

   _____ .

2. If two variables are independent, the covariance between them is _____ .

3. The 90% confidence interval on σ^2 for a sample size 10 with variance 5.4, is _____ .

4. Let H_0: $p \le 0.5$, H_1: $p > 0.5$. Critical point for $\alpha = 0.025$ level is _____ .

5. If the random number 0.8 is used then the simulated value of an exponential variate with parameter $\beta = 2$ is _____

Tick the correct answer:

6. If the power of a test is 0.93, the probability of committing Type-II error is
   a) 0.08   b) 0.03   c) 0.05   d) none of these

7. A researcher wants to find a 90% confidence interval of the parameter p, the proportion of defective products in a production system. He wants it to within 0.05 units. The minimum sample size required is
   a) 272   b) 269   c) 270   d) none of these

↓ SPACE FOR ROUGH WORKS ↓

A

BITS, PILANI – DUBAI
I YEAR – I SEMESTER
2010-2011

QUIZ – I(CB)

COURSE: Probability and Statistics
Max. Marks: 24   Weightage: 8%

COURSE NO.: AAOC C111
Date: 11-10-2010   Time: 20 mins.

NAME: ID. NO.:   SEC:

Attempt all the questions. No marks will be awarded for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks. No extra sheet will be given for rough works. Do rough works in the space provided.

Fill in the blanks with correct answers:

1. The density function $f(x)$ of a discrete random variable X is defined in the following table:

x	1	2	3	4
$f(x)$	m	$2m$	m	m

   The value of m is _____ and $E(X) =$ _____ .

2. If $f(x) = \dfrac{1}{8}$ defines density function of a discrete random variable X, then the number of values that X can take is _____ .

3. If the events A and B are mutually exclusive then $A \cap B =$ _____ .

4. 2-digit numbers are formed using *1,2,3,4,5* without repetition. The probability of the first digit selected is an even is _____ .

5. From a city population, the probability of selecting a smoker is 4/5, a male   smoker is 1/5. If a person randomly selected is a smoker, the probability of a male is _____ .

6. If X_1 and X_2 are independent random variables where X_1 has mean 5, variance 7 and X_2 has mean -4, variance 7, then $E[2X_1 - 4X_2 - 4] =$ _____ and $Var[2X_1 - 4X_2 - 4] =$ _____ .

7. Three news papers *A, B and C* are published in a city. It is estimated from a survey that of adult population: *20%* read *A*, *16%* read *B*, *14%* read *C*, 10% read both *A and B*, *5%* read both *A and C*, *4%* read both *B and C*, *2%* read all three. The percentage of adult population read at least one of the papers is _____ .

8. If X has Poisson distribution with $P(X = 0) = 0.36787$, the parameter k of the distribution is approximately _____ (*write the value in nearest integer*).

AA

QUIZ – I(CB)

COURSE: Probability and Statistics   **COURSE NO.:** AAOC C111
Max. Marks: 24   Weightage: 8%   Date: 11-10-2010   Time: 20 mins.

NAME: **ID. NO.:**   **SEC:**

Attempt all the questions. No marks will be awarded for overwriting and multiple answers. Do not use pencil. Each question carries 3 marks. No extra sheet will be given for rough works. Do rough works in the space provided.

Fill in the blanks with correct answers:

1. The density function $f(x)$ of a discrete random variable X is defined in the following table:

x	1	2	3	4
$f(x)$	m	$2m$	$3m$	$4m$

   The value of m is _____ and $E(X) =$ _____ .

2. If $f(x) = \dfrac{1}{2}$ defines density function of a discrete random variable X, then the number of values that X can take is _____ .

3. If the events A and B are mutually exclusive then $A \cap B =$ _____ .

4. 2-digit numbers are formed using *1,2,3,4,5* without repetition. The probability of the first digit selected is an odd is _____ .

5. From a city population, the probability of selecting a smoker is 3/5, a male smoker is 2/5. If a person randomly selected is a smoker, The probability of a male is _____

6. If X_1 and X_2 are independent random variables where X_1 has mean 5, variance 7 and X_2 has mean -4, variance 7, then $E[2X_1 - 3X_2 - 4] =$ _____ and $Var[2X_1 - 3X_2 - 4] =$ _____ .

7. Three news papers *A, B and C* are published in a city. It is estimated from a survey that of adult population: *20%* read *A*, *16%* read *B*, *14%* read *C*, *8%* read both *A and B*, *5%* read both *A and C*, *4%* read both *B and C*, *2%* read all three. The percentage of adult population read at least one of the papers is _____ .

B

QUIZ – I(CB)

COURSE: Probability and Statistics   **COURSE NO.:** AAOC C111

Max. Marks: 24   Weightage: 8%   Date: 11-10-2010   Time: 20 mins.

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Fill in the blanks with correct answers:

1. The density function $f(x)$ of a discrete random variable X is defined in the following table:

x	1	2	3	4
$f(x)$	$8m$	$4m$	$3m$	$5m$

The value of m is _____ and $E(X) =$ _____ .

2. If $f(x) = \dfrac{1}{3}$ defines density function of a discrete random variable X, then the number of values that X can take is _____ .

3. If the events A and B are mutually exclusive then $P(A \cap B) =$ _____ .

4. 2-digit numbers are formed using *1,2,3,4,6,8* without repetition. The probability of the first digit selected is an even is _____ .

5. From a city population, the probability of selecting a smoker is 4/5, a male smoker is *2/5*. If a person randomly selected is a smoker, the probability of a male is _____ .

6. If X_1 and X_2 are independent random variables where X_1 has mean 5, variance 5 and X_2 has mean -4, variance 7, then $E[2X_1 - 4X_2 - 4] =$ _____ and $Var[2X_1 - 4X_2 - 4] =$ _____ .

7. Three news papers *A, B and C* are published in a city. It is estimated from a survey that of adult population: *20%* read *A*, *16%* read *B*, *12%* read *C*, *8%* read both *A and B*, *5%* read both *A and C*, *4%* read both *B and C*, *4%* read all three. The percentage of adult population read at least one of the papers is _____ .

8. If X has Poisson distribution with $P(X = 0) = 0.04978$, the parameter k of the distribution is approximately _____ (*write the value in nearest integer*).

<u>↓ SPACE FOR ROUGH WORKS ↓</u>

BITS, PILANI – DUBAI
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QUIZ – I(CB)

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Fill in the blanks with correct answers:

1. The density function $f(x)$ of a discrete random variable X is defined in the following table:

x	1	2	3	4
$f(x)$	m	$2m$	$3m$	$2m$

    The value of m is _____ and $E(X) =$ _____ .

2. If $f(x) = \dfrac{1}{4}$ defines density function of a discrete random variable X, then the number of

    values that X can take is _____ .

3. If the events A and B are mutually exclusive then $P(A \cap B) =$ _____ .

4. 2-digit numbers are formed using *1,2,3,4,5* without repetition. The probability of the first digit selected is an even is _____ .

5. From a city population, the probability of selecting a smoker is 1/5, a male   smoker is *1/10*. If a person randomly selected is a smoker, the probability of a male is _____ .

6. If X_1 and X_2 are independent random variables where X_1 has mean 5, variance 5 and X_2 has mean -4, variance 7, then $E[2X_1 - 3X_2 - 4] =$ _____ and $Var[2X_1 - 3X_2 - 4] =$ _____ .

7. Three news papers *A, B and C* are published in a city. It is estimated from a survey that of adult population: *20%* read *A, 16%* read *B, 14%* read *C, 8%* read both *A and B, 5%* read both *A and C, 4%* read both *B and C, 4%* read all three. The percentage of adult population read at least one of the papers is _____ .

8. If X has Poisson distribution with $P(X = 0) = 0.01831$, the parameter k of the distribution is approximately _____ (*write the value in nearest integer*).