BITS, PILANI - DUBAI CAMPUS
INTERNATIONAL ACADEMIC CITY, DUBAI

FIRST YEAR – I SEMESTER (2010-11)

MATHEMATICS-I (MATH C191)

COMPREHENSIVE EXAMINATION (CLOSED BOOK)

Date: 26.12.2010	**Max. Marks: 120**
Time: 3 hours	**Weightage: 40 %**

Answer Part A, Part B and Part C in separate Answer Books.
Answer all the questions.

PART A

1. Find the area that lies between both the curves $r = \sqrt{3}\cos\theta$ and $r = \sqrt{3}\sin\theta$ (8)

2. Find the length of the polar curve $r = e^{\left(\frac{\theta}{2}\right)}$, $-\pi \le \theta \le \pi$ (6)

3. If $f(x) = x^2$, $x_0 = 3$, $\varepsilon = 0.1$, $L = 9$ find a number $\delta > 0$ such that for all x: $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$. Also find an open interval about x_0 such that the inequality holds? (5)

4. Find $\hat{T}$, $\hat{N}$, κ for the space curve $\vec{r}(t) = 2t\ \hat{i} + t^2\ \hat{j} + \left(\frac{t^3}{3}\right)\hat{k}$ at $t = 1$. (8)

5. If the position of a particle in space at time t is given by $\vec{r}(t) = (t^2 + 1)\ \hat{i} + t\ \hat{j}$ find the particles velocity and acceleration at $t = 1$ and sketch them as vectors on the path traced by the curve. (6)

6. Find the directional derivative of $f(x, y, z) = \cos(xy) + e^{yz} + \ln(zx)$ at $P_0\left(1, 0, \frac{1}{2}\right)$ in the direction of $\vec{A} = \hat{i} + 2\hat{j} + 2\hat{k}$. (7)

PART B

1. Find the points on the surface $z^2 = xy + 4$ closest to the origin. (8)

2. Evaluate by reversing the order of integration: $\int_0^1 \int_y^1 x^2\ e^{xy}\ dx\ dy$ (8)

(P.T.O.)

3. Evaluate $\int_0^{\sqrt{2}} \int_0^{3y} \int_{x^2+3y^2}^{8-x^2-y^2} dz\, dx\, dy$ (8)

4. Evaluate $\int_C (x - y + z - 2)\, ds$ where C is the straight line segment

$x = t, \quad y = 1 - t, \quad z = 1$ from $(0, 1, 1)$ to $(1, 0, 1)$ (8)

5. Change the Cartesian integral to polar integral and evaluate the polar integral:

$$\int_0^{2} \int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)}\, dy\, dx$$ (8)

PART C

1. Use Green's theorem to evaluate the integral $\int_C y^2\, dx + x^2\, dy$ where C: The triangle bounded by $x = 0, x + y = 1, y = 0$. (8)

2. Use Stoke's theorem to evaluate $\int_C \vec{F} \bullet dr$ if $\vec{F} = xz\,\hat{i} + xy\,\hat{j} + 3xz\,\hat{k}$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant. (8)

3. Show that $\vec{F} = \left(3x^2 y + z\sin(xz)\right)\hat{i} + \left(x^3 + z\right)\hat{j} + \left(y + x\sin(xz)\right)\hat{k}$ is a conservative field. Also find the scalar potential and the work done by $\vec{F}$ along any smooth curve C joining the points $(1, 1, 2)$ to $(3, 5, 0)$. (8)

4. Use the Divergence theorem to calculate $\iint_S F \bullet ds$, where $\vec{F} = 2x\,\hat{i} + 3y\,\hat{j} + 4z\,\hat{k}$ and S is the sphere given $x^2 + y^2 + z^2 = a^2$. (8)

5. Determine if the following series is convergent or divergent:

a) $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^n}$ b) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2\ (n+1)^2}$ (4+4)

<u>All the Best!</u>

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TEST – II (Open Book)

Date: 21.11.2010 | **Max. Marks: 60**
Time: 50 minutes | **Weightage: 20%**

Answer all the questions

1. Find the unit tangent vector and arc length parameter of the curve
$$\vec{r}(t) = (3\cos t)\hat{i} + (3\sin t)\hat{j} + 2\,(t)^{(3/2)}\,\hat{k}, \quad 0 \le t \le 3.$$ (7)

2. Find $\hat{T},\ \hat{N}\ \&\ \kappa$ for the space curve $\vec{r}(t) = \left(\cos^3 t\right)\hat{i} + \left(\sin^3 t\right)\hat{j} \quad ; 0 < t < \dfrac{\pi}{2}$ (7)

3. If $z = \dfrac{x^3 y^3}{x^3 + y^3}$, find $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y}$ in terms of z. (7)

4. If $u = x^2 - y^2 + \sin(yz)$ where $y = e^x$ & $z = \log x$, find $\dfrac{du}{dx}$ by chain rule and verify by direct method. (7)

5. If $f(x,y) = \cos^2 x - \sin^2 y$, find the derivative of f at the point $\left(\dfrac{\pi}{4}, \dfrac{\pi}{4}\right)$ in the direction from $P(1,1)$ to $Q(-1,2)$. (8)

6. Maximise $z = xy$ subject to the constraint $(x-2)(y-4) = 18$. (8)

7. Discuss the maxima and minima of $f(x,y) = x^3 y^2 (12 - 3x - 4y)$. (8)

8. Find the linearization of $f(x,y,z) = x^3 + y^3 - 3z^2 x + 4yz$ at the point $(1, -1, 3)$. (8)

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First Year – Semester I (2010-11)
MATHEMATICS-I (MATH C191)
TEST – I (Closed Book)

Date : 10.10.2010 **Max. Marks : 75**
Time: 50 minutes **Weightage : 25%**

Answer all the questions

1. Find the length of the curve given by the polar equation:
$$r = 8\sin^3\left(\frac{\theta}{3}\right) \quad , \quad 0 \le \theta \le \frac{\pi}{4}$$ (10)

2. Determine the area of the region shared by $r = 3 + 2\sin\theta$ & $r = 2$ (10)

3. If $e = \frac{1}{3}$ and $r\sin\theta = -6$ find the equation of the conic in polar form. Also sketch it by labeling the vertices, center, foci and directrix with the appropriate polar coordinates. (10)

4. Shade the region which satisfies the following inequalities:

(a) $\theta = \frac{3\pi}{4}, \quad 1 \le |r| \le 3$ (b) $2 \le r \le 2(1+\cos\theta)$ (8)

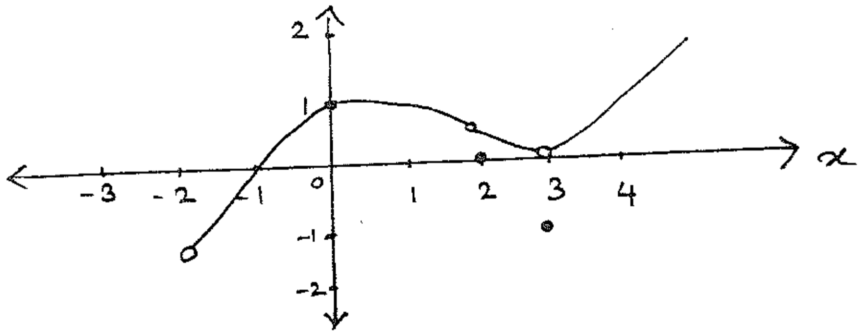
5. Find the velocity and acceleration at the given value of t and plot them on the path (cycloid): $\vec{r}(t) = (t - \sin t)\hat{i} + (1 - \cos t)\hat{j}, \quad t = \pi$ (9)

6. Solve the initial value problem for $\vec{r}(t)$ as a vector function of t:
$$\frac{d\vec{r}}{dt} = \frac{3}{2}(t+1)^{1/2}\,\hat{i} + e^{-t}\,\hat{j} + \frac{1}{t+1}\hat{k}$$
with the initial condition: $\vec{r}(0) = \hat{k}$ (9)

7. (a) If $f(x) = \sqrt{x-7}, \quad x_0 = 23, \quad \varepsilon = 1 \quad L = 4$ find a number $\delta > 0$ such that for all x: $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$. Also find an open interval about x_0 such that the inequality holds? (6)

(b) Evaluate the following: $$\lim_{h \to 0^-} \frac{\sqrt{6} - \sqrt{5h^2 + 11h + 6}}{h}$$ (4)

8. Given the graph of $f(x)$ shown below, answer the following questions:



a) Find $\lim_{x \to 0} f(x)$

b) Find $f(0)$ and determine whether $\lim_{x \to 0} f(x) = f(0)$

c) Discuss the continuity of $f(x)$ at $x = 2, 3$ in detail. (9)

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A

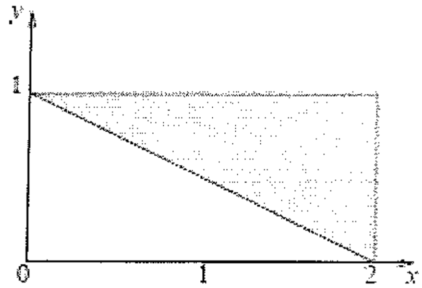
Quiz - 2

Time: 20 Minutes	**Max Marks: 21**	**Weightage: 7%**
Name:	**ID:**	**13.12.2010**

Answer all the question:

1. Below is the region in the plane. Fill in the limits on the double integrals and evaluate it over this region: (4)

$$\iint (x+y)\, dx\, dy$$



2. Sketch the regions of integration and reverse the order: $\int_0^1 \int_{\sqrt{y}}^{y^2} f(x,y)\, dx\, dy$. (4)

5. Determine whether or not $\vec{F}$ is a conservative vector field. If it is find the potential function: $\vec{F} = (2xy + z^2)\,\hat{i} + 2xy\,\hat{j} + (x^2 - 3z^2)\,\hat{k}$ (4)

BITS, Pilani - Dubai Campus
Dubai International Academic City, Dubai
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Mathematics I (MATH C191)

B

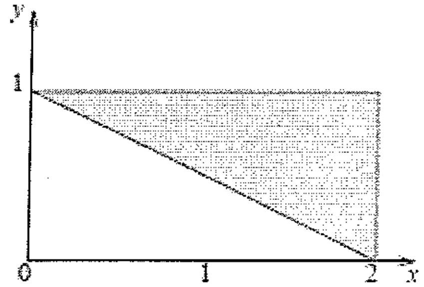
Quiz - 2

Time: 20 Minutes	**Max Marks: 21**	**Weightage: 7%**
Name:	**ID:**	**13.12.2010**

Answer all the question:

1. Below is the region in the plane. Fill in the limits on the double integrals and evaluate it over this region (4)

$$\iint (x+y)\, dy\, dx$$



2. Sketch the regions of integration and reverse the order: $\int_0^1 \int_{\sqrt{x}}^{x^2} f(x,y)\, dy\, dx$ (4)

5. Determine whether or not $\vec{F}$ is a conservative vector field. If it is find the potential function: $\vec{F} = (2xz - y^2)\,\hat{i} + xy\,\hat{j} + (x^2 + 3z^2)\,\hat{k}$ (4)

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Dubai International Academic City, Dubai
First year – Semester I 2010 – 2011
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A

Quiz - 1

Time: 20 Minutes **Max Marks: 24** **Weightage: 8%**

Name: **ID:** **1.11.2010**

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1. Without finding $\hat{T}$ & $\hat{N}$, write the acceleration of the motion $\vec{r} = t\cos t\,\hat{i} + t\sin t\,\hat{j} + t\,\hat{k}$ in the form $\vec{a} = a_T\,\hat{T} + a_N\,\hat{N}$ when $t = 0$. (5)

2. For the function $f(x, y) = e^{x+y}$ find the domain, range. Also find the equation of the level curve passing through the point (1, 2). (4)

3. By two path test, show that the function $f(x,y)=\frac{x^2}{x^2-y}$ is not continuous at the origin. (5)

4. Find $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$ for the function $f(x,y)=\log_y x$ (5)

5. Find $\frac{\partial u}{\partial z}$ if when $u=p-q+pr,\quad p=x+y+z,\quad q=x-y+z,\quad r=x+y-z$ at the point $(\sqrt{3},2,1)$. (5)

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Mathematics I (MATH C191)

B

Quiz - 1

Time: 20 Minutes | **Max Marks: 24** | **Weightage: 8%**

Name: | **ID:** | **1.11.2010**

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1. Without finding $\hat{T}$ & $\hat{N}$, write the acceleration of the motion $\vec{r} = t\cos t\ \hat{i} + t\sin t\ \hat{j} + t^2\ \hat{k}$ in the form $\vec{a} = a_T\ \hat{T} + a_N\ \hat{N}$ when $t = 0$. (5)

2. For the functions $f(x, y) = \ln(x^2 + y^2)$ find the domain, range. Also find the equation of the level curve passing through the point (1, 2). (4)

3. By two path test show that the function $f(x, y) = \frac{y}{x^2 - y}$ is not continuous at the origin. (5)

4. Find $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$ for the function $f(x, y) = \log_x y$ (5)

5. Find $\frac{\partial u}{\partial y}$ if $u = p - q + pr, \quad p = x + y + z, \quad q = x - y + z, \quad r = x + y - z$ at the point $(\sqrt{3}, 2, 1)$ (5)